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Problem-Solving Based Epistemic Learning Pattern: Optimizing Mathematical Representation Ability of Prospective Teachers and Pharmacists

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Abstract

Background/purpose. Although mathematical representation ability is a significant ability and an interesting topic to study, quite a few researchers have studied this ability, and even fewer have examined it through the implementation of epistemic learning patterns based on problem-solving. In fact, this learning pattern is epistemic and makes problems the core situation for students in constructing concepts or solutions. Therefore, this study aimed to optimize students' mathematical representation abilities through the implementation of this learning pattern.

Materials/methods. This study employed a didactical research design to achieve the research objectives. Participants were 56 prospective mathematics teachers and pharmacists (14 men and 42 women) aged between 20 and 24 years. The main instrument was the researcher, with several additional instruments, such as lecture design, mathematical representation ability forms, and documentation studies. Data were analyzed using qualitative data analysis methods.

Results. The results of the study revealed that all prospective teacher participants used verbal representation forms, and most others used symbolic and verbal representation forms when solving problems. All prospective pharmacist participants did not use visual representation forms but used symbolic and verbal representation forms when solving mathematical problems. This finding depended on the form of the task given.

Conclusion. The results of this study concluded that problem-solving-based epistemic learning patterns could be used during lectures to optimize students' mathematical representation abilities, noting that the form of the assignment should be designed in a way that facilitates visual representation.

1. Introduction

Mathematical representation ability is a person's ability to use various forms, such as images, symbols, verbal, or other forms, in presenting mathematical concepts (Bajo-Benito et al., 2023; F. G. Putra et al., 2024; Rahmawati & Anwar, 2020; Stieff et al., 2016). This ability is critical to strengthening students' understanding of content elements in mathematics (Novitasari et al., 2021). In addition, this ability makes one accustomed to conveying their ideas or concepts in a simple and interesting way (Isnawan et al., 2024a; Septiani et al., 2020).

Ideally, students' mathematical representation abilities should be good, but the facts are different. Several of the previous studies revealed that students' mathematical representation abilities tended not to develop optimally because students were not used to presenting ideas in a structured and systematic manner and using illustrations when solving problems (Arefaine et al., 2022; F. G. Putra et al., 2024; R. W. Y. Putra et al., 2023; Son & Lee, 2016). Given the importance of mathematical representation abilities and the existence of problems related to these abilities, quite a few studies attempted to examine mathematical representation abilities, especially in the educational context. As shown in Table 1, several systematic literature reviews (SLR) were conducted to summarize the findings of previous studies. Table 1 also shows the differences between the current and several previous studies. In addition, previous SLR studies by Lutfi and Juandi (2023) and F. G. Putra et al. (2024) also revealed a decrease in the number of publications and citations related to mathematical representation skills despite its significance.

Table 1. Differences between current research and previous research

SLR Articles	Research Approach/Design	Learning Strategy	Technology Integration	Participants
R. W. Y. Putra et al. (2023)	Dominance of case studies, interviews, observations, and phenomenology.	Problem-solving, android challenge-based learning, REACT, RME, PMRI, collaborative learning, and discovery learning.	MIKIMOM, GeoGebra, visual heuristic tools, and Carom comic game.	-
Lutfi and Juandi (2023)	Quantitative	-	-	Secondary school students
F. G. Putra et al. (2024)	-	-	Zoom Meetings, Google Meet, Google Classroom, Edmodo, GeoGebra, Kahoot, and Moodle.	Students at school
Current Research	Didactical design research (DDR) using lesson study activities.	Epistemic learning patterns based on problem-solving.	Quizizz and QR Code.	Prospective mathematics teachers and pharmacists.

Concerning the previously extended reasons, this study aims to analyze the impact of problem-solving-based epistemic learning patterns on the mathematical representation skills of students in higher education. The epistemic learning pattern was chosen because the learning material was arranged based on how to acquire knowledge, which made it more relevant to the characteristics of students (Isnawan et al., 2024a). The problem-solving-based epistemic learning pattern is a systematic learning pattern that follows the didactic situation pattern in arranging activities during lectures and uses problems as an initial situation to trigger students to understand concepts or formulas or find solutions (Sukarma et al., 2024). Problem-solving then becomes the basis of the learning pattern because problems provide challenges to students so that student competencies develop optimally (Barham, 2020; Lillo, 2023; Rahayuningsih et al., 2021). Problems are also used to facilitate students' ability to find concepts or formulas while learning mathematics (Idawati et al., 2020; Isnawan et al., 2024a; Nindiasari et al., 2024). Several research questions were then formulated to achieve the research objectives, including:

RQ1: What is the form of problem-solving-based epistemic learning patterns in higher education?

RQ2: What is the description of the implementation process of problem-solving-based epistemic learning patterns in higher education?

RQ3: What is the description of mathematical representation ability after implementing problem-solving-based epistemic learning patterns in higher education?

2. Literature Review

As described earlier, representational ability refers to a person's capacity to use various forms of representation in conveying ideas or goals. (Lutfi & Juandi, 2023). There are at least three types of representations commonly used: visual, symbolic, and verbal (Novitasari et al., 2021). Visual representation relates to a person's ability to convey ideas or goals through forms that engage vision. A person with good visual representation skills typically uses visible forms, such as pictures, diagrams, or graphs, to communicate ideas or goals. Meanwhile, someone with symbolic representation skills employs various forms of symbols to convey ideas or goals. Verbal representation skills, on the other hand, involve a person's ability to use language or words in conveying ideas or goals. In this context, the language includes both spoken and written forms (Arcavi, 2003; Novitasari et al., 2021; Septiani et al., 2020).

There are several ways to optimize a person's representational ability. In mathematics learning, the concept of the epistemic learning pattern is recognized. (Sukarma et al., 2024). This learning pattern is developed based on the theory of didactical situations. (Brousseau, 2002; Isnawan, 2023). Didactic situations refer to situations where students take action on scenarios presented by lecturers. Epistemic learning patterns use didactic situations as the main flow of learning. These situations include action, formulation, validation, and institutionalization situations. In this study, the initial situation presented by lecturers is in the form of a problem, and the didactic situation consists of a series of activities designed to solve the problem. Therefore, the epistemic learning pattern used in this study is called a problem-solving-based epistemic learning pattern. The epistemic learning pattern is an example of a didactic design that emerged after conducting several studies with the DDR type. DDR is a research design that aims to analyze the factors causing students to experience learning obstacles and to develop a didactic design based on these causal factors (Isnawan et al., 2024b; Suryadi, 2019a, 2019b). These patterns involve a series of learning activities that are repeated to help students construct knowledge. The series encompasses complete learning stages, starting from initial activities to the conclusion of learning. (Isnawan, 2023).

The initial activities include setting learning objectives, motivating students to learn, and exploring prerequisite material needed before addressing the main topic. The core activities then follow, consisting of several steps. First, students are presented with a problem to solve independently. This problem serves as an initial situation that encourages students to take action during the learning process. Once solved, students present their solutions in various representational forms (visual, symbolic, or verbal). Students also respond to each other's presentations to validate the solutions, which helps them acquire new knowledge. This knowledge can take the form of mathematical concepts or formulas (Isnawan et al., 2024b).

