Ed Process Int J | www.edupij.com
ISSN 2147-0901 | e-ISSN 2564-8020
2023 | ÜNİVERSİTEPARK

EDUCATIONAL PROCESS
INTERNATIONAL JOURNAL

ARTICLE HISTORY
Received *January 23, 2023*
Accepted *May 01, 2023*
Published Online *May 22, 2023*

CORRESPONDENCE
Konstantinos Lavidas
Lavidas@upatras.gr
Department of Educational Sciences and Early Childhood Education, University of Patras, Greece.

AUTHOR DETAILS
Additional information about the author is available at the end of the article.

How to cite: *Eleftheriadi, A., Lavidas, K., Koustourakis, G., & Papadakis, S. (2023). Misconceptions about numbers and operations. A case study of preschoolers. Educational Process: International Journal, 12(2): 59-75.*

RESEARCH ARTICLE

Misconceptions about Numbers and Operations–A Case Study of Preschoolers

Artemis Eleftheriadi · Konstantinos Lavidas · Gerasimos Koustourakis · Stamatis Papadakis

ABSTRACT

Background/purpose – Investigation into the misconceptions of preschool students in mathematics and their differences between the ages of 4-5 and 5-6 years old helps form appropriate developmental mathematics teaching programs. However, although several studies have been conducted examining preschoolers' previous knowledge and misconceptions about mathematics, no corresponding research has been found in Greece. This study aims to investigate preschoolers' misconceptions about numbers and operations and to reveal differences between preschoolers aged 4-5 and 5-6 years old.

Materials/methods – Data were collected through semi-structured interviews and analyzed according to content analysis methodology.

Results – The study's results showed that 5-6-year-old preschoolers perform better than those aged 4-5 years old. Most misconceptions of the latter group appeared to be related to reverse counting, identifying arithmetic symbols and their matching quantities, adding and removing numbers without using auxiliary objects and multiplication. On the contrary, some preschoolers aged 5-6 years old needed help adding or subtracting two-digit numbers without the use of auxiliary objects. There were also a few cases where cardinality, division, and multiplication were observed.

Conclusion – The misconceptions identified in the two groups of students regarding numbers and operations and their distinct needs that emerged through the research will allow teachers to offer differentiated instruction and personalize teaching support.

Keywords – math, preschool education, numbers; operations, knowledge skills, misconceptions

To link to this article– https://dx.doi.org/10.22521/edupij.2023.122.4

1. INTRODUCTION

Early mathematical education has been recognized as significant since it foreshadows future educational success (Eleftheriadi et al., 2021; Lavidas et al., 2023; McGuire et al., 2011; Sarama & Clements, 2009). Studies have shown that early mathematics ability is a more influential factor in later academic success than early reading ability and socioemotional skills (Alkhadim et al., 2021; Claessens et al., 2009; Engel et al., 2013; Watts et al., 2014). This assumption is supported by the fact that in the early years of life, children are able to master fundamental concepts and skills related to mathematics (Charlesworth & Leali, 2012; McDonald & Murphy, 2019). However, preschool students enter the classroom possessing a rich set of stimuli and knowledge that, according to researchers, appears to influence their mathematical abilities (Lee & Md-Yunus, 2016; McDonald & Murphy, 2019).

A large number of researchers have been involved in identifying the pre-existing knowledge of preschoolers, mainly based on researching numbers and operations (Aubrey, 1993; Clements & Sarama, 2007; Copley, 2009; Lee, 2014; Lee & Md-Yunus, 2016; Muthukrishnan et al., 2019). Some have focused on specific skills such as counting, cardinals, matching quantities with numbers or verbal expressions and even comparing quantities (Hurst et al., 2016; Lee & Md-Yunus, 2016; McDonald et al., 2021; Nguyen et al., 2017). Other studies have opted to focus on controlling the abilities of preschool students regarding the operations of addition, subtraction, and multiplication as a process of continuous addition and division in the sense of sharing (Aubrey, 1993; Muthukrishnan et al., 2019). However, although several studies have been carried out on preschoolers' previous knowledge and misconceptions about mathematics (e.g., in the United Kingdom, United States, Canada, and Malaysia), no corresponding research has been found to have been undertaken in Greece. In addition, most of the research studies have investigated mathematical knowledge and the misconceptions of students without providing evidence on how they are distributed across individual age groups.

The objective of the current study is twofold. First, we aim to investigate Greek preschoolers' knowledge and misconceptions about the two primary standards of mathematics, numbers and operations (Ohio Department of Education, 2004). Second, we aim to identify the differences between two preschool age groups, those of 4-5-year-olds and 5-6-year-olds, as this may provide a clearer image of which mathematical skills preschoolers can acquire and at what age (Litkowski et al., 2020). In Greek public kindergartens, preschool children of 4-5 years old are defined as those who have reached the age of 4 years old as of December 31 of their enrolment year in kindergarten. The need to compare two age groups has emerged from the fact that public kindergarten classrooms in Greece, as well as in many other countries, are mixed, including students from both age groups. Therefore, a need exists to further investigate the mathematical knowledge (of numbers and operations) and misconceptions that are common problems among preschool students (Muthukrishnan et al., 2019). Highlighting these differences is considered of interest since this may lead to better organization in the teaching of mathematics to mixed age group classes in Greek kindergartens. Furthermore, the findings of this study could provide valuable insight regarding discussions about teaching mathematics at the preschool level both in Greece and worldwide.

