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Author for correspondence:
Yelitza Freitas
yelitzamafreitas@gmail.com
Polytechnic University of Coimbra

Using Serious Games with Exploratory Teaching to Develop Mental Arithmetic in Primary Education

Yelitza Freitas, Beatriz Guiomar, Sofia Gonçalves, Ricardo Pinto, Elisabete Pinto, Virgílio Rato, Fernando Martins

Abstract

Background/purpose. This article aims to analyze the use of serious games from the Hypatiamat Platform to promote mental arithmetic in a first-grade primary school class in an Exploratory Teaching environment.

Materials/methods. This study is mixed, interpretative, and action-research in design. It quantitatively analyses the students' Level of Knowledge and Global Performance and uses multimodal narratives to explain the data in Detail.

Results. The results show that using Serious Games from the Hypatiamat Platform through Exploratory Teaching favored student learning. There was a significant improvement in the students' Level of Knowledge and Global Performance and an increase in student motivation and involvement.

Conclusion. It is concluded that the adequate integration of serious games through Exploratory Teaching created a motivating and challenging learning environment for the students. This approach promoted greater active participation and involvement in 1st-grade primary school students and contributed to a significant improvement in their mental arithmetic development. Serious Games must be integrated into the classroom in an appropriate way to favor the learning of mathematics. We suggest integration through Exploratory Teaching so that students develop essential skills and improve their learning.

1. Introduction

Mental arithmetic is considered one of the basic skills of mathematics (Berticelli & Zancan, 2023), so it is important to be stimulated from an early age (Shavkatovna & Gulbahor, 2021).

Games are renowned for impacting students' motivation, enthusiasm, and ability to generate individual and social activity. Their motivational value has raised the question of how to harness them in teaching and learning (Mohammed et al., 2024; Plass et al., 2015; Schrader, 2023). Schrader (2023) points out that game-based student learning can benefit students' cognitive outcomes, depending on the type of game, the characteristics of the student, and the learning approach generated. Games can be used simply for entertainment or a specific purpose (Becker, 2021).

Using games allows for creating interactive and challenging learning environments where students actively experiment and apply knowledge (Mohammed et al., 2024). It also encourages autonomous learning and allows students to learn through error (Albahiri, 2025). Plass et al. (2015) state that games should aim to make the learning environment inherently attractive. Games have been shown to positively affect students' mathematical performance, motivation, and engagement (Maryana, 2024; Umboh et al., 2021). In the study by Umboh et al. (2021), the Kahoot game application increased students' active engagement, generated good teacher-student and student-student collaboration, and significantly improved learning outcomes. Maryana (2024) found that using a gamified learning platform increased students' engagement during learning and improved their mathematical performance.

Serious Games (SGs) are games created for a specific purpose. Their inclusion in the classroom makes it possible to take advantage of the fun nature of games to facilitate student learning (Schrader, 2023). The Hypatiamat Platform (PH) offers a set of digital artifacts geared toward the mathematics curriculum, including SGs (Pinto et al., 2022). The more than 60 SGs on the platform aim to develop mental arithmetic and mental calculation and promote transversal skills in problem-solving, calculation, memory, and attention (Pinto et al., 2022). This platform is recognized for improving students' results in mathematics (Escaroupa, 2023; Freitas, 2024; Gomes, 2023; Freitas et al., 2024).

Actively using digital artifacts enhances student engagement and leads to improved performance outcomes (Jahnke et al., 2022). Exploratory teaching is an interactive pedagogical method focused on active student learning (Freitas et al., 2024). Freitas et al. (2024) state that this method facilitates the integration of digital artifacts into the classroom. They suggest integrating other digital artifacts to see if this method facilitates integration.

Considering the learning opportunities that SG can offer and the current trend of integrating game-based learning approaches (Schrader, 2023), four SGs from HP were selected. This study, therefore, aims to answer the following research question: How does the use of Serious Games from the Hypatiamat Platform promote the development of mental arithmetic in students in the 1st grade of primary school? To address the research question, we accomplished the following objectives:

i) identify the students' difficulties related to the addition operation;

ii) analyze the influence of Serious Games from the Hypatiamat Platform on the development of mental arithmetic in students in the 1st-grade of primary school.

Liu et al. (2024) referee that primary school students often struggle with calculations. This study was conducted in a first-grade class with mental arithmetic difficulties. In this way, we present an exploratory teaching practice using Serious Games that allows of 1st year of primary school students to develop their mental arithmetic with significant improvements. This practice is a way of integrating Serious Games into the classroom.

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2. Theoretical Background

2.1. Mental arithmetic

Mental arithmetic is how we operate digits to arrive at a result/solution quickly and flexibly (Buys, 2008). Involves performing calculations using one's cognitive skills, relying solely on thought and memory to derive answers without the aid of external tools (Liu et al., 2024). Mental arithmetic encompasses various arithmetic operations, including the arithmetic operation of addition. Berticelli (2017) defines mental arithmetic as the ability to perform arithmetic operations mentally. Students' difficulties in understanding basic mathematical concepts, such as arithmetic, represent an obstacle to their future learning, preventing them from progressing (Fritz et al., 2019; Liu et al., 2024). Heuvel-Panhuizen (2008) states that mental arithmetic is a skillful and versatile calculation based on known numerical relationships and numerical characteristics. It is characterized by how we use the properties of operations and their numerical relationships.

According to DeStefano and LeFevre (2004), mental arithmetic is a well-defined domain governed by explicit rules, facts, and principles. They say that mental arithmetic studies have primarily focused on how single-digit arithmetic (e.g., 2 + 5; 6 x 9) is represented and processed across operations. Liu et al. (2024) referee that multi-digit mental arithmetic is important for how numbers work, how to select appropriate procedures, and how to develop various strategies to solve mathematical problems. Mental arithmetic strategies entail computational procedures without external tools that are important to proficiency and performance in mental calculation (Liu et al., 2024). Mental calculation helps of basic arithmetic facts and enhances one's understanding of mathematical concepts.

The arithmetic operation of addition combines two numbers to form a third, called a sum (Parker & Baldridge, 2004). This operation is defined as let A and B be two disjoint finite sets, if, #A=a e #B=b then, the sum of a with b (a + b) is given by (a + b) =#(A∪B), where a and b are referred to as portions and the number (a + b) is referred to as the sum (Pimentel & Vale, 2004). The meanings of add and joining of addition are called changing together and combining, respectively. The meaning of add corresponds to increased quantity, while the meaning of joining corresponds to the transformation of two added quantities into another quantity (Ponte & Serrazina, 2000).

The process of solving arithmetic problems involves the application of various cognitive processes and strategies (Hubber et al., 2014). In mental arithmetic, students are free to use the strategies they consider most effective when solving a problem (Buys, 2008).

Simple calculations, such as those involving numbers up to 20, often rely on the use of memorization strategies and retrieval of number facts, while solving more complex problems, such as those involving two digits, requires mental calculation strategies, such as decomposition or counting (Beishuizen et al., 1997; Hubber et al., 2014). The decomposition strategy and the counting strategy play a central role in the addition of two-digit numbers. Taking the arithmetic operation of addition as an example, the adder and the added are worked on separately in decomposition. The adder is broken down into tens and units and then added, order by order, to the adder (e.g., 26+12 is determined from 20+10=30 and 6+2=8, answering 30+8=38). The tens are counted by adding or subtracting them from the addition (e.g., 26+12 is determined from 26+10=36, 36+2=38) (Beishuizen et al., 1997).