After validating their solutions, students reinforce the knowledge they have gained by solving new problems in different contexts or situations using the acquired knowledge. During this stage, students utilize various representational forms to assist in solving the problems. This process is designed to help students enhance their representational abilities and internalize their acquired knowledge. Finally, the learning process concludes with reflection activities. There are three components students reflect on. First, they reflect on the knowledge they have gained, answering simple questions related to the content learned. Second, they reflect on their feelings after the learning activities, ensuring that their social-emotional state has improved. Third, they reflect on the values, character traits, or positive lessons they have learned during the process. In summary, this problem-solving-based epistemic learning pattern aims to optimize students' representational abilities, benefiting not only future mathematics teachers but also professionals such as pharmacists (Sukarma et al., 2024).

3. Methodology

3. 1. Research Design

This study used DDR to produce a lecture design that was in accordance with the characteristics of students in higher education. In addition, DDR was used to qualitatively explore students' mathematical competence. DDR was developed based on a qualitative approach because it was expected to describe theories or conjectures after the research ended (Suryadi, 2019b, 2019a). In this context, these descriptions were related to several research questions previously stated. This research was conducted at a private university in Mataram, Indonesia, for two semesters, from September 2023 to August 2024. The courses that were the focus of this study were related to mathematics, specifically the course on developing a mathematics learning program for prospective mathematics teachers and the mathematics course for prospective pharmacists.

Since this study used a DDR design, the research procedure also followed DDR steps: prospective, *metapedadidactic*, and retrospective analysis. (Marfuah et al., 2022). Prospective analysis was used to answer RQ-1. In this step, the researcher designed a lecture design that followed a problem-solving-based epistemic learning pattern. *Metapedadidactic* analysis was used to answer RQ-2 because the activity carried out in *metapedadidactic* analysis involved the researcher, through the model lecturer, implementing the previously prepared lecture design. In this step, the researcher ensured the relevance, unity, and flexibility of the lecture design. Finally, retrospective analysis was used to answer RQ-3, as this stage involved the researcher reflecting on the learning process and outcomes. The researcher analyzed the mathematical representation abilities of students in higher education to obtain an overview of these competencies. All stages in this study were carried out using lesson study activities involving several lecturers at the university during the plan, do, and see stages. (Isnawan et al., 2024a; Joubert et al., 2020). In simple terms, the research procedure is shown in Figure 1.

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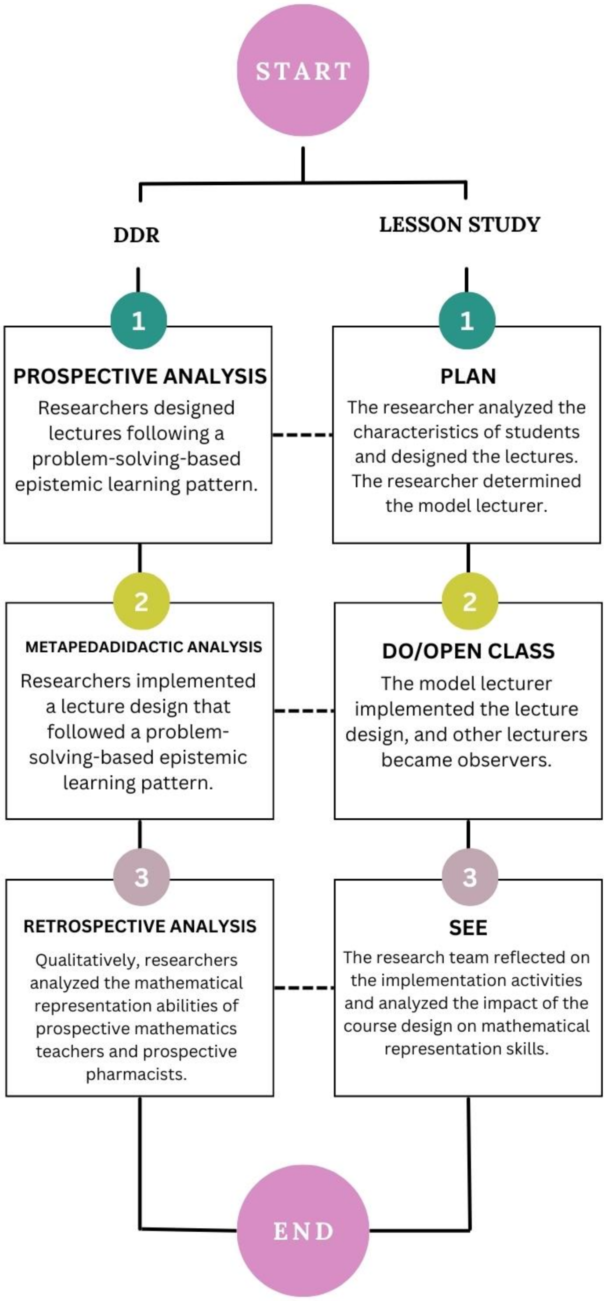

Figure 1. Research Procedures

3. 2. Participants and Data Collection

The participants in this study were 56 university students, 28 of whom were in the mathematics education study program (7 males and 21 females), and the other 28 were in the pharmacy study program (7 males and 21 females). The participants' ages ranged from 20 to 24 years. Additionally, most of the students came from the Sasak Tribe (an indigenous tribe in Mataram), and the majority of their parents were farmers. Participants were selected using criteria-based sampling as the sampling method. They were chosen based on two main criteria: students taking mathematics courses and having experienced obstacles in performing mathematical representation. As previously described, this study used a qualitative approach, with the researcher acting as the main instrument. However, several additional instruments were also used, such as lecture design, mathematical representation ability forms, and documentation studies. The lecture design was used to obtain data on the implementation process of problem-solving-based epistemic learning patterns in higher education. The mathematical representation ability form was used to gather data on the description of participants' mathematical representation ability, adapted from research conducted by (Hadiastuti & Soedjoko, 2019). Meanwhile, documentation studies were used to collect data on student activities during the lecture process.

3. 3. Analyzing of Data

Data regarding the implementation of problem-solving-based epistemic learning patterns were analyzed using qualitative data analysis, following several stages; namely, data reduction, data display, and conclusion (Miles et al., 2014). Data reduction was conducted by picking out data that was less relevant to the research question. Data display was conducted by creating various forms of representing information obtained from the data. Meanwhile, conclusions were drawn by formulating answers to research questions based on the results of the analysis. The mathematical representation ability data were analyzed using thematic analysis. The steps of thematic analysis included getting familiarized with the data (reading the data repeatedly), determining the initial codes (based on the general characteristics of the data), determining the theme (combining initial codes with the same characteristics), reviewing the theme (re-reading the theme with all initial codes), and drawing conclusions (finding answers to research questions) (Benavides-lahnstein & Ryder, 2019; Nowell et al., 2017). Researchers also triangulated data sources and data collection methods to strengthen the results of data analysis.

4. Results

4. 1. What Is the Form of Problem-Solving-Based Epistemic Learning Patterns in Higher Education? (RQ-1)

This study used a problem-solving-based epistemic learning pattern. This learning pattern facilitated learning activities from the initial activities to the final activities. The initial activities included lecture objectives, learning motivation, and confirming prerequisite materials. The core activities consisted of action-formulation situations, validation, and institutionalization. The final activities consisted of reflection. The action-formulation situation refers to when students solve problems in groups. The validation situation refers to student activities in concluding concepts, formulas, or solutions to problems being solved. The institutionalization situation is related to student activities when using concepts, formulas, or problem solutions to solve new problems.