2. LITERATURE REVIEW

Theoretical Framework

Math has a high status in the Greek curriculum since it is considered an essential subject (Koustourakis, 2007; Koustourakis & Zacharos, 2011; Lavidas et al., 2022; Shiakalli et al., 2017; Zacharos et al., 2014). The Greek kindergarten curriculum for teaching mathematics is based on guidance from the National Council of Teachers of Mathematics (Lavidas et al., 2022; National Council of Teachers of Mathematics, 2001). According to the Greek curriculum for kindergarten ([Greek] Ministry of Education, 2014) on natural numbers, students are required to recite, read, and write numbers up to 10 (as numerical symbols). In addition, they must be able to recognize numerical quantities using strategies such as immediate recognition. Specifically, in terms of counting quantities, students are required to count both natural and pictured objects, as well as other forms of symbolic representations up to the value of 10, to compare and order quantities and numbers, and to represent them on the number line. At the same time, when arranging quantities and numbers, preschoolers should be familiar with analyzing and synthesizing quantities by 10. Regarding operations, preschoolers are required to approach addition and subtraction and to explore the stable relationship of numbers up to 10. In the next stage, students should be able to group objects in pairs, triads, and groups of five, and vice versa, and to distribute objects into pairs, triads etc. in order to become familiar with multiplication and division (Zacharos et al., 2014).

According to the Greek contemporary kindergarten curriculum, child-centered pedagogical practices must be applied in the kindergarten classroom (Dafermou et al., 2006; Koustourakis, 2013, 2014, 2018). Thus, preschoolers must be encouraged to work individually or in pairs or groups so as to reveal their personality, cognitive abilities, and cognitive development (Koustourakis, 2013, 2018). In this way, as Bernstein (2000) stated, preschoolers are transformed into “texts,” and kindergarten teachers must use one of the available developmental psychology theories to succeed in reading and interpreting their students’ behavior. The application of Piaget’s theory is proposed to kindergarten teachers by the Greek state as the most suitable developmental theory to understand their students’ cognitive evolution during their daily pedagogical interaction with them (Dafermou et al., 2006; Koustourakis, 2013, 2018).

Regarding the cognitive development of preschoolers, Piaget explained that during their given age, children develop their symbolic ability through the use of images and words as symbols in order to understand the natural world. Children aged from 2 to 4 years old may be able to create an image but still not think logically (Babark et al., 2019; Piaget et al., 1995). Therefore, these are said to be the stages of preparation for logical thinking. Piaget also examined the rational development of children, stating that logical mathematical knowledge is derived from a child’s interaction with objects and the resulting reasoning (Piaget et al., 1995). A child’s transition through the different stages, and their evolution throughout each stage, may reveal differences in cognitive development and, therefore, such differences could be detected between two different age groups of preschoolers, i.e., those aged 4-5 years old and those aged 5-6 years old, as has been investigated by previous research. In addition, the presence of kindergarteners’ pre-existing knowledge in combination with their school knowledge and any lack of connection between these two can often lead to teaching problems (Sarama & Clements, 2009). Therefore, in order for teachers to teach young students math in a developmentally appropriate way, it is necessary for them to understand and evaluate what their students already know and what components of a particular mathematical skill they are capable of acquiring so that they can organize, develop, and apply appropriate methods of teaching mathematics (Copley, 2009; Eleftheriadi et al., 2021; Lee, 2014; Lee & Md-Yunus, 2016; Litkowski et al., 2020). Apart from students’ pre-existing knowledge and cognitive developmental level, their age distribution in classes should also be taken into account.

It has been observed that in countries outside Greece, there is a variable age distribution of preschoolers in kindergartens; the exception being in Poland, where class organization is strictly based on student age (European Education and Culture Executive Agency, n.d.). More specifically, countries such as Austria, Denmark, and Lithuania follow a method of organizing classes where the students' age varies from 3 to 6 years old. In the example of Hungary, children over 3 years old attend kindergarten, and their classrooms can be either mixed or age-based according to a decision reached by of the head of the school, in conjunction with the parents and teachers. In Greece, kindergarten classes are of mixed ages, meaning that children can be aged from 4 to 6 years old. Both private and public kindergartens must follow the same curriculum and are all under the jurisdiction of the Greek Ministry of Education. In private Greek kindergartens, how the students are distributed into classes is left at the discretion of the kindergarten owner. For example, in the private school where the current research was conducted, students were distributed to classes based on age.

Previous works

Mathematics forms an essential element of curricula worldwide. As such, providing preschool students with mental obstacles in mathematics learning is of significant interest (Sarama & Clements, 2009). Many researchers have shown concern to discover the cognitive obstacles of preschoolers with the ultimate goal of addressing these obstacles and better developing students' cognitive skills through their teaching (Aubrey, 1993; Claessens et al., 2009; Copley, 2009; Lee & Md-Yunus, 2016; Muthukrishnan et al., 2019). Students' misconceptions can be organized according to the five standards of mathematics (Zacharos et al., 2014). These standards are a) numbers and operations, b) algebra, c) geometry, d) measurement, and e) data analysis/probabilities.

Numbers and operations are of the greatest concern to researchers, since numbers and their understanding are key to doing any form of mathematics (Lavidas et al., 2022; Ohio Department of Education, 2004). Children should be able to understand how numbers are used in daily life. Through time and experimentation, children will evolve in their mathematical thinking, externalize, and discover new mathematical concepts through numbers such as counting, comparison, quantities, or even operations (Ohio Department of Education, 2004). When it comes to numbers, researchers highlight misconceptions when, for example, students are asked to identify random numbers from zero to 13, either in writing numbers from one to 10 or in finding the previous or next number (Aubrey, 1993). However, counting is a skill where preschoolers often need to correct mistakes, as they often stop counting when they reach numbers ending at nine or zero or adopt the wrong arithmetic order (Aubrey, 1993; McGuire et al., 2011; Nguyen et al., 2017).