2.2. Serious Games

The game is defined by Salen et al. (2004) as a structured environment defined by predefined rules where participants engage in simulated conflicts that culminate in quantifiable results. It is recognized for its motivational function and ability to motivate students to remain engaged in tasks for extended periods (Plass et al., 2015). In game-based learning, the interactivity of games is used

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to enhance learning and skills development. This approach generates collaboration between students and games and between students and teachers (Hu, 2024).

The SGs go beyond the pure entertainment characteristics of games and include both digital and non-digital games. However, the increasing development of technology has oriented SG towards a digital point of view (Schrader, 2023). SGs are games that have a specific purpose. They are designed for learning purposes (Jääskä & Aaltonen, 2022). The SGs are interactive learning tools that provide educational experiences through animated elements that students can manipulate and interact with. Their purpose focuses on harnessing the fun nature of games to facilitate learning in the development of new knowledge and essential skills (Schrader, 2023). The SGs include incentive systems such as stars, points, badges, and trophies that arouse students' interest and satisfy their recognition needs (Albahiri, 2025; Plass et al., 2015). The immediate feedback games provide can encourage students to reflect on their learning (Schrader, 2023). Games can help guide students through educational tasks. However, it is important to know that using games casually can distract students from learning (Plass et al., 2015). In this sense, the teacher is decisive in including this digital artifact in the teaching and learning processes (Umboh et al., 2021). When handling digital artifacts, it is important for students to reflect and construct knowledge (Clements & McMillen, 1996), and they are used as epistemic tools (Costa et al., 2021; Lopes & Costa, 2019). When digital artifacts are used as epistemic tools, the quality of student learning increases (Silva et al., 2021). It should be noted that learning can benefit even more when digital artifacts are used with other epistemic artifacts, such as an exploration guide (Lopes & Costa, 2019).

Schrader (2023) states that the term Serious Games is often referred to as a synonym for Game-based learning. In a Game-based learning environment, learning and skill development occur through the intentional use of games (Becker, 2021; Mohammed et al., 2024; Hu, 2024). In this type of environment, learning can occur in three ways: through games, with games, or by creating games. In learning through games, games are used to teach a specific curriculum content (Compto, 2023). Learning with games occurs when games are used as an example to teach concepts and methods (Compto, 2023). When handling digital artifacts, students need to reflect and construct knowledge (Clements & McMillen, 1996), and they are used as epistemic tools (Costa et al., 2021; Lopes & Costa, 2019). When digital artifacts are used as epistemic tools, the quality of student learning increases (Silva et al., 2021). It should be noted that learning can be beneficial even more when digital artifacts are used with other epistemic artifacts, such as an exploration guide (Lopes & Costa, 2019). Learning through game creation occurs when students are challenged to develop games to learn about the content in the game creation (Compto, 2023). This approach aligns with the idea that learning through digital artifacts should be balanced by teachers to facilitate and promote interesting and meaningful learning for students (Umboh et al., 2021). The integration of technology into teaching and learning processes has made this process more student-centered, with teachers taking on the role of learning facilitators. This has prompted studies aimed at creating different teaching and learning approaches aimed at improving student performance and increasing active participation (Mohammed et al., 2024). Epistemic appreciation also plays an important role in student participation (Rodrigues et al., 2022).

2.3. Exploratory Teaching

Exploratory Teaching is a student-centered teaching approach in which students work actively and autonomously on challenging mathematical tasks (Canavarro et al., 2012; Jesus et al., 2020). An Exploratory Teaching lesson is structured in four phases, according to Canavarro et al. (2012): Introduction of the task, Development of the task, Discussion of the task, and Systematization of mathematical learning. In the Introducing the task phase, the teacher proposes the task to the students. In this phase, students must understand the context and objectives of the task (Canavarro et al., 2012) so that they can be autonomous during the next phase (Canavarro et al., 2012). In the

Task Development phase, students usually work in groups, while the teacher monitors their learning. During this monitoring, the teacher must be careful not to validate the correctness of the students' strategies or answers (Canavarro et al., 2012), guiding them through questions (Canavarro et al., 2012; Guerreiro et al., 2015). According to Narváez and Cañadas (2023), the teacher's mediation plays an important role in students' learning, motivating them to respond, justify, correct mistakes, and overcome difficulties. Also, in this phase, the teacher must select the students' resolutions that will be explored in the next phase. The Task Discussion phase focuses on the collective sharing and discussion of the different resolutions selected in the previous phase. This phase is important for understanding mathematical content and producing knowledge (Freitas et al., 2024; Guerreiro et al., 2015; Ponte, 2017). In the study by Gomes et al. (2023), we observed that the collective discussion was an important moment for sharing different resolution strategies and contributing to the construction of new meaningful knowledge. Freitas et al. (2025) found that the TT primarily adopted questioning actions and requests for clarification. In the Systematization of the mathematical learning phase, the teacher adopts a more directive role, seeking to guide this moment to systematize the content worked on during the lesson (Canavarro et al., 2012). This phase is also essential for constructing mathematical knowledge (Guerreiro et al., 2015).

3. Material and Methods

3.1. Study Design

This study is of a mixed nature (Creswell & Clark, 2018), interpretive (Amado, 2017), and action research design (Bogdan & Biklen, 2013). In this way, data collection and analysis used techniques and instruments characteristic of quantitative and qualitative research (Cohen et al., 2018). Qualitative data was used to explain quantitative data in detail by describing and interpreting the data in detail (Amado, 2017).

3.2. Participants

The study participants 14 students from a mixed class in an elementary school in Portugal. The class had 7 female and 7 male students (aged between six and seven). The data was collected as part of an internship carried out by a teacher trainee (TT) on a teacher training course at a higher education institution in Portugal. During the sessions of the pedagogical intervention, the students were grouped into seven pairs. The implementation of the research had permission and parental approval obtained, as well as the consent of all students and educational entities involved. This study was conducted following the Declaration of Helsinki and approved by the Ethics Committee of Instituto Politécnico de Coimbra (reference: 101 CEIPC/2022 approved on June 24, 2022).