In the context of this study, the initial activities involved listening to learning objectives (Let's Listen), watching inspirational videos to arouse students' interest in learning (Let's Watch), and answering several questions related to prerequisite materials (Let's Play). The core activities consisted of student activities in solving problems (Let's Find Out), students presenting solutions or problem-solving processes (Let's Tell Stories), concluding concepts obtained when solving problems (Let's Conclude), and using the concept in solving problems in a different context than before (Let's Practice). In this core activity, students were expected to be able to use various forms of representation, both when solving problems and when presenting solutions or problem-solving processes. The final activity consisted of reflection activities related to the reflection of the concepts obtained, students' social-emotional development, and the lecture process that had been carried out (My Reflection).

Figures 2 and 3 show examples of activities for action situations. A complete description of the epistemic learning patterns used in this study can be accessed at https://shorturl.at/pUF1G for prospective teacher students and https://shorturl.at/vcyIx for prospective pharmacist students.

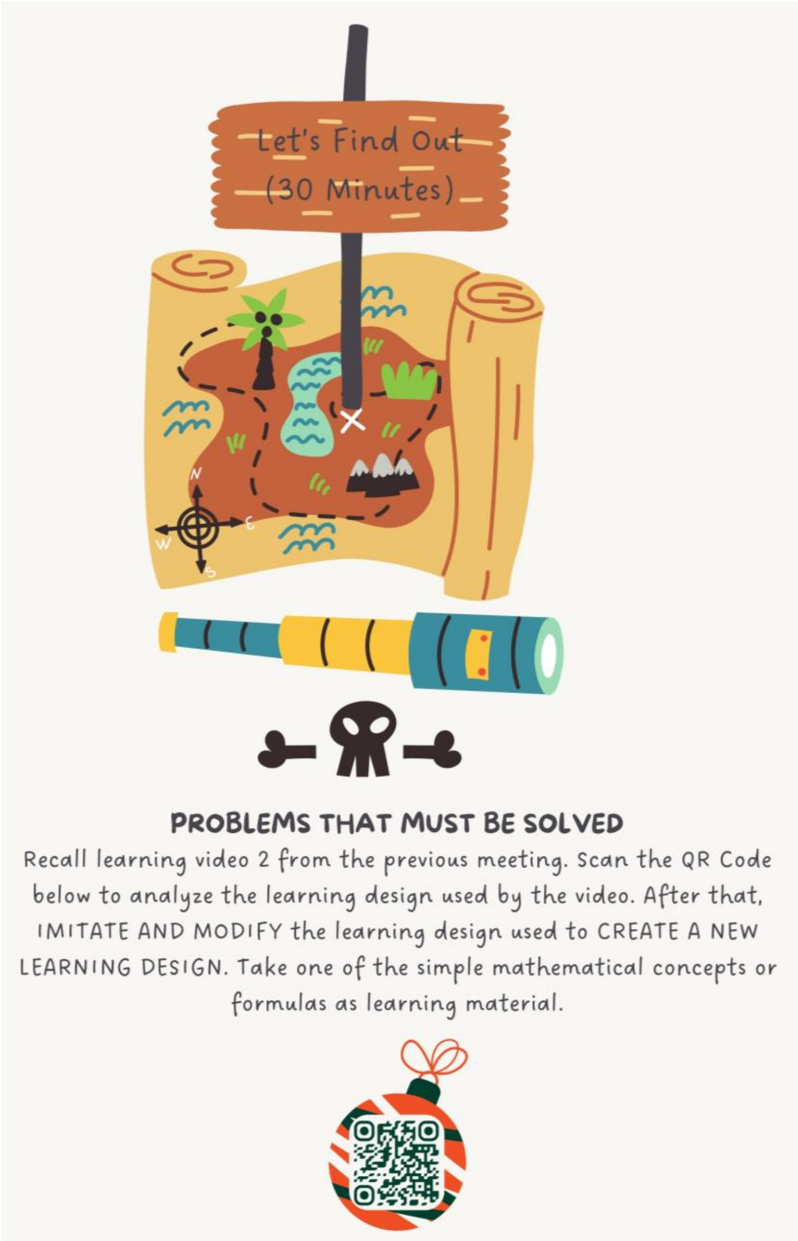

Figure 2. Example Problems for Prospective Mathematics Teachers

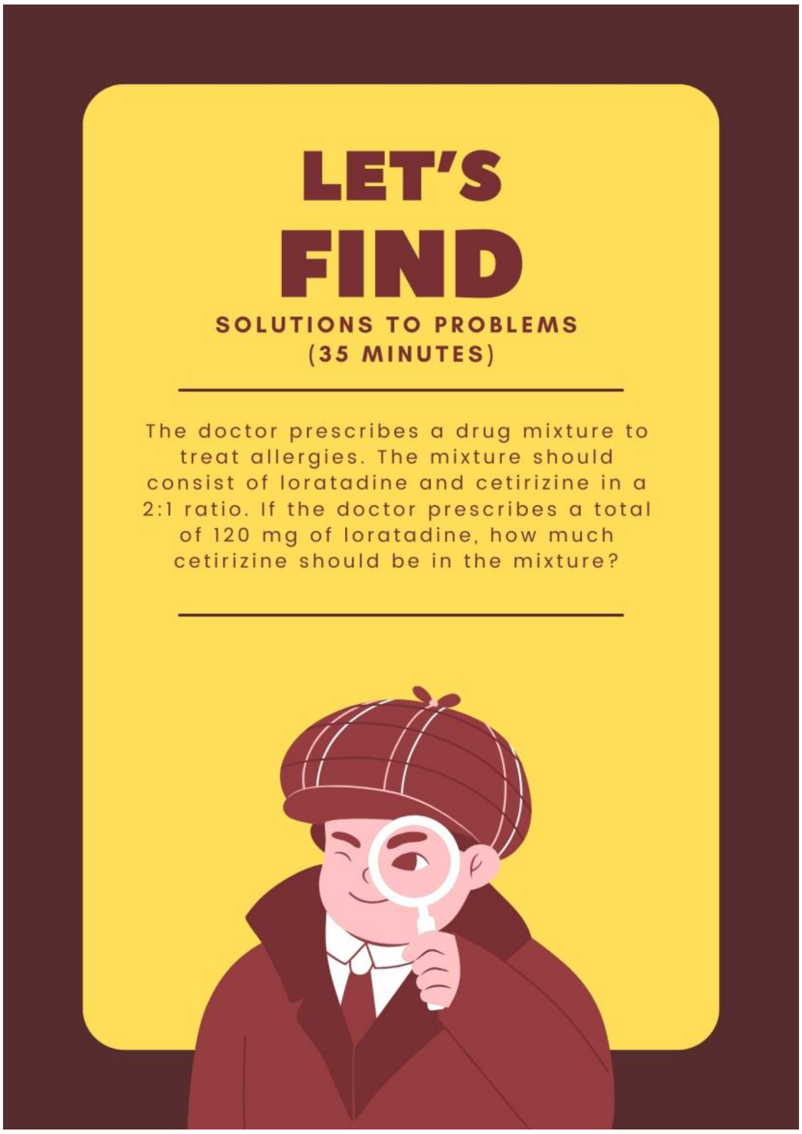

Figure 3. Example Problems for Prospective Pharmacists

4. 2. What Is the Description of the Implementation Process of Problem-Solving-Based Epistemic Learning Patterns in Higher Education? (RQ-2)

As previously described, the lectures in this study followed a problem-solving-based epistemic learning pattern. In general, the lecture activities followed the activities that had been planned in the design. Table 2 provides descriptions of the two groups of students and briefly describes the important activities carried out by students during lectures. These activities are the responses given by students, starting from the initial activities of lectures to the final activities of lectures.