This standard of mathematics includes object counting, a skill known for various student misconceptions (Alkhadim et al., 2021; Aubrey, 1993; McGuire et al., 2011). Students tend to respond incorrectly when asked for the number of many objects presented to them (Clements & Sarama, 2007; McGuire et al., 2011). In similar activities, preschoolers may omit numbers or objects (Lee & Md-Yunus, 2016; McGuire et al., 2011; Nguyen et al., 2017), while sometimes they may repeat numbers during counting or count objects more than once (McGuire et al., 2011; Nguyen et al., 2017). An equally important skill linked to counting and where misconceptions occur is the ability of students to count by indicating objects and consequently to count rationally following cardinality (McDonald et al., 2021; McGuire et al., 2011; Nguyen et al., 2017). In the next stage, more misconceptions were recorded in activities where students are required to match many objects with the correct number or vice versa, as well as when they are required to identify a number after someone else provides a description (Nguyen et al., 2017). In addition, in research by Hurst et al. (2016), it was observed that students performed better in matching quantity with words, rather than the opposite; words with quantity. However, when matching quantity–number and word–number, children showed similar effects to the reverse matches. In the analysis of their study's results, Lee and Md-Yunus (2016) found that besides counting, number misconceptions were also found to be related to the ability to compare, where students would confuse the concepts of "more" and "less."

Kindergarten children can face difficulties in understanding the abstract nature of mathematics, resulting in the appearance of errors in simple numerical calculations. Muthukrishnan et al. (2019) tried to explore the common mistakes that preschool children make when adding up and the underlying misconceptions among children who make factual, conceptual, or procedural errors when attempting addition. They found that factual misconceptions were based on a lack of knowledge or misinformation, such as a lack of vocabulary or problems with digit recognition. Conceptual errors were revealed to be mostly due to an insufficient understanding of the concepts which are critical pillars for problem solving and a need to understand the relationship between the fundamental concepts of problem solving. Procedural errors can make it difficult to understand the steps in solving a problem, and the majority of mistakes made during addition were found to be conceptual errors. The findings from their interviews showed that children needed to become more familiar with the terminologies used in the operation of adding. Specifically, misconceptions were spotted in concepts such as position values and the grouping of digits according to their position (Muthukrishnan et al., 2019). In similar research, students made mistakes in the addition and subtraction of numbers because they needed more data during the count (Aubrey, 1993). Related problems were also highlighted in research published by Clements and Sarama (2007), where a large percentage of students did not respond to a problem where they were required to add a few more objects to an initial amount by using objects given to them. Similar results were found when students were called upon to solve problems by adding two numbers smaller than five with the option to use objects close to them; but where the researchers gave no suggestions. Corresponding obstacles were observed when students were asked to solve division and multiplication problems (Aubrey, 1993).

However, a study by Litkowski et al. (2020) developed more detailed trajectories of children's numeracy abilities across the preschool age groups in order to enrich existing developmental research on children's early mathematics knowledge. In their study, the researchers investigated age in 6-month brackets in order to reveal which specific components of mathematics skills are acquired by children across eight different early numeracy skills. Their results provided a clearer picture of at what age children can acquire specific numeracy skills. The findings indicated that nearly half of children aged 3 years old were capable of counting up to 10, whereas children aged 5 years old were capable of counting up to 20. Moreover, 71.9% of 4-year-old children could complete one-to-one correspondence of six objects, whilst 71.1% of 5-year-olds could do so for 11 objects. Through investigating children's calculation abilities it was indicated that their performance on addition problems was lower at the age of 5.5 years old than for any of the other numeracy tasks (Litkowski et al., 2020).

In the majority of previous studies, we observed that the researchers focused on the mathematics skills of preschoolers. Some studies involved counting and omission, repetition of numbers, or even omitting objects during counting (Lee & Md-Yunus, 2016; McGuire et al., 2011; Nguyen et al., 2017), whilst others examined the matching of quantities with numbers (Hurst et al., 2016). Another group of researchers focused on misconceptions related to mathematical operations. They examined the misconceptions of preschool students when they come into contact with addition, subtraction, multiplication, or division (Aubrey, 1993; Clements & Sarama, 2007; Muthukrishnan et al., 2019). The researchers opted to conduct semi-structured interviews with their sample in order to investigate their math knowledge and misconceptions; however, their findings were not analyzed according to the preschoolers' age, such as grouping them as 4-5-year-olds and 5-6-year-olds.

In the current study, we aimed to answer the following research question: "Are there any differences between preschoolers aged 4-5 years old and 5-6 years old in their understanding and misconceptions of mathematical knowledge?"

3. METHODOLOGY

This study investigated the mathematical knowledge and misconceptions of preschool students regarding numbers and operations between two age groups; 4-5-year-old and 5-6-year-old children. The comparative study of these two groups allows the emergence of shared and different misconceptions. The research was conducted in a private kindergarten in Patras (Greece) during the last months of the 2021-2022 school year. In total, 15 students participated in the research, with nine aged 5-6 years old, and six aged 4-5 years old. The specific school was chosen since its class organization met the research needs as mixed aged groups were not employed as is customary in Greek public kindergartens. On the contrary, the students were separated into a class for children aged 4-5 years old and another for 5-6-year-olds. However, the curriculum followed in both classes of the private kindergarten was the same as in public Greek kindergartens.