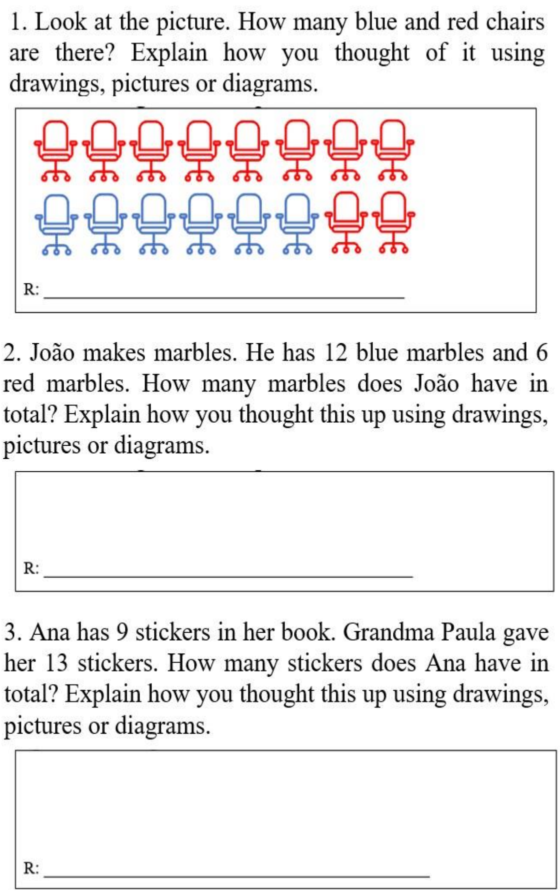
3.3. Data Collection

The data collection took place between March and April 2022, spanning three phases: pre-intervention, intervention, and post-intervention phases, as illustrated in Table 1. All data collected (written records, photographs, audio recordings) were used exclusively for this study, following the acquisition of informed, freely given and clarified consent from all Guardians of Education and authorization from the School Group. Furthermore, the anonymity of all involved participants was maintained in strict compliance with the Declaration of Helsinki and approved by the Ethics Committee of the Polytechnic Institute of Coimbra (reference: 101 CEIPC/2022, approved on June 24, 2022). The anonymity of the participants was maintained in strict compliance with the Declaration of Helsinki and approved by the Ethics Committee of the Polytechnic Institute of Coimbra (reference: 101_CEIPC/2022 approved on June 24, 2022).

Table 1. Research sessions

Pre-intervention phase	**Intervention phase**				**Post-intervention phase**
Initial Task	Session 1	Session 2	Session 3	Session 4	Final Task
March 8	March 14	March 21	March 28	April 4	April 5

In the pre-intervention and post-intervention phases, initial and final tasks were administered (Figures 1 and 2), aimed at obtaining feedback on students' knowledge regarding the arithmetic operation addition through the assessment of tasks (Andrade, 2000; Brookhart & Chen, 2015). The initial and final tasks were constructed by the research team. Task 1 involved counting elements. Task 2 involved addition in the meaning of adding, and task 3 in the meaning of adding.



1. Look at the picture. How many blue and red chairs are there? Explain how you thought of it using drawings, pictures or diagrams.

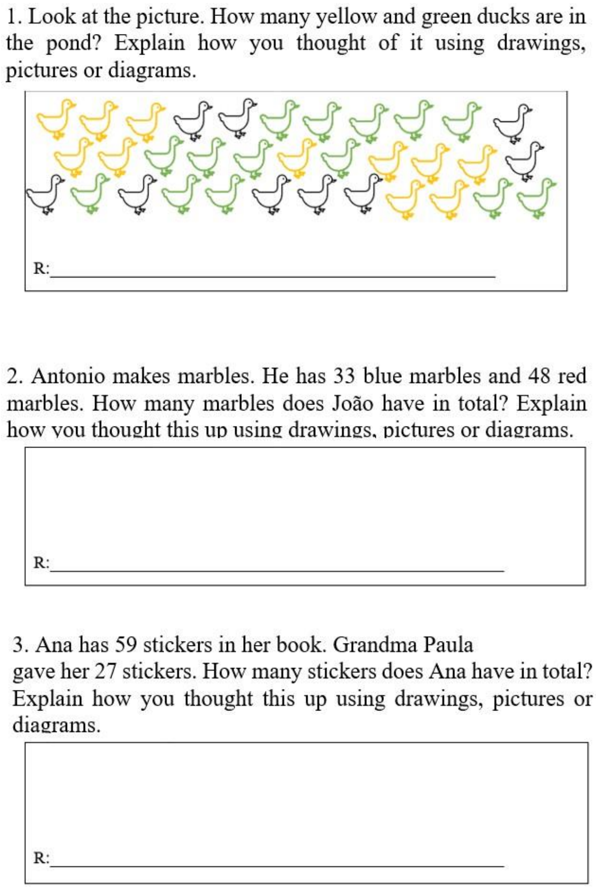
R: ______________________

2. João makes marbles. He has 12 blue marbles and 6 red marbles. How many marbles does João have in total? Explain how you thought this up using drawings, pictures or diagrams.

R: ______________________

3. Ana has 9 stickers in her book. Grandma Paula gave her 13 stickers. How many stickers does Ana have in total? Explain how you thought this up using drawings, pictures or diagrams.

R: ______________________



1. Look at the picture. How many yellow and green ducks are in the pond? Explain how you thought of it using drawings, pictures or diagrams.

R: ______________________

2. Antonio makes marbles. He has 33 blue marbles and 48 red marbles. How many marbles does João have in total? Explain how you thought this up using drawings, pictures or diagrams.

R: ______________________

3. Ana has 59 stickers in her book. Grandma Paula gave her 27 stickers. How many stickers does Ana have in total? Explain how you thought this up using drawings, pictures or diagrams.

R: ______________________

Figure 1. Pre-intervention phase **Figure 2.** Post-intervention phase

The evaluation of the initial and final tasks was based on general criteria (Table 2) and specific descriptions (Table 3) constructed by the research team, considering the mathematical content involved in the tasks. The use of rubrics aimed to identify the students' difficulties in order to build an action plan based on the difficulties identified (Brookhart & Chen, 2015). According to Panadero et al. (2023), using specific criteria increases the reliability of the assessment. This evaluation using rubrics also aimed to analyze the students' progress before and after the intervention (Petkov & Petkova, 2006).

These general criteria and specific descriptions were based on Andrade (2000), Escaroupa (2022), and Freitas et al. (2024) were validated by three PhD professors. The evaluation of the initial tasks made it possible to identify the discrepancy levels of each student and group them into seven pairs, according to the conditions of the Zone of Proximal Development (Vygotsky, 1980). This resulted in the formation of level 1 and 2 pairs, level 2 and 3 pairs, and level 3 and 4 pairs.

Table 2. Criteria with four levels of knowledge

Level 1 (L_1)	Level 2 (L_2)	Level 3 (L_3)	Level 4 (L^4)
The resolution does not demonstrate knowledge of the mathematical concepts involved or does not answer.	The resolution demonstrates limited knowledge of the mathematical concepts involved and contains many inaccuracies.	The resolution shows some knowledge of the mathematical concepts involved and contains some inaccuracies.	The resolution demonstrates a complete knowledge of the mathematical concepts involved in the task.

Table 3 illustrates the specific descriptions adapted to the mathematical content of the tasks. The specific descriptions focused on three specific objectives for each task. Task 1 has the following objectives: Recognize the need to count (objective 1); Identify that the order in which objects are listed is irrelevant (objective 2); Count (objective 3); Identify the result (objective 4). These objectives aimed to improve the students' mathematical knowledge of the arithmetic operation of addition. The objectives of tasks 2 and 3 are to understand the meaning of addition (objective 1), Recognize the added and the adder in the context of the task (objective 2), Perform addition (objective 3), and identify the result operation in the context of the task (objective 4).