Table 2. Implementation of epistemic learning patterns based on problem-solving

Activities	Prospective Mathematics Teachers	Prospective Pharmacists
Let's Listen	Students read the objectives of the lecture and were able to identify the target of the lecture at the meeting, namely, to design a mathematics learning design.	Students identify the objectives of the lecture, namely, to be able to apply the concept of ratio in pharmaceutical work.

Activities	Prospective Mathematics Teachers	Prospective Pharmacists
Let's Watch	Students watched an inspirational video about a life journey, and student representatives responded that they did not want to be ordinary teachers.	Students watched an inspirational video and responded that mathematics was close to life and useful for everyday life.
Let's Play	Students played *Quizizz,* which was related to important components of learning. Students looked enthusiastic while playing.	Students scanned the QR code to play *Quizizz* on the concept of fractions. Students looked happy while playing.
Let's Find Out	Students discuss in small, heterogeneous groups. In this activity, students have prepared several devices needed to design learning. In fact, students have printed the learning design. Therefore, the researcher asked students to discuss their learning designs because, in the previous meeting, the researcher asked students to prepare the materials to use while designing their learning. Examples of students' work can be seen in Figure 4.	Students work in small groups to solve problems. In this activity, students discuss problems and share roles when solving them, including appointing one group member as a presenter. Figure 5 shows examples of solutions provided by students.
Let's Tell Stories	In this activity, group representatives presented the results of problem solving, and others responded.	Group representatives presented the results of problem-solving in front of the class, and other students responded.
Let's Conclude	Students concluded several activities that had to be included in the initial, core, and final activities in mathematics learning.	Students concluded that around 60 mg of *Cetirizine* was needed to be mixed into a drug prescription.
Let's Practice	Students concluded several contents that had to be included in the initial, core, and final learning activities.	This activity was done at home because the allocated time for learning was limited. Students shared the results of their work via *WhatsApp.*
My Reflection	Students could adequately convey the components that had to be present in mathematics learning. All students also felt happy with the activities carried out during the lecture.	Students felt happy with the lecture activities and considered that the concept of ratio was quite important for pharmacists.

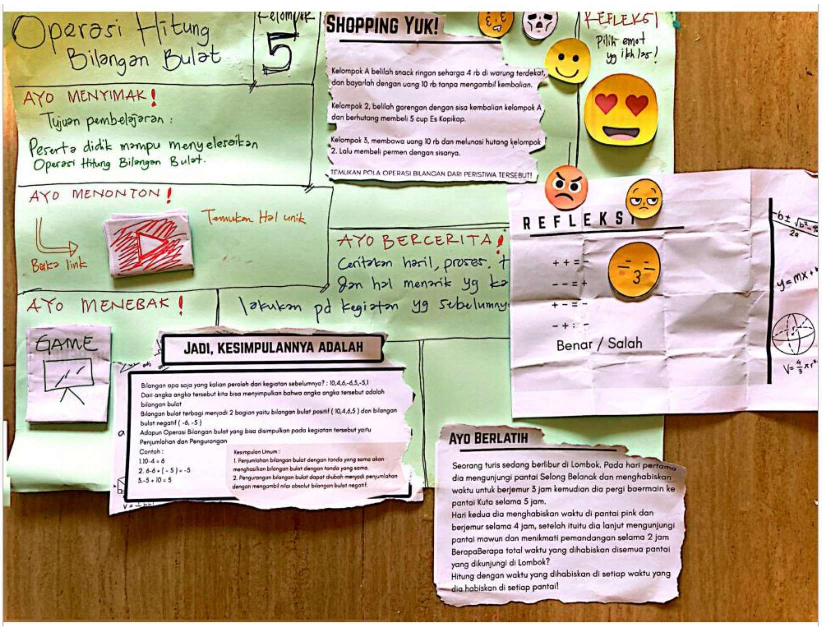

Figure 4. Example of Answers from Prospective Mathematics Teachers *(Let's Find Out)*

Figure 4 is an example of the work by prospective mathematics teachers while solving problems. It shows that students used various forms of representation to convey ideas. They used emoticons and several images as tangible examples of visual representation. Additionally, students used various symbols or mathematical formulas as a form of symbolic representation. They also showed verbal representation by providing various descriptions when conveying ideas. Meanwhile, Figure 5 is an example of the work by prospective pharmacy students when solving problems. It shows that these students demonstrated verbal and symbolic representation skills, as they tended to provide explanations in the form of descriptions and mathematical formulas. No visual representation is observed in the work of prospective pharmacist students, as displayed in Figure 5.

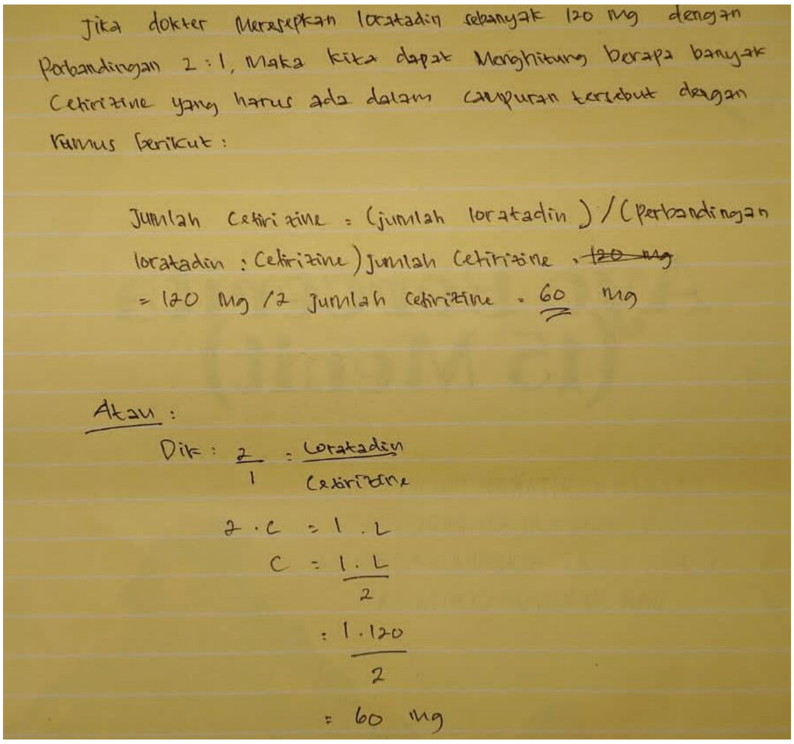

Figure 5. Example of Answers for Prospective Pharmacists *(Let's Find Out)*

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4. 3. What Is the Description of Mathematical Representation Ability After Implementing Problem-Solving-Based Epistemic Learning Patterns in Higher Education? (RQ-3)

As previously described, this study involved two groups of participants; namely, a group of prospective mathematics teachers who took the didactic design research course and a group of prospective pharmacists who took the mathematics course. For the group of prospective mathematics teachers (G1), the results of the representation ability analysis can be seen in Table 3. Table 3 shows the results of the analysis of the responses from all groups of students related to the problems presented. A score of "1" indicates the emergence of indicators of mathematical representation ability when students solve problems, while a score of "0" indicates that mathematical representation ability does not appear. Table 3 reveals that all groups can demonstrate verbal representation, and most groups demonstrate visual and symbolic representation ability.