Research tool and research process

The semi-structured interview was the research method employed for data collection in the study. The interview protocol was based on the research goal and consisted of 30 questions divided into two groups (see Appendix). This protocol was created by taking into account previous research related to the exploration and recording of preschool students' misconceptions (Aubrey, 1993; Charlesworth & Leali, 2012; Lee & Md-Yunus, 2016), as well as the learning objectives of the Greek preschool mathematics curriculum. Additionally, the way that young children think was also taken into account, as it is relatively specific and takes note of their abstract ideas concerning mathematical concepts (Piaget, 1952). For example, they can generally compare two sets of objects while knowing that addition will always give us more, but that subtraction will give us less. Therefore, in order to conduct a valid and reliable recording of the students' knowledge, they were asked to explain their answers since it is essential to examine the reasoning behind the skills demonstrated by children (Lee & Md-Yunus, 2016).

The first group of questions aimed to investigate the students' skills with numbers and in counting. By contrast, the second category consisted of games and questions that aimed to examine the participants' abilities in addition, subtraction, multiplication, and division. Prior to the research, the interview protocol was first piloted with two students in order to determine if any problematic points existed and to make appropriate revisions. The two students who took part in the pilot application were subsequently excluded from the final sample.

Prior to commencing the investigation, approval was received from the participant school's head teacher and the participant children's parents and also from the Research Ethics Board (REB) designated by the Department of Educational Science and Early Childhood Education of the University of Patras (protocol code 4553, approved January 23, 2023). In preparation for the interviews, we asked the teachers to inform their students about their involvement in the research and gave them some information about the games they would play during their interviews. The semi-structured interviews were then conducted in the children's classrooms after they had completed the compulsory student schedule. The interviews were individually carried out, with each lasting about 15 to 20 minutes. During the interviews, the arguments presented by the students when asked to explain their ways of thinking were recorded so as to increase the credibility of the measurement (Bryman, 2016; Katsidima et al., 2023; Pogiatzi et al., 2022;).

Data analysis

Content analysis was employed to analyze the qualitative research findings. Specifically, the collected data were analyzed according to the coding schedule method (Bryman, 2016) in order to categorize the students' responses accordingly. The researcher developed the coding format based on a schedule created as the students answered the interview questions. The form was based on the students' individual mathematical skills, including numbers and operations, which form the central objectives of the research. The coding schedule was designed to facilitate the analysis and discussion of the interview data. Consequently, the research data were categorized as either numbers or operations.

4. RESULTS

This section presents the results of the interview analysis which compared the two groups of students.

Numbers

Table 1 shows the readiness of the participant preschoolers and their misconceptions concerning numbers. Among the answers received from both sample groups, it was observed that all of the students were able to correctly count the objects during the first activity (see Appendix, Q. 1.1.1.). Nevertheless, upon checking the ability of their cardinality, it was revealed that two students from each group found it difficult to answer the question "So, how many are there all together?" having counted the objects again from the beginning. In addition, one student from the group of 4-5-year-old preschoolers appeared unable to justify their answer. Specifically, when asked how they'd known that the objects were as many as stated, the response was:

"[...] Hmm… I don't know."

The rest of the students came up with the same number, with one explaining that "I counted them before." In the next counting activity, students from both groups, except for one, counted appropriately by showing the objects. However, when the process was reversed and they were asked to count the objects, only the 5-6-year-olds achieved 100% success. From the group of 4-5-year-olds, four out of the six children were unable to succeed, either because they could not remember a number (i.e., three) or needed clarification with the reverse order of numbers.

In both matching numbers with quantities and the reverse, all of the students in the 5-6-year-olds group accomplished both activities (see Appendix, Q. 1.2.1). However, in the group of 4-5-year-olds, only 50% did both tasks correctly. In matching numbers with quantities, three out of the six preschoolers in this group needed clarification or failed to remember the numbers six, seven, and nine. Specifically, one of the students showed six but said nine and showed nine but said seven.

- Student: "[...] How much should I put here?"

- Researcher: "[...] What number do you see above the basket?" (the number six was in the basket)

- Student: "[...] Nine."

- Researcher: "[...] And what is this number?" (shows the number nine)

- Student: "[...] It's six."

In the reverse procedure (see Appendix, Q. 1.2.4), three of the six students from the 4-5-year-old group needed help identifying the numbers. In a similar activity, including recognizing the number 10, all the students of both groups responded correctly. Similarly, in another, where the students were asked to fill a basket with Lego bricks, they also responded to the researcher's prompts.

In the next activity (see Appendix, Q. 1.1.1-1.1.3), the students were asked to compare each tower separately (red, blue, and yellow) to their own (green) to find out which one was the biggest, and why. In counting and comparing quantities, all of the children counted and compared them appropriately. In general, both groups observed a spontaneous, intuitive comparison based on the height of the towers. The researcher insisted and urged the students to think of a safer way of comparing, thus resulting in some counting in order to answer the question, hence there were several variations in the explanations provided. In the group of 4-5-years-olds, one of the six students found it challenging to explain the result of two out of three comparisons they made and thus did not give a clear enough answer. In addition, three out of the six counted the objects to answer the comparison question, whilst four out of the six compared only the height of the brick tower or combined it with counting. Similarly, four out of the nine students in the 5-6-year-olds group compared only the height of the brick tower or combined with counting, while five of them counted the bricks to respond appropriately.

When comparing two given quantities by checking abstract ability (see Appendix, Q. 2.2.1.), it was observed that most of the students had acquired this skill with a few exceptions. Specifically, the researcher initially distributed an equal number (four) of bricks of the same color (red) to both themself and the student. However, when the students were given three more bricks that were colored blue, all of the children in the 5-6-year-old group responded correctly. Most explained that they had more bricks than the researcher since they also had different colored bricks, while one student counted the bricks to arrive at their answer.

- Researcher: "[...] Who has the most bricks, you or me?"
- Student: "[...] I do"
- Researcher: "[...] Why? How did you figure that out?"
- Student: "[...] Because I have blue bricks, and you do not."