Table 3. Specific descriptions (task 1 involves counting, and tasks 2 and 3 involve the adding and adding senses of addition, respectively)

Tasks	LK	Objective 1	Objective 2	Objective 3	Objective 4
1	L_1	Not presenting a solution or presenting a solution that has nothing to do with the problematic situation.	Not presenting a solution or presenting a solution that has nothing to do with the problematic situation.	Not presenting a solution or presenting a solution that has nothing to do with the problematic situation.	Not presenting a solution or presenting a solution that has nothing to do with the problematic situation.
	L_2	Recognizing the need to count, making many mistakes or showing extreme difficulty in completing the task.	Partial counting, showing serious errors.	Demonstrate that you want to count, even if you can't.	Developing a response that is partially inadequate to the problematic situation
	L_3	Recognizing the need to count, showing some understanding of the pre-numerical activity, making mistakes or showing difficulties in completing the task.	Present the count, showing errors in the representation of the calculation.	Count the two quantities, even though it is not shown correctly.	Come up with an appropriate response to the problematic situation, even if the result is incorrect.
	L_4	Using counting, through verbal, symbolic or visual representations, as evidence of their understanding of the meaning of counting.	Present the count correctly, bearing in mind that the order of counting does not influence the result.	Count correctly and get the result.	Come up with a correct answer according to the statement of the problem situation, or just indicate the result.
		Objective 1	Objective 2	Objective 3	Objective 4
2 & 3	L_1	Not presenting a solution or presenting a solution that has nothing to do with the problematic situation.	Not presenting a solution or presenting a solution that has nothing to do with the problematic situation.	Not presenting a solution or presenting a solution that has nothing to do with the problematic situation.	Not presenting a solution or presenting a solution that has nothing to do with the problematic situation.
	L_2	Recognize the need to use the addition operation, making many mistakes or showing extreme difficulty in completing the task.	Partially presenting addition, showing serious errors in the representation of the calculation.	Demonstrate that you want to do the addition, even though you can't do it.	Presenting a partially inadequate response to the problematic situation.
	L_3	Recognize the need to use the addition operation, showing some understanding of the meaning of adding, making mistakes or having difficulties in completing the task.	Presenting addition, showing errors in the representation of the calculation.	Carrying out the addition, with some errors in the calculation.	Present the answer properly, even if the result is incorrect.
	L_4	Using verbal, symbolic or visual representations of the addition operation, showing an understanding of the meaning of addition involved.	Present the addition correctly.	Solve the addition correctly and get the result.	Come up with a correct answer according to the problem statement or indicate the result.

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The evaluation of the initial and final tasks made it possible to analyze the students' Level of Knowledge (LK) and Global Performance (GP), taking into account the specific descriptors. First, the median for each task was calculated, followed by the median of the medians for each task, the latter being used to assess the students' Global Level of Knowledge (GLK). The LK was defined by calculating the median for each task, and the GLK was calculated using the median of the medians for each task. The student's GP was obtained based on the percentage values assigned to each specific objective in each task (Table 4). The final value of the students' GP was calculated by adding the percentage values obtained in each task. These final values were then organized into classes: [0; 25[, [25; 50[, [50; 75[and [75; 100].

Table 4. Percentage values for each specific objective of each task

Tasks	Percentage values	LK	Objective 1	Objective 2	Objective 3	Objective 4
1	20%	1	0%	0%	0%	0%
		2	1%	1%	2%	1%
		3	3%	3%	6%	3%
		4	4%	4%	8%	4%
2	40%	1	0%	0%	0%	0%
		2	2%	2%	6%	2%
		3	5%	5%	17%	5%
		4	6%	6%	18%	6%
3	40%	1	0%	0%	0%	0%
		2	2%	2%	6%	2%
		3	5%	5%	17%	5%
		4	6%	6%	22%	6%

Written records, audio recordings, and photographs were also collected throughout the intervention phase. These data enabled the construction of Multimodal Narratives (MN), following the protocol of Lopes et al. (2018).

3.4. Pedagogical intervention

In the intervention phase, access to HP was supported by a guide to accessing the platform. Each session followed the Exploratory Teaching approach (Canavarro et al., 2012) and was organized into four phases: Introduction of the task, Exploration of the SGs and carrying out the task, Discussion of the task, and Systematization of mathematical learning. The four sessions lasted one and a half hours each. Throughout the intervention, the students worked in pairs with computers while the TT circulated the room to monitor the students' work and guide them whenever necessary (Canavarro et al., 2012; Guerreiro et al., 2015). In the introductory phase of the task, TT demonstrated the steps to be taken by the students to access the SGs on the classroom projector and clarified essential aspects of the game. This was to ensure that the students understood so that they would be autonomous in the next phase (Canavarro et al., 2012). During the phase of exploring the SGs and carrying out the task, the TT circulated the room to monitor and clarify the students' doubts. In the four sessions, four SGs of HP were explored, as shown in Table 5.

Table 5. Serious Games explored in the four sessions

Intervention phase			
Session 1	Session 2	Session 3	Session 4
Game 10	Addition wheel (levels 1 and 2)	Ball Add	Addition game

Game 10 (Figure 1) aims to compose and decompose the number 10, improve mental arithmetic, develop attention, concentration, and observation, make sums equal to 10 in executive squares horizontally and/or vertically, and, finally, make sums with two more parts. To get sums equal to 10, you must move the numbers in the squares.

Figure 1. Game 10 (Source: Hypatiamat platform)

The Addition wheel (Figure 2) consists of six levels. The objectives are to make sums up to 10 and 20, with two parts, compose and decompose numbers, and develop attention, observation, and concentration. Here, we must insert the sum corresponding to the addition operation generated by the wheel.

Figure 1. Addition wheel (Source: Hypatiamat platform)

Ball Add (Figure 3) aims to add up to 20, add two or more parts, make up numbers from 8 to 20, and develop attention, observation, and concentration. During the game, we must get the sum indicated in a row or column (from 8 to 20) by directing the ball toward the desired column.

Figure 3. Ball Add (Source: Hypatiamat platform)

The Addition game (Figure 4) consists of two levels: "addition with natural numbers" and "addition with whole numbers". It also has two game modes: "practice" and "play". The first level aims to do sums up to 18, develop and improve mental arithmetic, develop attention, concentration observation, and the composition and decomposition of a number. You must eliminate all the numbers on the board. At each step, we must click on two numbers, which, using addition, allows us to obtain the result indicated.

Figure 2. Addition game (Source: Hypatiamat platform)

The first session aimed to work on “Friends of ten,” doing sums equal to ten in executive squares horizontally and/or vertically. The main aim of the second session was to do sums up to 10 and 20, with two parts. The main objective of the third session was to add up to 20 with two or more parts. Finally, the fourth session aimed to do sums up to 18 in a given time.

Once the game had been explored, each pair was given an exploration guide, and the aim of the task was explained. The exploration script (Figure 5) contained instructions for accessing the game and a table where each pair had to indicate the addition shown on the game screen and explain the mathematical reasoning used.

While carrying out the task, the students recorded the addition operation involved in the game on the exploration guide. They explained how they thought of it using diagrams, drawings, or words.