Table 3. Results of mathematical representation ability analysis for prospective teachers

Indicators	G1-1	G1-2	G1-3	G1-4	G1-5
Visual					
Creating general types of drawings.	1	0	1	1	1
Using various geometric shapes.	1	1	0	1	0
Symbolic					
Creating mathematical equations.	0	1	1	1	0
Creating other mathematical expressions.	0	1	0	1	1
Verbal					
Writing interpretations using words.	1	1	1	1	1
Writing problem-solving steps using words.	1	1	1	1	1

Table 4 shows the results for the prospective pharmacy students (G2) group. It indicates that none of the groups produced visual representations when conveying ideas or concepts while solving problems. However, all groups of prospective pharmacists can convey ideas or concepts using symbolic and verbal representations.

Table 4. Results of mathematical representation ability analysis for prospective pharmacists

Indicators	G2-1	G2-2	G2-3	G2-4	G2-5
Visual					
Creating general types of drawings.	0	0	0	0	0
Using various geometric shapes.	0	0	0	0	0
Symbolic					
Creating mathematical equations.	1	1	1	1	1
Creating other mathematical expressions.	1	1	1	1	1
Verbal					
Writing interpretations using words.	1	1	1	1	1
Writing problem-solving steps using words.	1	1	1	1	1

An example of how students represented ideas can be seen in Figure 6. Figure 6 illustrates a prospective mathematics teacher's answer when solving a problem. It shows that the group of students demonstrated visual representation skills by displaying various flat shapes, such as squares, rectangles, circles, trapezoids, parallelograms, kites, rhombuses, and combinations of different plane geometric shapes. Additionally, Figure 6 highlights students' writing area and circumference formulas for each geometric shape, which manifests their symbolic representation skills. The group also includes written descriptions in Figure 6 to strengthen or clarify their ideas.

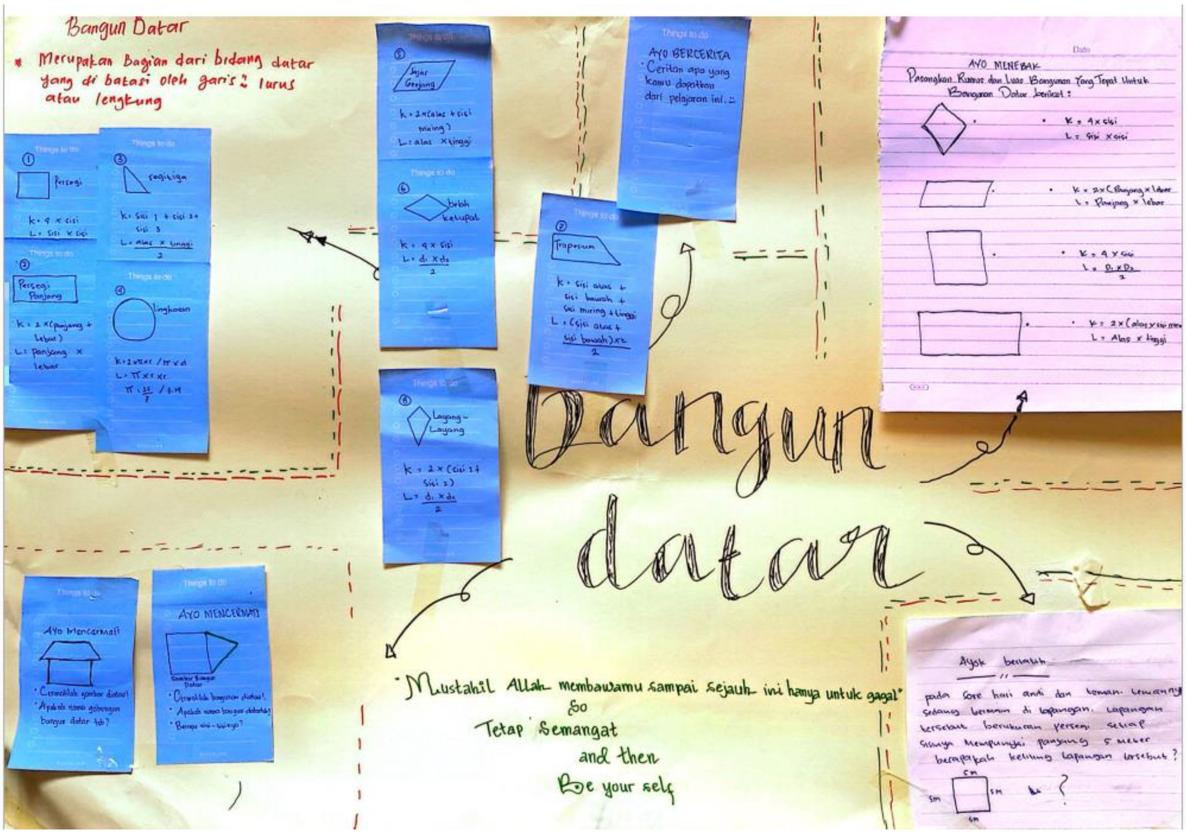

Figure 6. Example of Representation of Prospective Mathematics Teachers

5. Discussion

5. 1. What Is the Form of Problem-Solving-Based Epistemic Learning Patterns in Higher Education? (RQ-1)

The lecture design used in this study followed an epistemic learning pattern. The epistemic learning pattern is structured based on the theory of didactic situations (Isnawan et al., 2024a). It used didactic situation steps; namely, action, formulation, validation, and institutionalization situations (Arslan et al., 2011; Brousseau, 2002; Prabowo et al., 2022) in developing lecture activities for students. The action situation begins with the lecturer giving a problem as an initial situation (Marfuah et al., 2022) for students to learn the concept. In this situation, students are given the maximum opportunity to explore ideas and use various forms of representation when solving problems. After that, students formulate a strategy to obtain a solution to the problem being solved. In the validation situation, students conclude a concept, formula, or solution to the problem being solved (Suryadi, 2019a). Finally, students use concepts, formulas, or solutions to solve new problems in different contexts or situations (Sukarma et al., 2024).

The problem-solving-based didactic situation in the context of this study refers to the core activities students are engaged with during lectures. To enable a complete learning process, lecture activities must facilitate initial and final activities. This study uses an epistemic learning pattern to facilitate this because it contains a series of lecture activities from beginning to the end. The epistemic learning pattern is also developed based on the didactic situation. In addition, it facilitates learning systematically and sequentially. During the initial learning activities, the epistemic learning pattern generates focus and motivation and explores the prerequisite material needed during learning. The final stage activities reflect the learning activities that have been carried out (Isnawan et al., 2024a; Sukarma et al., 2024).

The results of this study are supported by Sukarma et al. (2024), which revealed that the epistemic learning pattern contained several student-oriented activities, following the steps of didactic situations and using problems as an initial situation to trigger students to use previous knowledge and experience when solving problems during learning. These results are also in line with the findings of Isnawan et al. (2024a), who used an epistemic learning pattern by following the steps of didactic situations—action situations, formulation, validation, and institutionalization—in optimizing the mathematical representation abilities of prospective mathematics teachers. Based on these findings, it can be concluded that the problem-solving-based epistemic learning pattern contains a variety of didactic situations and complete learning activities in order to optimize students' representation abilities.