In the group of 4-5-year-olds, five out of the six students responded correctly, while one needed to pay attention to the color before responding correctly.

- Researcher: "[…] Who has the most bricks, you or me?"
- Student: "[...] We have the same."
- Researcher: "[...] Why? How did you figure that out?"
- Student: "[...] (only counts the red bricks) I counted them... It is the same."

Of the 4-5-year-old preschoolers, 50% explained their answer by stating that they had the most bricks or bricks of different colors, while the remaining 50% had to count the bricks in order to provide their answer.

Table 1. Results for numbers and their relative misconceptions

Skills – Misconceptions	Preschoolers (4-5-year-olds) *n* = 6	Preschoolers (5-6-year-olds) *n* = 9
<u>**Count**</u>		
Objective: Count from 1 to 10	6	9

Skills – Misconceptions	Preschoolers (4-5-year-olds) *n* = 6	Preschoolers (5-6-year-olds) *n* = 9
Cardinality		
Objective: Understanding the rule	4	7
Misconception: Count again to find all objects	2	2
Reverse counting		
Objective: Reverse count from 10 to 1	2	9
Misconception: Omit numbers, confusion with the reverse order of numbers	4	0
Number recognition		
Objective: Identification of symbolic numbers: Matching numbers with quantity	3	9
Misconception: Confusion of symbolic representations of numbers, e.g., confused 6 with 9, or 9 with 7	3	0
Objective: Identification of symbolic number representation: Match quantity with number	3	9
Misconception: Confusion of symbolic representations of numbers	3	0
Comparison of quantities		
Objective: Compare two quantities	6	9
Misconception: Difficulty finding a non-intuitive way of comparing quantities	2	4
More – Less		
Objective: Abstraction principle	4	9
Misconception: Confusion in comparison (failed to ignore color property)	2	0

Operations

In order to examine the students' misconceptions of math, we integrated them into a scenario where they were in an empty city and houses were gradually being built. Bricks were used for the houses, with one brick representing one house. After every process combined with a narrative, a question of related cognitive content was posed and then one which was removed from the context of the story. Regarding the one-digit addition activities (see Appendix, Q. 3.1) using objects (see Table 2), it was observed that 100% of students from both age groups responded correctly, each counting the objects one by one. During the addition of single digits without the help of objects (see Appendix, Q. 3.1.1), only the 5-6-

year-old preschoolers managed to correctly respond to everything by counting using their fingers to achieve the correct result. In the preschool group of 4-5-year-olds, two of the six students responded correctly by counting with their fingers, while two of the six gave the wrong answer, although they had also used their fingers. The rest of the sample from the same group failed to provide the researcher with an answer.

A similar pattern was observed in the next activity, where the students were tasked with adding two-digit numbers using objects (see Appendix, Q. 5.1), and the entire research sample reached the correct answer by utilizing the objects in front of them. However, significant variations appeared in another activity where certain objects were absent (see Appendix, Q. 5.1.1). In the group of preschoolers aged 5-6 years old, seven out of the nine students managed to provide a correct answer and explained that they put the biggest number in their head and the smallest on their fingers or put the biggest number in their head and counted accordingly using the smaller number.

- Researcher: “[...] Can you now think about; if I have 11 houses and put three more, how many houses will there be all together?”

-Student: “[...] 14.”

- Researcher: “[...] How did you come up with that number?”

- Student: “[...] I put 11 in my head, and then I added three more with my fingers... and I counted from the 11, three times.”

The remainder of the children gave the wrong answers, explaining that they thought about it but without giving further clarification. The students from the group of 4-5-year-olds were expected to have provided a clear answer; however, they expressed being unable to do the math problem.

In subtracting single digits using objects (see Appendix, Q. 4.1), all of the students from both sample groups responded correctly by counting the objects. In the subtraction of one-digit without the objects (see Appendix, Q. 4.1.1), from the group of 5-6-year-olds, seven out of the nine children responded correctly, with one of the nine having counted using their fingers, and the remainder explained that they just thought about it. In addition, one of the nine students gave the wrong answer, and explained having first added the two numbers and then removed the second from the result.

- Researcher: “[...] If I have four and remove two, how much is left?”

- Student: “[...] Four.”

- Researcher: “[...] How did you come up with that number?”

- Student: “[...] I put four and two together and then I took two.”

On the other hand, all of the preschoolers in the group of 4-5-year-olds responded correctly with the help of counting by using their fingers. Then, in subtracting two-digit numbers using objects (see Appendix, Q. 6.1), all of the students from both groups responded correctly, except for one. In the subtraction of two-digit numbers without objects (see Appendix, Q. 6.1.1), six out of the nine students in the group of 5-6-year-olds responded correctly, explaining that from 13, they subtracted two to arrive at their answer. However, two of the nine students gave the wrong answer, saying they performed subtraction in their heads, whilst only one was unable to provide an answer. In comparison, no preschoolers in the 4-5-year-olds group responded correctly, with five of the six explaining that they did not know, and with just one giving a wrong answer, explaining that they had thought about it.

During another activity, the mayor had decided that there should be five city neighborhoods, that the total number of houses would be 10, and that no one neighborhood could have more or fewer houses than any other. The students were asked to distribute houses to each neighborhood, executing the division operation (see Appendix, Q. 7.1). In the group of 5-6-year-old students, eight out of the nine children responded correctly. However, only six distributed the bricks on a one-by-one basis. The others needed to sufficiently explain their thinking, but only stated that they did it without providing any further explanation. Finally, one student made a quick attempt but then quit the task and gave up having placed as many bricks as the neighborhood number in the first attempt. In practice, the student put one brick in the first neighborhood, two in the second, and three in the third, but as soon as they realized that there were not enough bricks to continue with the same pattern, the student allocated two bricks in the fourth and fifth neighborhoods. From the team of preschoolers aged 4-5 years old, almost all of the students (five out of six) responded correctly, distributing the bricks one-by-one, except for one student who failed to do so.