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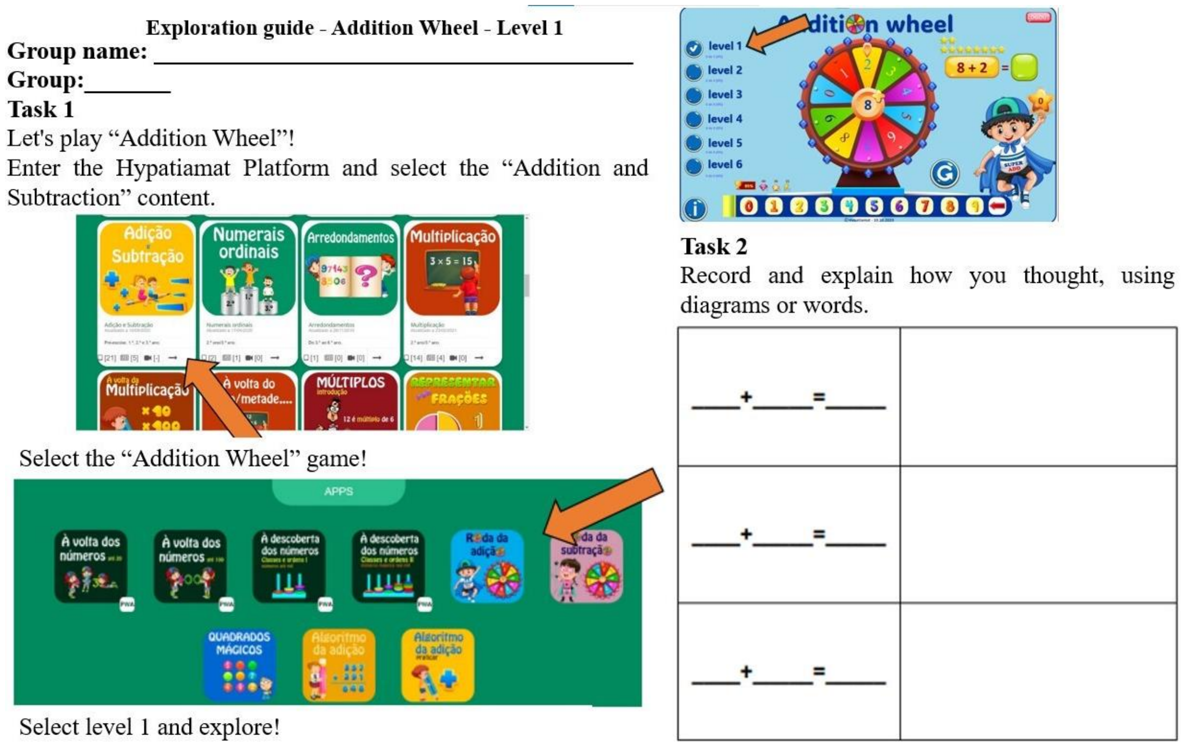

Figure 3. Exploitation guide (Source: Authors' own elaboration)

Finally, a summary of the mathematical results obtained was made in the mathematical learning systematization phase. In this phase, the EP emphasized the importance of mathematical knowledge and worked on the difficulties identified out in the pre-intervention phase, such as distinguishing sum from parcel.

3.5. Statistical analysis

Statistical analysis was carried out using descriptive statistics based on the description of the students' LK, GLK, and GP obtained in the pre-intervention and post-intervention phases. LK, GLK, and GP were characterized using frequency tables, mean (M), and standard deviation (SD). Levels 1 and 2 were considered for the negative trend, and levels 3 and 4 for the positive trend of the students' LK and GLK. As for the GP, we considered the intervals [50; 75[and [75; 100] as a positive trend and the intervals [0; 25[and [25; 50[as a negative trend.

The Student's t-test for paired samples was used to compare the GLK and GP obtained in the pre-intervention and post-intervention phases after validating its assumption (Field 2018; Marôco, 2021). The assumption of normality for each dependent variable was assessed using the Shapiro-Wilk test (Field, 2018; Marôco, 2021). In cases where normality was not verified, symmetry was analyzed using the following conditions (Pestana & Gageiro, 2020); Tabachnick et al., 2021):

$$\left|\frac{skewness\ coefficient}{error\ of\ the\ sckewness\ coefficient}\right| \leq 1.96$$

The effect size of the Student's t-test for paired samples is obtained using Cohen's d, and the effect size was classified as follows (Marôco, 2021; Pallant 2020): small ($d \leq 0.2$), medium ($0.2 < d \leq 0.5$), high ($0.5 < d \leq 0.8$) and very high ($d > 0.8$).

All statistical analyses were carried out using IBM SPSS Statistics software (version 28, IBM USA), with a significant level of 5%.

4. Results

4.1. Quantitative results

4.1.1. Levels of Knowledge

Table 6 shows the absolute and relative frequency distribution of the students' LK per task and the students' Global Knowledge Level. The table shows the students' LK evolution in all the tasks. In Task 1 (T1), students had to count elements. Table 2 shows that in T1, 85.7% of the students were in the negative levels (levels 1 and 2) before the intervention. After the intervention, 92.8% of the students were at the positive levels (levels 4 and 5). In Task 2 (T2), students had to perform addition in the sense of adding. Table 4 shows that in this task, 47.2% of the students were at negative levels, and after the intervention, there was a reduction in negative levels to 21.4%. In Task 3 (T3), students had to understand the meaning of addition and solve the operation. Table 2 shows that the majority of students (64.2%) are at negative levels, with only 35.7% of students at positive levels. After the intervention, there was a significant reduction in negative levels and an increase in positive levels, with 85.7% of the students. It should also be noted that in all the tasks, there were no students at level 1 and a low percentage of students at level 2 after the intervention.

Table 6. Distribution of absolute (*n*) and relative frequencies (%) of LK

	Pre-intervention				Post-intervention			
Task	L_1	L_2	L_3	L_4	L_1	L_2	L_3	L_4
	Negative		Positive		Negative		Positive	
T1	14.3% (2)	71.4% (10)	7.1% (1)	7.1% (1)	0% (0)	7.1% (1)	35.7% (5)	57.1% (8)
T2	14.3% (2)	42.9% (6)	35.7% (5)	7.1% (1)	0% (0)	21.4% (3)	28.6% (4)	50.0% (7)
T3	7.1% (1)	42.9% (6)	35.7% (5)	14.3% (2)	0% (0)	7.1% (1)	28.6% (4)	64.3% (9)
GLK	7.1% (1)	57.1% (8)	35.7% (5)	0% (0)	0% (0)	14.3% (2)	35.7% (5)	50.0% (7)

Table 7 shows statistically significant differences (t (13) = - 5.980; p = 0.001; d = 1.374; effect size - very high) in the students' GLK between the pre- and post-intervention phases. There was a significant increase between the mean value in the pre-intervention phase (2.39) and the post-intervention phase (3.36). Initially, the majority of students (13 students) were at level 2 and after the intervention they progressed to a level of 3 or higher. This means that the students, taking into account the criteria defined, show an improvement in the development of mental arithmetic.