5. 2. What Is the Description of the Implementation Process of Problem-Solving-Based Epistemic Learning Patterns in Higher Education? (RQ-2)

Referring to the *metapedidactic* theory (Suryadi, 2019a), the implementation activity aims to observe the unity of the design, the coherence between didactic situations, and the flexibility of the design, especially when solving problems. The unity of design can be seen in the formation of a complete learning process that facilitates students from the beginning to the end of learning activities. (Marfuah et al., 2022). In the context of this study, this unity was evident in the problem-solving-based epistemic learning pattern, which supported students throughout the lecture activities. However, the validation situation *(Let's Practice)* could not be fully implemented due to time constraints. Coherence between didactic situations refers to the interconnection of each step or activity, allowing students to construct concepts or solutions when solving problems (Suryadi, 2019b). In this study, coherence was observed through the action-formulation situation *(Let's Find Out)*, which served as the foundation for students to validate their findings *(Let's Conclude)*. However, institutionalization activities were not run optimally due to time limitations. The termination of the didactic contract often determines flexibility in the lecture design and allows for adjustments when students solve problems (Arias & Araya, 2009). Ideally, learning should avoid terminating the didactic contract during problem-solving. No such termination occurred in this study because students could solve problems using potential *didactic* contracts.

The findings of this study align with the results of Isnawan et al.'s (2024b) study, which revealed that epistemic learning patterns created unity, coherence, and flexibility between didactic situations during lecture implementation. Students actively participated in lecture activities; discussions went smoothly, and they expressed satisfaction at the end of the lecture. This was likely due to the freedom given to students to construct concepts or formulas using various forms of representation. Additionally, integrating ICT tools like *Google, AI, ChatGPT,* and *Quizizz* enhanced engagement during lectures. Previous research has shown that using ICT significantly boosts student interest and motivation in learning (Bencivenga, 2017; Dockendorff & Solar, 2018; El Messaoudi, 2024; Fernández-Cézar et al., 2024; Khasanah & Lestari, 2021; Zainudin & Zulkiply, 2023).

Sukarma et al. (2024) also noted that implementing epistemic learning patterns provided a unique experience for students. They could follow the learning process independently, from the beginning to the end of the activities, and even engaged with mathematics learning on their own. Other studies (Rønning, 2021; Supriadi, 2019) further suggested that using didactic situation theory could enhance students' competence in mathematics learning. In conclusion, the implementation of problem-solving-based epistemic learning patterns successfully fostered unity, coherence, and flexibility between didactic situations. However, the institutionalization situation for prospective pharmacists was not resolved during the lectures but after the lecture activities concluded.

5. 3. What Is the Description of Mathematical Representation Ability After Implementing Problem-Solving-Based Epistemic Learning Patterns in Higher Education? (RQ-3)

Table 2 and Table 3 highlighted the representation abilities of the two groups of students: prospective mathematics teachers and prospective pharmacists. All groups received a score of 1 for verbal indicators, indicating that all participants could interpret and verbally explain the steps to solve problems. No prospective pharmacist student group used visual representation as a visual indicator when solving problems. However, most prospective mathematics teacher students used visualizations, such as pictures and geometric shapes, to express ideas or concepts. This distinction points to a key difference in how the two groups approach problem-solving visually. Looking at the symbolic indicator, all prospective pharmacist students used equations and mathematical expressions when solving problems, while one group of prospective mathematics teachers did not. This suggests that while symbolic representation is commonly used by prospective pharmacists, not all prospective mathematics teachers rely on it.

Based on these findings, all prospective mathematics teacher participants used verbal representation and most used visual and symbolic representations. On the other hand, all prospective pharmacist participants used symbolic and verbal representation but did not incorporate visual representation. The theory of representation abilities states that verbal representation is a common method for conveying ideas (Novitasari et al., 2021). Verbal skills are often displayed when students write explanations requiring minimal tools or skills. For instance, conveying an idea verbally only needs simple writing tools, and advanced drawing or mathematical skills are unnecessary. These results align with previous studies indicating that verbal representation is a frequently used method among students when learning mathematics, despite the varying quality of their representations (Novitasari et al., 2021; Rahmawati & Anwar, 2020; Septiani et al., 2020). The study also reveals differences in visual representation abilities between students. The researcher analyzed the types of tasks students worked on to investigate these differences further. The prospective mathematics teachers were tasked with designing a mathematics lesson, which required visualization to make the lesson more engaging. On the other hand, the prospective pharmacists focused on solving pharmacy-related problems involving mathematical equations and symbols. This analysis supports the conclusion that the type of task influences students' use of representation, particularly visual skills.

Visual representation abilities are tied to a person's capacity to create illustrations or pictures to convey ideas (Bronkhorst et al., 2021; Novitasari et al., 2021). Previous research has also shown that visual representation is an essential skill when solving problems, as it captures attention and makes the concepts more understandable (Arcavi, 2003; Bronkhorst et al., 2021; Cooper & Alibali, 2012). When Table 2 and Table 3 are reanalyzed, it becomes clear that symbolic representation is closely tied to verbal representation. To better understand this relationship, the researcher conducted an additional analysis of student worksheets. This analysis showed that students often started by using mathematical equations or symbols when solving problems. However, to enhance clarity for readers, students added verbal explanations, describing the steps of their solution process and interpreting the final result.

These findings align with the theory that mathematical problems typically require symbolic representation. Mathematical problems often include various mathematical forms, such as symbols or geometric shapes, which play a key role in the problem-solving process (Jupri & Sispiyati, 2021). These results also support earlier studies indicating that students tend to rely on symbolic representation skills when solving mathematical problems (Novitasari et al., 2021; Ünal et al., 2023).

Overall, the study's findings are consistent with findings by Isnawan et al. (2024a) and Sukarma et al. (2024), which highlighted that problem-based epistemic learning patterns can optimize students' mathematical competencies, including representation skills. The epistemic learning approach allows students to use multiple forms of representation when expressing ideas or concepts during lectures. The integration of various forms of ICT also plays a crucial role in engaging students during learning activities. Printed modules and ICT tools were especially helpful for students in solving problems, even in a hybrid learning environment.

Several prior studies have also shown that problem-based learning improves students' mathematical abilities (Guo, 2023; Idawati et al., 2020; Mudhofir, 2021), particularly when ICT is used to support the learning process (Dockendorff & Solar, 2018; Tossavainen & Faarinen, 2019; Yanuarto & Hastinasyah, 2022). The challenges presented by problem-based tasks often require collaboration, further enhancing students' mathematical competencies. Moreover, using ICT tools makes learning more interactive and enjoyable, helping capture students' attention and increase their interest in the material.

Furthermore, the emergence of various forms of mathematical representation abilities has a direct impact on the implementation of problem-solving-based epistemic learning patterns in students. This occurs because the epistemic learning pattern provides students with the freedom to convey ideas or concepts when solving problems *(Let's Find Out)*. Students are given the opportunity to use various forms of images, symbols, and descriptions while solving problems. During *Let's Tell Stories*, students are also encouraged to use various forms of representation to make their ideas or concepts more accessible to others.

The results of this study align with the findings of several previous studies (Isnawan et al., 2024a, 2024b; Sukarma et al., 2024), which revealed that the epistemic learning pattern optimized students' representation abilities by giving them the freedom to solve problems systematically within their groups and to use diverse representations, making their ideas or concepts easier to understand. However, a significant challenge in implementing problem-solving-based epistemic learning patterns was found to be the allocation of lecture time, which often exceeds the planned time. Students tend to take longer when solving problems, affecting other learning activities. Additionally, creating various forms of representation takes more time than verbal explanations. These findings are consistent with the results of Sukarma et al.'s (2024) study, which highlighted pedagogical challenges in problem-based learning, particularly the fact that students needed extended time to accomplish problem-solving activities. Addressing this challenge should be a focus for future mathematics researchers.