The final activity involved multiplication (see Appendix, Q. 8.1). The city's mayor changed their mind and decided on three neighborhoods with three houses in each. This time round, the students had to think of a way to calculate the number of houses without actually counting them. The preschoolers' answers varied considerably. In the group of 5-6-year-olds, a different pattern of answers was observed, with seven of the nine students responded correctly, explaining that they added the same three numbers and thus found the result was nine (3 + 3 + 3 = 9). Another student responded correctly, but without justifying their answer, whilst the remaining student responded correctly and explained having quickly counted the objects based on their formation. In the preschool group of 4-5-year-olds, one of the six students responded correctly, but when asked to explain how they achieved it, the student stated that they had counted the objects. Three of the remaining students failed to give a clear answer for their reasoning, while another two gave the wrong answer with an insufficient explanation.

Table 2. Results about operations and relative misconceptions

Skills – misconceptions	Preschoolers (4-5-year-olds) *n* = 6	Preschoolers (5-6-year-olds) *n* = 9
<u>Addition</u>		
Addition of one-digit numbers with objects		
Objective: Addition using objects	6	9
Addition of one-digit numbers without objects		
Objective: Addition without the use of objects (counting by fingers)	2	9
Misconception: Difficulty in counting	4	0
Addition of two-digit numbers with objects		
Objective: Addition using objects	6	9
Addition of two-digit numbers without objects		
Objective: Addition without the use of objects (counting by fingers)	0	7
Misconception: Difficulty in counting	6	2

Skills – misconceptions	Preschoolers (4-5-year-olds) *n* = 6	Preschoolers (5-6-year-olds) *n* = 9
<u>Subtraction</u>		
Subtraction of one-digit numbers with objects		
Objective: Subtraction using objects	6	9
Subtraction of one-digit numbers without objects		
Objective: Subtraction without objects (counting by fingers)	6	8
Misconception: Difficulty in counting	0	1
Subtraction of two-digit numbers with objects		
Objective: Subtraction using objects	5	9
Misconception: Difficulty in counting	1	0
Subtraction of two-digit numbers without objects		
Objective: Subtraction without objects (counting by fingers)	1	6
Misconception: Difficulty in counting	5	3
<u>Division</u>		
Objective: Perception of division as a fair sharing process	5	8
Misconception: Difficulty in implementing the operation	1	1
<u>Multiplication</u>		
Objective: Perception of multiplication as a continuous addition	0	8
Misconception: Difficulty in implementing the operation (counted, unaware, incorrect answer)	6	1*

* no other way of calculating the whole other than by counting

5. DISCUSSION

The purpose of this study was to present the differences between the mathematical knowledge and misconceptions of preschoolers aged 4-5 years old and 5-6 years old regarding numbers and operations. In terms of the students' counting skills, it was apparent that enumeration is a skill that had been acquired by both groups of children. However, in the cardinality and the counting down from 10 to one, some cognitive barriers appeared mainly within the 4-5-year-olds group. The students' misconceptions aligned with related studies from the literature (Litkowski et al., 2020; McDonald et al., 2021; McGuire et al., 2011;

Nguyen et al., 2017), where students presented difficulty in counting with the use of objects and, therefore, were not counting rationally.

As for corresponding numbers with quantities and vice versa, and the identification of numbers when preceding verbal formulation, it was observed that only preschoolers aged 4-5 years old encountered difficulties in the matching process, having appeared to experience confusion regarding the numbers (Hurst et al., 2016). On the other hand, even though existing research has recorded students' misconceptions about identifying numbers after verbal formulation (Nguyen et al., 2017), in the current study, neither of the student groups faced any obstacles in this area.

In terms of counting and comparing quantities, all of the participant students were able to perform comparisons properly, and were able to detail their justification. At the same time, when checking their abstract capacity during their comparing of quantities, the results showed that cases of misconception were minimal and mainly occurred with students from the group of 4-5-year-old students. This was in contrast to previous research, which stated that preschool students can face difficulties and confuse the concepts of "more" and "less" (Lee & Md-Yunus, 2016). In the addition and subtraction operations, it was observed that the use of objects helped the students answer the various questions posed to them. They represented the numbers using the objects and thus managed to count and calculate them accordingly (Muthukrishnan et al., 2019). In addition, students from both groups excelled when adding or removing small (one-digit) numbers compared to the larger (two-digit) numbers (Litkowski et al., 2020; Muthukrishnan et al., 2019).

One essential finding of the current study is that some of the participant students need to become more familiar with the terms used in addition and subtraction operations (Muthukrishnan et al., 2019). This issue was most prominent in the 4-5-year-olds student group, with incidences of incorrect answers having been given by those in the other group (5-6-year-olds) being significantly less.

Finally, regarding the operation of division, it was observed that almost all of the students understood the concept of fair sharing with only a few having made a mistake (Aubrey, 1993). The operation of the division was a difficult concept for both student groups. With the multiplication, it was observed that most students in the 4-5-year-olds group failed to solve the problem or gave insufficient reasoning (Aubrey, 1993). By comparison, most students in the 5-6-year-olds group were able to explain multiplication as a continuous addition process through the utilization of objects.

6. CONCLUSION

The results of the current research emphasize that differences exist in the cognitive abilities of 4-5-year-olds compared to 5-6-year-olds regarding numbers and mathematical operations. These differences could be explained by taking into account the cognitive evolution of children at their various stages of cognitive development (Dafermou et al., 2006; Koustourakis, 2018; Litkowski et al., 2020; Piaget, 1952; Piaget et al., 1995; Zacharos et al., 2014). However, the distribution of children in the current study's participant school where the research was conducted should also be taken into account when comparing differences to any previously published research.