Table 7. Descriptive statistics and pre- and post-intervention comparison in GLK

	M	*SD*	*t*	*p*	*d*	*Effect size*
Pre-intervention	2.39	0.6557	- 5.980	0.001	1.374	*Very high*
Post-intervention	3.36	0.745				

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4.1.2. Global performance

Table 8 compares the distribution of absolute and relative frequencies of students' GP between the pre-and post-intervention phases. There was considerable improvement in the students' GP after the intervention, with no students at the negative levels. Before the intervention, 35.6% of the students had a negative score of less than 50%. After the intervention, all the students scored above 50%, with 100% of the students in the positive levels.

Table 8. Distribution of absolute and relative frequencies of the GP

	Pre-intervention	Post-intervention
[0;25[	7.1% (1)	0% (0)
[25;50[	28.6% (4)	0% (0)
[50;75[	42.9% (6)	35.7% (5)
[75;100]	21.4% (3)	64.3% (9)

Table 9 shows statistically significant differences in the students' average GP results in all the tasks, with a medium effect size in T2 and very high effect sizes in T1 and T3.

Table 9. Descriptive statistics and pre-intervention and post-intervention comparison of the GP

		M	*SD*	*t*	*p*	*d*	*Effect size*
T1	Pre-intervention	7.83	0.35	-6.02	0.001	2.286	*Very high*
	Post-intervention	16.21	5.18				
T2	Pre-intervention	24.64	11.45	-1.63	0.128	0.462	Medium
	Post-intervention	29.64	10.16				
T3	Pre-intervention	24.86	12.48	-3.61	0.003	1.250	*Very high*
	Post-intervention	36.64	4.67				
DG	Pre-intervention	57.64	22.70	-5.05	0.001	1.178	*Very high*
	Post-intervention	81.21	16.89				

This positive development is evident in the strategies used by the students during the tasks carried out in the pre- and post-intervention phases, as shown below. Excerpts from the NM will also be presented, showing in detail the students' reasoning, the EP's actions, and the moments in the lesson that may have contributed to this positive evolution.

4.2. Qualitative results Pre-intervention phase - Tasks 1

4.2.1. Pre-intervention phase - Tasks 1

Figure 4 shows Student 1's (A1) solution to task 1 of the pre-intervention phase. In this task, the student had to count elements, explaining how he counted. The student gave an answer to the problem situation and did not explain his reasoning.

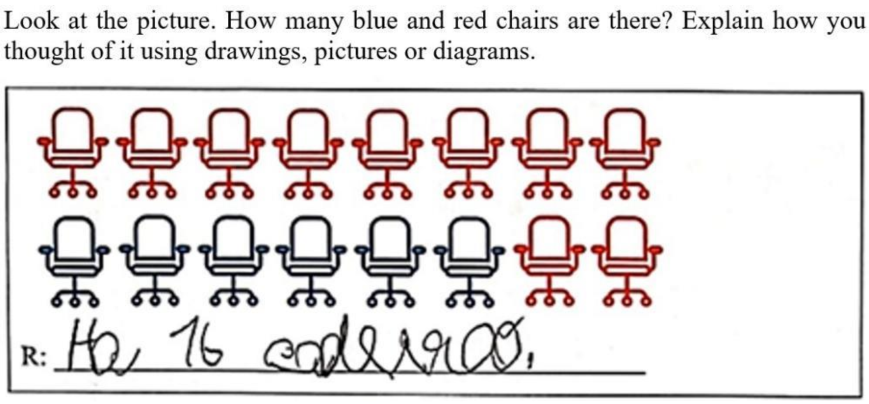

Figure 3. S1's solution to task 1, in the pre-intervention phase

4.2.2. Post-intervention phase - Tasks 1

In the post-intervention phase, his resolution shows that he counted using dots and presented the correct result.

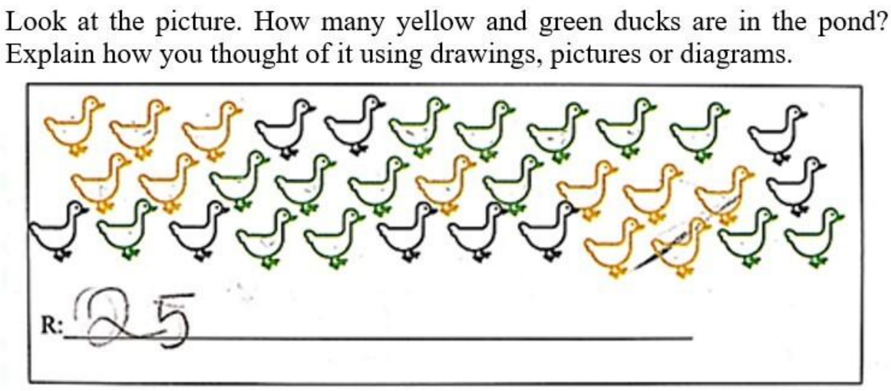

Figure 5. S1's solution to task 1, in the post-intervention phase (Source: Authors' own elaboration)

4.2.3. Pre-intervention phase - Tasks 2

Figures 6 and 7 show S1's and student 2's (S2) resolutions in the pre-intervention phase. In this task, the students had to perform addition in the join direction. S1 gives the correct result but does not show how he made the addition. S2 presents an incomplete solution, with difficulties in correctly completing the task. In the pre-intervention phase, students were at level 2 according to the established criteria.

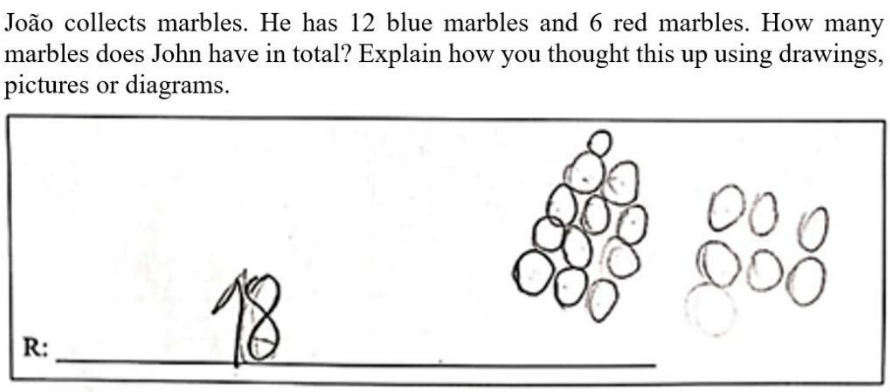

Figure 4. S1's solution to task 2 in the pre-intervention phase

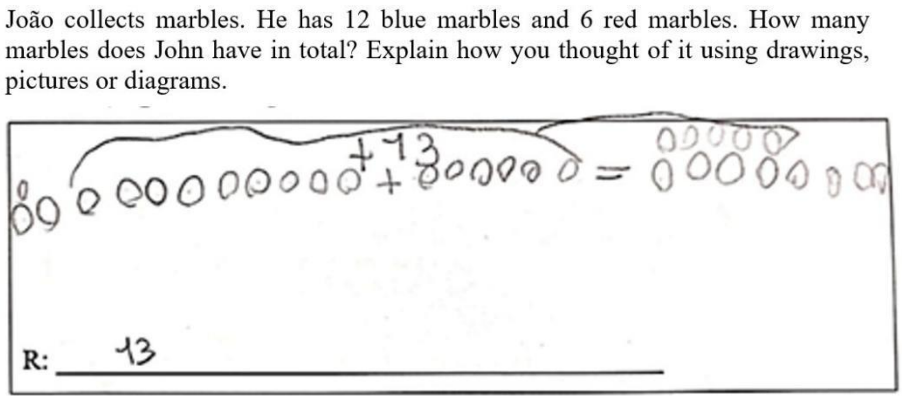

Figure 5. S2's solution to task 2 in the pre-intervention phase (Source: Authors' own elaboration)

4.2.4. Post-intervention phase - Tasks 2

In the post-intervention phase, students S1 and S2 presented the solutions shown in Figures 8 and 9, showing that the students could use the addition operation. A1 presented the addition operation correctly, used a visual representation, and presented the correct result at level 4 according to the established criteria. S2 also presented a visual representation and the correct result; however, he did not correctly present the order of the addition parts at level 3 according to the established criteria.