6. Conclusion and Limitations

The findings of this study imply several significant conclusions. First, the problem-solving-based epistemic learning pattern facilitates a variety of didactic situations in order to optimize the representational abilities of prospective mathematics teachers and prospective pharmacists. Second, the implementation of the learning pattern forms a unity in learning, from the initial activities to the end of learning. The activities involved in the learning pattern are interrelated, between one didactic situation and another, and there is flexibility between didactic situations. For example, students can do the institutionalization situation (practice activities) well, even though it is not done directly during lectures. Third, all prospective mathematics teacher participants used verbal representation, and most of the others used visual and symbolic representation forms. Meanwhile, all prospective pharmacist participants did not use visual representation abilities but used symbolic and verbal representation abilities. Fourth, the form of visual representation is influenced by the type or form of the task given. Fifth, symbolic and verbal representation abilities usually appear when solving mathematical problems.

This study has several limitations. First, the video documentation of lectures for prospective pharmacists could not be fully recorded, so observation notes were used to describe the results of implementing problem-solving-based epistemic learning patterns. Second, this study used a qualitative perspective in the analysis process, so using quantitative analysis in the future is highly recommended. Third, the researcher paid less attention to allocating time when implementing problem-solving-based epistemic learning patterns for prospective pharmacists. Fourth, there is no report on the results of the analysis of students' mathematical representation abilities before learning. Hence, the impact of problem-solving-based epistemic learning patterns is not very visible.

8. Implications

The results of the current study suggest several implications. First, problem-solving-based epistemic learning patterns are recommended to enhance mathematics lectures, especially when researchers or lecturers want to optimize students' representational abilities. Second, researchers in the future must pay more attention to time allocation when implementing problem-solving-based epistemic learning patterns so that students can optimize institutionalization situations or practice activities. Third, researchers or lecturers should design mathematical assignments that can facilitate students' visual representation ability. Moreover, visual representation forms become forms of representation that attract the interest or attention of others to the ideas or concepts given. Fourth, researchers can use problem-solving-based epistemic learning patterns in schools, both at elementary and secondary school levels. Fifth, further quantitative research needs to be conducted to examine the impact of problem-solving-based epistemic learning patterns. Researchers should also use various forms of statistical analysis to arrive at more generalizable results.

Declarations

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References

Arcavi, A. (2003). The role of visual representations in the learning of mathematics. *Educational Studies in Mathematics*, *52*(3), 215–241. https://doi.org/10.1023/A:1024312321077

Arefaine, N., Michael, K., & Assefa, S. (2022). GeoGebra assisted multiple representations for enhancing students' representation translation abilities in calculus. *Asian Journal of Education and Training*, *8*(4), 121–130. https://doi.org/10.20448/edu.v8i4.4309

Arias, F., & Araya, A. (2009). Analysis of the didactical contracts in 10th grade math classes. *Quaderni Di Ricerca in Didattica (Matematica)*, *4*(19), 155–163. http://www.math.unipa.it/~grim/TSG24_ICIMI_Arias-Araya_QRDM_Supl4_09.pdf

Arslan, S., Baran, D., & Okumus, S. (2011). Brousseau's theory of didactical situations in mathematics and an application of adidactical situations. *Necatibey Faculty of Education Electronic Journal of Science and Mathematics Education*, *5*(1), 204–224. https://www.academia.edu/813560/Brousseaus_Theory_of_Didactical_Situations_in_Mathematics_and_An_Application_of_Adidactical_Situations

Bajo-Benito, J. M., Gavilán-Izquierdo, J. M., & Sánchez-Matamoros García, G. (2023). The concept of number sequence in graphical representations for secondary school students. *European Journal of Educational Research*, *12*(1), 159–172. https://doi.org/10.12973/eu-jer.12.1.159

Barham, A. I. (2020). Investigating the development of pre-service teachers' problem-solving strategies via problem-solving mathematics classes. *European Journal of Educational Research*, *9*(1), 129–141. https://doi.org/10.12973/eu-jer.9.1.129

Benavides-lahnstein, A. I., & Ryder, J. (2019). School teachers' conceptions of environmental education: Reinterpreting a typology through a thematic analysis. *Environmental Education Research*, *26*(1), 43–60. https://doi.org/10.1080/13504622.2019.1687649

Bencivenga, R. (2017). The 'digital curious': first steps towards a new typology for mapping adults' relationships with others when using ICT. *European Journal for Research on the Education and Learning of Adults*, *8*(1), 55–76. https://doi.org/10.3384/rela.2000-7426.rela9117

Bronkhorst, H., Roorda, G., Suhre, C., & Goedhart, M. (2021). Student development in logical reasoning: Results of an intervention guiding students through different modes of visual and formal representation. *Canadian Journal of Science, Mathematics and Technology Education*, *21*(2), 378–399. https://doi.org/10.1007/s42330-021-00148-4

Brousseau, G. (2002). *Theory of didactical situations in mathematics*. Kluwer Academic Publishers. https://doi.org/10.1007/0-306-47211-2

Cooper, J. L., & Alibali, M. W. (2012). Visual representations in mathematics problem-solving: effects of diagrams and illustrations. *Proceeding of the 34th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, 281–288. https://files.eric.ed.gov/fulltext/ED584910.pdf

Dockendorff, M., & Solar, H. (2018). ICT integration in mathematics initial teacher training and its impact on visualization: The case of GeoGebra. *International Journal of Mathematical Education in Science and Technology*, *49*(1), 66–84. https://doi.org/10.1080/0020739X.2017.1341060

El Messaoudi, M. (2024). The impact of a blended learning model on undergraduate university students' digital literacy skills: empirical evidence from higher education. *Educational Process International Journal*, *13*(4). https://doi.org/10.22521/edupij.2024.134.5

Fernández-Cézar, R., Prada-Núñez, R., & Pinto, N. S. (2024). Collaborative online international learning: Experiences in higher education. *Educational Process International Journal*, *13*(4). https://doi.org/10.22521/edupij.2024.134.1

Guo, W. (2023). Solving word problems involving triangles by transitional engineering students: learning outcomes and implications. *European Journal of Science and Mathematics Education*, *11*(2), 249–258. https://doi.org/10.30935/scimath/12582

Hadiastuti, D. I., & Soedjoko, E. (2019). Analysis of mathematical representation ability based on students' thinking style in solving open-ended problems. *Unnes Journal of Mathematics Education*, *8*(3), 195–201. https://doi.org/10.15294/ujme.v8i3.34189

Idawati, I., Setyosari, P., Kuswandi, D., & Ulfa, S. (2020). The effects of problem-solving method and cognitive flexibility in improving university students' metacognitive skills. *Journal for the Education of Gifted Young Scientists*, 657–680. https://doi.org/10.17478/jegys.652212

Isnawan, M. G. (2023). *Pola pembelajaran epistemik: Sebuah pemikiran awal untuk pendidikan*. Nashir Al-Kutub Indonesia.