7. SUGGESTIONS

The differences observed from the current study's two age groups may be used by teachers to better organize their teaching in mixed-age classes that dominate preschool education worldwide. In the case of mixed-age classes, through utilizing the findings of the current study, teachers could provide more individualized teaching to students and make better use of the educational materials on offer. Emphasis on mastering mathematical skills should be given to preschoolers aged 4-5 years old and the teaching of complex mathematical skills should be considered as better delivered separately for preschoolers aged 4-5 and 5-6 years old. In this direction, teachers could organize separate educational games for these two distinctive age groups. Moreover, these findings could provide insight for various professional development programs for teachers that address the teaching of mathematics at the preschool level (Eleftheriadi et al., 2021; Lavidas et al., 2022, 2023; Shiakalli et al., 2017).

In closing, the authors recognize that the small sample size in the current study presents a difficulty in generalizing the results (Bernstein, 2000). Moreover, the gap identified regarding the mathematical knowledge and skills between these two age groups of preschool children should not be presented as absolute, since these age groups are obviously very close. Therefore, further research is required to investigate the math knowledge of preschoolers of various age groups in order to reaffirm the differences revealed in the current study. Additional research with a more representative sample would help confirm or refute these findings. Furthermore, additional research could investigate which teaching practices kindergarten teachers use to teach mathematics to preschoolers aged 4-5 and 5-6 years old, and thereby to identify any differences in their practices.

DECLARATIONS

Author Contributions: Conceptualization, K.L. and A.E.; Methodology, K.L., G.K., A.E., and S.P.; Writing of original draft, K.L. and A.E.. All authors have read and agreed to the published version of the manuscript.

Conflicts of Interest: The authors declare no conflict of interest.

Funding: This research received no external funding.

Ethical Approval: The study was conducted in accordance with the Declaration of Helsinki, and approved by the Institutional Review Board designated by the Department of Educational Science and Early Childhood Education of the University of Patras (protocol code 4553 and date of approval January 23, 2023).

Data Availability Statement: The data is unavailable due to privacy or ethical restrictions.

Acknowledgments: The authors would like to thank both the parents who permitted their children to participate in this research as well as the children themselves.

REFERENCES

Alkhadim, S. G., Cimmeta, D. A., Marx, W. R., Gutshaw, A. C., & Yaden, B. D. (2021). Validating the research-based early math assessment (REMA) among rural children in southwest United States. *Studies in Educational Evaluation*, *68*, Article 100944. https://doi.org/10.1016/j.stueduc.2020.100944

Aubrey, C. (1993). An investigation of the mathematical knowledge and competencies which young children bring into school. *British Educational Research Journal*, *19*(1), 27-41. https://doi.org/10.1080/0141192930190103

Babark, H. Z., Mohamedamin, P., & Kakamad, K. (2019). Piaget's cognitive developmental theory: critical review. *Education Quarterly Reviews*, *2*(3), 517-524. https://doi.org/10.31014/aior.1993.02.03.84

Bernstein, B. (2000). *Pedagogy, Symbolic control and identity. Theory, Research, Critique*. (revised ed.). Rowman & Littlefield.

Bryman, A. (2016). *Social research methods*. Oxford University Press.

Charlesworth, R., & Leali, A. S. (2012). Using problem solving to assess young children's mathematics knowledge. *Early Childhood Education Journal*, *39*, 373-382. https://doi.org/10.1007/s10643-011-0480-y

Claessens, A., Duncan, G., & Engel, M. (2009). Kindergarten skills and fifth-grade achievement: Evidence from the ECLS-K. *Economics of Education Review*, *28*(4), 415-427. https://doi.org/10.1016/j.econedurev.2008.09.003

Clements, H. D., & Sarama, J. (2007). Effects of a preschool mathematics curriculum: Summative research on the "Building Blocks" project. *Journal for Research in Mathematics Education*, *38*(2), 136-163. https://pubs.nctm.org/view/journals/jrme/38/2/article-p136.xml

Copley, J. V. (2009). *The young child and mathematics* (2nd ed.). National Association of Education for Young Children.

Dafermou, C., Koulouri, P., & Mpasagianni, E. (2006). *Kindergarten Teachers' Guide*. OEDB (in Greek).

Eleftheriadi, A., Lavidas, K., & Komis, V. (2021). Teaching Mathematics in Early Childhood Education with ICT: The Views of Two Contrasting Teachers' Groups. *Journal of Digital Educational Technology*, *1*(1), Article ep2103. https://doi.org/10.21601/jdet/11117

Engel, M., Claessens, A., & Finch, A. M. (2013). Teaching students what they already know? The (Mis)Alignment between mathematics instructional content and student knowledge in kindergarten. *Educational Evaluation and Policy Analysis*, *35*(2), 157-178. https://doi.org/10.3102/0162373712461850

European Education and Culture Executive Agency. (n.d.). *National Policies Platform*. https://national-policies.eacea.ec.europa.eu/

Hurst, M., Anderson, U., & Cordes, S. (2016). Mapping among number words, numerals, and nonsymbolic quantities in preschoolers. *Journal of Cognition and Development*, *18*(1), 41-62. https://doi.org/10.1080/15248372.2016.1228653

Katsidima, M., Lavidas, K., Kornelaki, A., Kaliampos, G. (2023). An investigation on alternative ideas on thermal phenomena of pupils with and without learning difficulties, SN Soc Sci, 3(15). https://doi.org/10.1007/s43545-022-00603-5