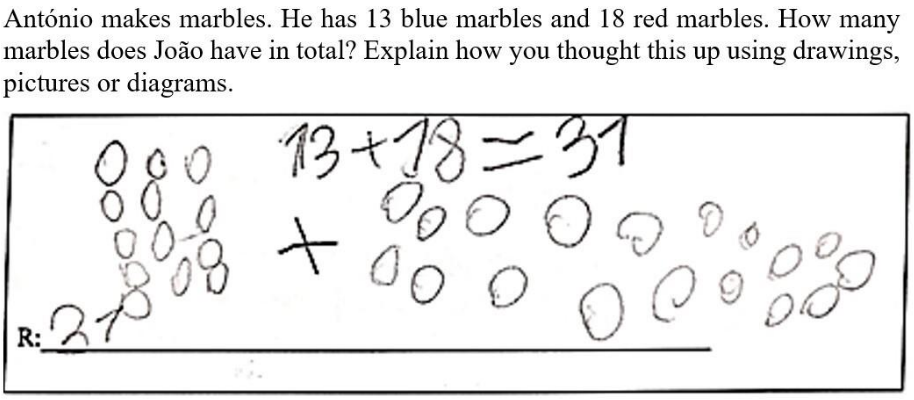

Figure 6. S1's solution to task 2 in the post-intervention phase

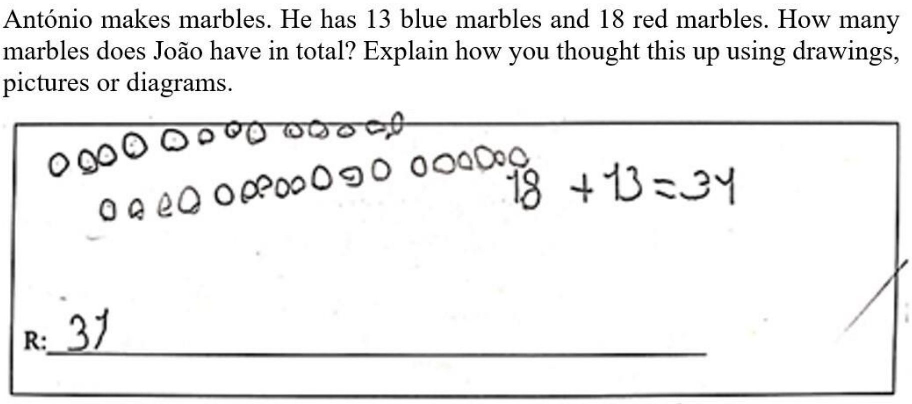

Figure 7. S2's solution to task 2 in the post-intervention phase (Source: Authors' own elaboration)

4.2.5. Pre-intervention phase - Tasks 3

In Task 3, Student 3's (S3) resolution in the pre-intervention phase (Figure 10) shows that he used a visual representation, demonstrating that he does not understand the meaning of addition. This resolution places the student at level 3 according to the established criteria.

Ana has 9 stickers in her book. Grandma Paula has given her 13 stickers. How many stickers does Ana have in total? Explain how you thought using drawings, pictures or diagrams.

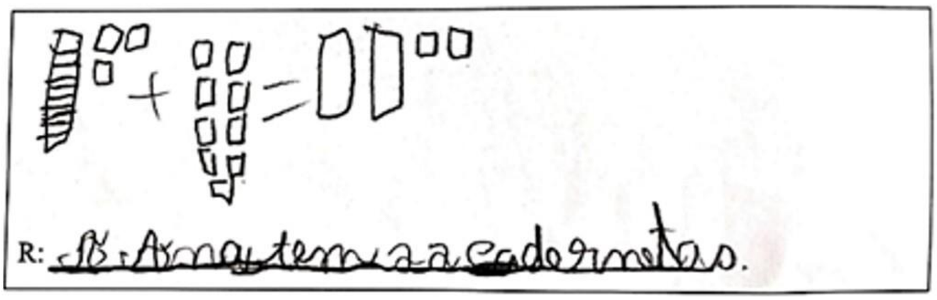

Figure 8. S3's solution to task 3 in the pre-intervention phase

4.2.6. Post-intervention phase - Tasks 3

After the intervention, the student uses the symbolic representation of the addition operation, correctly representing the addend, the adder, and the result. According to the criteria established, this places him at level 4 (Figure 11).

Ana has 19 stickers in her book. Grandma Paula gave her 17 stickers. How many stickers does Ana have in total? Explain how you thought this up using drawings, pictures, or diagrams.

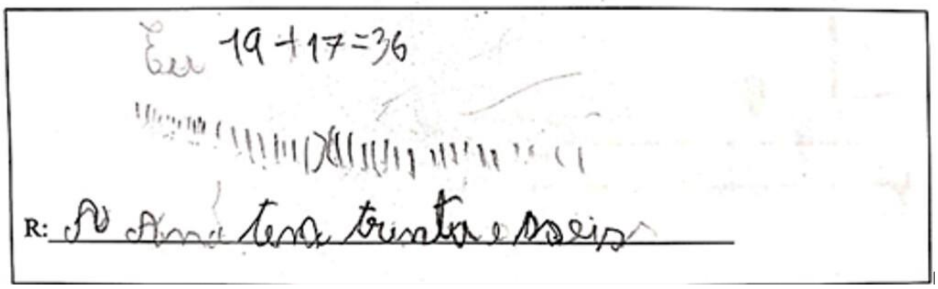

Figure 9. S3's solution to task 3 in the post-intervention phase (Source: Authors' own elaboration)

4.2.7. Intervention phase

The following excerpts from the MN show aspects of the intervention that contributed to this positive development.

Throughout the intervention, the students were challenged to perform calculations quickly and flexibly when using the SGs throughout the intervention. In addition, TT also encouraged them to do so during the systematization phase of mathematical learning, as the following excerpt shows. In the same excerpt, we see the involvement and active participation of students 2 and 3 (S2 and S3) and student 4 (S4) and TT's epistemic appreciation of the students' answers.

TT: Student 3, tell me a friend of ten. Fast, very fast, very fast!

S3: Two plus eight.

TT: Very good! (PE snaps her fingers) Student 4, tell me a friend of ten!