Isnawan, M. G., Alsulami, N. M., & Sudirman, S. (2024a). Optimizing prospective teachers' representational abilities through didactical design-based lesson study. *International Journal of Evaluation and Research in Education (IJERE)*, *13*(6), 4004. https://doi.org/10.11591/ijere.v13i6.29826

Isnawan, M. G., Alsulami, N. M., Yanuarto, W. N., & Sukarma, I. K. (2024b). Exploring the use of mobile technologies in Indonesian mathematics lectures. *Jurnal Matematika Kreatif-Inovatif*, *15*(2), 528–547. https://doi.org/10.15294/s3jcep43

Joubert, J., Callaghan, R., & Engelbrecht, J. (2020). Lesson study in a blended approach to support isolated teachers in teaching with technology. *ZDM-Mathematics Education*, *52*(5), 907–925. https://doi.org/10.1007/s11858-020-01161-x

Jupri, A., & Sispiyati, R. (2021). Symbol sense characteristics for designing mathematics tasks. *Journal of Physics: Conference Series*, *1806*(1). https://doi.org/10.1088/1742-6596/1806/1/012051

Khasanah, K., & Lestari, A. (2021). The effect of Quizizz and learning independence on mathematics learning outcomes. *Tadris: Jurnal Keguruan Dan Ilmu Tarbiyah*, *6*(1), 63–74. https://doi.org/10.24042/tadris.v6i1.7288

Lillo, E. A. G. (2023). Tutors' and students' views on learning and feedback in problem-based learning. *European Journal of Educational Research*, *12*(2), 705–717. https://doi.org/10.12973/eu-jer.12.2.705

Lutfi, J. S., & Juandi, D. (2023). Mathematical representation ability: a systematic literature review. *Union: Jurnal Ilmiah Pendidikan Matematika*, *11*(1), 124–135. https://doi.org/10.30738/union.v11i1.14048

Marfuah, M., Suryadi, D., Turmudi, T., & Isnawan, M. G. (2022). Providing online learning situations for in-service mathematics teachers' external transposition knowledge during Covid-19 pandemic: case of Indonesia. *Electronic Journal of E-Learning*, *20*(1), 69–84. https://doi.org/10.34190/ejel.20.1.2388

Miles, M. B., Huberman, A. M., & Saldana, J. (2014). *Qualitative data analysis: A methods sourcebook*. (3rd ed.). SAGE Publications, Inc. https://id.id1lib.org/book/3593988/83e08f

Mudhofir, A. (2021). Effect of problem-based learning model combination flipped classroom against problem solving ability. *The International Journal of High Education Scientists (IJHES)*, *2*(2), 11–26. www.ijhes.com

Nindiasari, H., Pranata, M. F., Sukirwan, S., Sugiman, S., Fathurrohman, M., Ruhimat, A., & Yuhana, Y. (2024). The use of augmented reality to improve students' geometry concept problem-solving skills through the steam approach. *Infinity Journal*, *13*(1), 119–138. https://doi.org/10.22460/infinity.v13i1.p119-138

Novitasari, P., Usodo, B., & Fitriana, L. (2021). Visual, symbolic, and verbal mathematics representation abilities in junior high school's students. *IOP Conference Series: Earth and Environmental Science*, *1808*(1). https://doi.org/10.1088/1742-6596/1808/1/012046

Nowell, L. S., Norris, J. M., White, D. E., & Moules, N. J. (2017). Thematic analysis: striving to meet the trustworthiness criteria. *International Journal of Qualitative Methods*, *16*, 1–13. https://doi.org/10.1177/1609406917733847

Prabowo, A., Suryadi, D., Dasari, D., Juandi, D., & Junaedi, I. (2022). Learning obstacles in the making of lesson plans by prospective mathematics teacher students. *Education Research International*, *2022*, 1–15. https://doi.org/10.1155/2022/2896860

Putra, F. G., Lengkana, D., Sutiarso, S., Nurhanurawati, N., Saregar, A., Diani, R., Widyawati, S., Suparman, S., Imama, K., & Umam, R. (2024). Mathematical representation: a bibliometric mapping of the research literature (2013–2022). *Infinity Journal*, *13*(1), 1–26. https://doi.org/10.22460/infinity.v13i1.p1-26

Putra, R. W. Y., Sunyono, S., Haenilah, E. Y., Hariri, H., Sutiarso, S., Nurhanurawati, N., & Supriadi, N. (2023). Systematic literature review on the recent three-year trend mathematical representation ability in Scopus database. *Infinity Journal*, *12*(2), 243–260. https://doi.org/10.22460/infinity.v12i2.p243-260

Rahayuningsih, S., Sirajuddin, S., & Ikram, M. (2021). Using open-ended problem-solving tests to identify students' mathematical creative thinking ability. *Participatory Educational Research (PER)*, *8*(3), 285–299. https://doi.org/10.17275/per.21.66.8.3

Rahmawati, D., & Anwar, R. B. (2020). Translation of mathematical representation: Characteristics of verbal representation unpacking. *Journal of Education and Learning (EduLearn)*, *14*(2), 162–167. https://doi.org/10.11591/edulearn.v14i2.9538

Rønning, F. (2021). Opportunities for language enhancement in a learning environment designed on the basis of the theory of didactical situations. *ZDM-Mathematics Education*, *53*(2), 305–316. https://doi.org/10.1007/s11858-020-01199-x

Septiani, D., Riyadi, & Triyanto. (2020). Students' verbal representation abilities in solving geometry problems. *Journal of Physics: Conference Series*, *1594*(1). https://doi.org/10.1088/1742-6596/1594/1/012046

Son, J.-W., & Lee, J.-E. (2016). Pre-service teachers' understanding of fraction multiplication, representational knowledge, and computational skills. *Mathematics Teacher Education and Development*, *18*(2), 5–28. https://eric.ed.gov/?id=EJ1113959

Stieff, M., Scopelitis, S., Lira, M. E., & Desutter, D. (2016). Improving representational competence with concrete models. *Science Education*, *100*(2), 344–363. https://doi.org/10.1002/sce.21203

Sukarma, I. K., Isnawan, M. G., & Alsulami, N. M. (2024). Research on nonroutine problems: A hybrid didactical design for overcoming student learning obstacles. *Human Behavior and Emerging Technologies*, *2024*(Article ID 5552365), 1–12. https://doi.org/10.1155/2024/5552365

Supriadi, S. (2019). Didactic design of sundanese ethnomathematics learning for primary school students. *International Journal of Learning, Teaching and Educational Research*, *18*(11), 154–175. https://doi.org/10.26803/ijlter.18.11.9

Suryadi, D. (2019a). *Landasan filosofis penelitian desain didaktis (DDR)*. Pusat Pengembangan DDR Indonesia.

Suryadi, D. (2019b). *Penelitian desain didaktis (DDR) dan implementasinya*. Gapura Press.

Tossavainen, T., & Faarinen, E. C. (2019). Swedish fifth and sixth graders' motivational values and the use of ict in mathematics education. *Eurasia Journal of Mathematics, Science and Technology Education*, *15*(12). https://doi.org/10.29333/ejmste/108533

Ünal, Z. E., Ala, A. M., Kartal, G., Özel, S., & Geary, D. C. (2023). Visual and symbolic representations as components of algebraic reasoning. *Journal of Numerical Cognition*, *9*(2), 327–345. https://doi.org/10.5964/jnc.11151

Yanuarto, W. N., & Hastinasyah, P. D. (2022). Gamification: Quizizz in mathematical game learning for secondary students. *Indonesian Journal of Mathematics Education*, *5*(2), 64–73. https://doi.org/10.31002/ijome.v5i2.6588

Zainudin, Z. A., & Zulkiply, N. (2023). Gamification in learning: Students' motivation and cognitive engagement in learning business using Quizizz. *Asian Journal of University Education*, *19*(4), 823–833. https://doi.org/10.24191/ajue.v19i4.24928

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