Koustourakis, G. (2007). The new educational policy for the reform of the curriculum and the change of school knowledge in the case of Greek compulsory education. *International Studies in Sociology of Education*, *17*(1-2), 131-146. https://doi.org/10.1080/09620210701433878

Koustourakis, G. (2013). A Sociological Approach to the Official Discourse on Space in the case of Greek Kindergarten Classrooms. *The International Journal of Early Childhood Learning*, *19*(3), 1-13. https://doi.org/10.18848/2327-7939/CGP/v19i03/48399

Koustourakis, G. (2014). A Sociological Approach to Painting Teaching according to the Contemporary Greek Kindergarten Curriculum. *The International Journal of Early Childhood Learning*, *20*(1), 23-37. https://doi.org/10.18848/2327-7939/CGP/v20i01/48408

Koustourakis, G. (2018). Classroom space and kindergarten curriculum: a sociological approach to teachers' discourse on the status of space and its use in teaching. *European Journal of Alternative Education Studies*, *3*(2), 47-65. https://doi.org/10.5281/zenodo.1422864

Koustourakis, G., & Zacharos, K. (2011). Changes in School Mathematics Knowledge in Greece: a Bernsteinian Analysis. *British Journal of Sociology of Education*, *32*(3), 369-387. https://doi.org/10.1080/01425692.2011.559339

Lavidas, K., Apostolou, Z., & Papadakis, S. (2022). Challenges and Opportunities of Mathematics in Digital Times: Preschool Teachers' Views. *Education Sciences*, *12*(7), Article 459. http://dx.doi.org/10.3390/educsci12070459

Lavidas, K., Skopeliti, I., Zacharos, K., & Panagiotounakos, E. (2023). Preservice preschool teachers' mathematics experience and math anxiety on their beliefs about and attitudes toward teaching mathematics, *Journal of Early Childhood Teacher Education* (advance online publication). https://doi.org/10.1080/10901027.2023.2196943

Lee, J. (2014). Is children's informal knowledge of mathematics important? Rethinking assessment of children's knowledge of mathematics. *Contemporary Issues in Early Childhood*, *15*(3), 293-294. https://doi.org/10.2304/ciec.2014.15.3.293

Lee, J., & Md-Yunus, S. (2016). Investigating children's abilities to count and make quantitative comparisons. *Early Childhood Education Journal*, *44*, 255-262. https://doi.org/10.1007/s10643-015-0707-4

Litkowski, C. E., Duncan, J. R., Logan, A. R. J., & Purpura, D. J. (2020). When do preschoolers learn specific mathematics skills? Mapping the development of early numeracy knowledge. *Journal of Experimental Child Psychology*, *195*, Article 104846. https://doi.org/10.1016/j.jecp.2020.104846

McDonald, A. J., Merkley, R., Mickle, J., Collimore, L., Hawes, Z., & Ansari, D. (2021). Exploring the implementation of early math assessments in kindergarten classrooms: A research-practice collaboration. *Mind Brain and Education*, *15*(4), 311-321. https://doi.org/10.1111/mbe.12293

McDonald, A., & Murphy, S. (2019). Mathematics education for children under four years of age: a systematic review of the literature. *Early Years*, *41*(5), 522–539. doi: https://doi.org/10.1080/09575146.2019.1624507

McGuire, P., Kinzie, B.M., & Berch, B.D. (2011). Developing number sense in Pre-K with five-frames. *Early Childhood Educational Journal*, *40*, 213-222. https://doi.org/10.1007/s10643-011-0479-4

Ministry of Education. (2014). *Curriculum for kindergarten [New school. School of 21st century. New Curriculum].* YPEPTH/P.I. [in Greek]

Muthukrishnan, P., Kee, M. S., & Sidhu, G. K. (2019). Addition Error Patterns Among the Preschool Children. *International Journal of Instruction*, *12*(2), 115-132. https://doi.org/10.29333/iji.2019.1228a

National Council of Teachers of Mathematics. (2001). *Principles and standards for school mathematics*.

Nguyen, H. N., Laski, V. E., Thomson, L. D., Bronson, B. M., & Casey, M. B. (2017). More Than Counting: Learning to Label Quantities in Preschool. *YC Young Children*, *72*(3), 22-29. https://www.jstor.org/stable/90013678

Ohio Department of Education. (2004). *Early learning content standards*.

Piaget, J. (1952). *The origin of intelligence in children*. International University Press.

Piaget, J., Gruber, H. E., & Vonèche, J. J. (1995). *The essential Piaget: an interpretive reference and guide*. Aronson.

Pogiatzi, M., Bardoutsou, I., Lavidas, K., & Komis, V. (2022). Interviewing preschool children in Greece about their usage of mobile devices at home. *SN Social Sciences*, *2*, Article 215. https://doi.org/10.1007/s43545-022-00522-5

Sarama, J., & Clements, H. D. (2009). *Early childhood mathematics education research: Learning trajectories for young children*. Routledge.

Shiakalli, M. A., Zacharos, K., & Lavidas, K. (2017). Preparing preschool teachers for teaching probability. *Review of Science, Mathematics and ICT Education*, *11*(2), 93-117. https://doi.org/10.26220/rev.2810

Watts, W. T., Duncan, J. G., Siegler, S. R., & Davis-Kean, E. P. (2014). What is past is prologue: Relations between early mathematics knowledge and high school achievement. *Educational Researcher*, *43*(7), 352-360. https://doi.org/10.3102/0013189X14553660

Zacharos, K., Koustourakis, G., & Papadimitriou, K. (2014). Analyzing the implemented curriculum of Mathematics in Preschool Education. *Mathematics Education Research Journal*, *26*, 151-167. https://doi.org/10.1007/s13394-013-0086-3