S4: A friend of ten?

TT: A friend of ten. Student 3 said two plus eight.

S4: Nine plus one.

TT: Nine plus one, very good! Student 5, tell me a friend of ten.

S4: Five plus five.

TT: Very good!

In the following excerpt, we can also see the involvement of the students when they do quick additions on the friends of 10.

PE: How many dots do I have here? (pointing to the first part of the operation 6 + 4 written on the board)

Students: Six.

PE: And here I have? (pointing to the second part of the operation 6 + 4 written on the board)

Students: Four.

Trainee teacher: It's different. (2-second pause) Calm down, calm down! But we still have to find two more (referring to the operations whose sum is equal to ten)

S5: Seven plus three and three plus seven.

PE: Student F.

S4: That's it.

PE: Student F, Student J (7-second pause as the teacher draws the students' attention) Student F, tell me another friend of ten. (2-second pause)

S2: Hi... eight.

PE: Okay. Eight plus two, it can't be that, but it could be something else, but it could be the other way around.

S2: Seven plus three.

In addition to encouraging fast and flexible calculation, different forms of representation were explored during the intervention. The following excerpt shows that student 1 (S1) was involved in sharing representations during the discussion phase of the task.

TT: Ten's friends. We've discovered a few here. Eight plus two, five plus five, six plus four and we can represent them in various ways. We can use the... (2-second pause) as?

S1: A line.

TT: The line. We can make it out of... we can draw pictures, or we can make lines, for example. (pause) There are various ways of drawing.

Still, in the discussion phase, it is possible to observe Student 3's participation in understanding the meaning of addition. In the following excerpt, we see that the student answered correctly, meaning that he understood the meaning of addition being worked on. Also, in this excerpt, you can see how student 3 collaborated during the guidance given by the TT to student 6 (S6).

TT: Well, if it is four, I have four fingers, and I add zero more fingers. How old am I? I have four fingers, plus... I have four fingers, I do not add... more, I add zero fingers, don't add... add any fingers?

S3: No.

TT: So, how many fingers am I left with?

S6: Four.

TT: So, four plus zero equals?

S6: Five.

TT: Ouch!

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S3: It is four.

During the discussion, TT encouraged the students to distinguish parcels from sums, as shown below.

PE: Very good! What do we call these two numbers (pointing to the adder and the added)? Sums or portions? (3-second pause) Par...?

S3: ...cels. (completing what trainee teacher A was saying)

Trainee teacher A: Parcels. So here (pointing to the addend and the adder) we have the par...?

S7: ...cels. (completing what trainee teacher A was saying)

It is also possible to identify moments in the discussion where the TT questioned the students about the distinction between the parts and the sum of the addition, as seen in the following excerpt, with the participation of student 7 (S7).

TT: I have got eight plus eight, what is the sum?

S7: Sixteen.

TT: And the parts?

S7: Eight and eight.

TT: Very good!

In the excerpts above, it can be seen that during the discussion TT adopted a mediating role by asking questions.

In the following excerpt, referring to the phase of exploring the SGs and carrying out the task, we can see that S2 found it difficult to correctly carry out the addition operation. However, the TT's mediation of S2 during the use of the SGs encouraged him to think and use the decomposition strategy to build knowledge.

TT: So, how much is it? Eight plus eleven?

S2: Twenty-one. (2-second pause)

TT: Are you sure? (pauses for 3 seconds) So let us think, let's think. Now, I've got, I've got what? I've got eight plus eleven, I'm not going to touch eleven. The eight, how much do I make... (TT corrected herself) eleven equals how much? Ten plus? (3-second pause)

S2: Eight.

TT: Eleven equals what? Ten more?

S2: Eleven.

TT: Ten more?

S2: Eleven.

TT: One. How much do I make eleven? (referring to the operation 10 + 1) A4?

S4: It's one and eleven.

TT: Let's think here. Eleven equals what? Ten plus?

S2: One.

TT: Very good! So, look, I've got ten plus one, what if I do now, one plus eight? How much is that?

S2: Nine.

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TT: Nine! And I've still got ten (referring to the fact that number ten has yet to be added) So what about ten plus nine?

S2: Ten plus nine?

TT: That's how much? (2-second pause) Deza....

S2: Nineteen.

TT: That's how much?

S4: Nineteen.

TT: Nineteen. So, eight plus eleven is how much? Deza...

S2: Nineteen.

TT: Is it nineteen? (Group 6 enters the answer into the game and a “plim” sound is heard) Yes!

The intentional inclusion of HP's SGs through Exploratory Teaching generated a motivating Game-based learning environment, where students were challenged to learn and develop skills with games, agreeing with Becker (2021), Compto (2023), Mohammed et al. (2024) and Plass et al. (2015). The quantitative results allow us to state that the use of SGs has contributed to a significant improvement in the level of knowledge and overall performance of the students, as also seen in the study by Maryana (2024). The intentional inclusion of SGs through Exploratory Teaching favored the students' learning, allowing us to agree that the teacher has a determining role in including digital artifacts in the teaching and learning processes (Umboh et al., 2021). So, this study shows that SGs, when integrated with exploratory teaching, increase not only performance but also motivation to learn.

5. Discussion

This study aimed to analyze how using PH SGs promotes the development of mental arithmetic in students in the first year of elementary school. Analysis of the initial tasks revealed that the students had difficulty adding arithmetic operations. The use of rubrics made it possible to identify better and analyze the students' difficulties, corroborating Freitas et al. (2024). These difficulties corroborate the idea that students have difficulties understanding basic mathematical concepts, such as arithmetic (Fritz et al., 2019). It was observed that the students had difficulties demonstrating how they performed the addition and recognizing the added and the adder in the task context. Rubrics also made it possible to select the SGs according to the students’ difficulties. In this way, we agree that this use makes it possible to define an action plan aligned with the students' difficulties (Brookhart & Chen, 2015).

The statistical results affirm that SG through Exploratory Teaching has contributed to a significant improvement in the students' LK and GP, as verified in Gomes et al. (2023) and Maryana's study (2024). The results of the descriptive statistics on the students' LK and GP show statistically significant differences, indicating a positive evolution of the students after the intervention. In addition, a statistically significant improvement was observed in the students' LGP, indicating an improvement in the student's mental arithmetic development. Likely, the improvements observed were partly the result of the students' interaction with the SGs. The intentional inclusion of HP's SGs through Exploratory Teaching generated a motivating Game-based learning environment, where students were challenged to learn and develop skills with games, agreeing with Becker (2021), Compto (2023), Mohammed et al. (2024) and Plass et al. (2015). The SGs made it possible to guide the student through the tasks, in agreement with Plass et al. (2015). The students learned actively, developing their autonomy and collaboration, as Freitas et al. (2024) and Jahnke et al. (2022) point out. It is also considered within the framework of national education policy that the Student’s Profile by the End of Compulsory Schooling (Martins et al., 2017) highlights the importance of students' active participation in the learning process. In this study, students' active participation and engagement, particularly in the phases of discussion and systematization of mathematical learning, are aspects that contribute to the improvement of mental arithmetic development. This aligns with the profile's guidance to promote competencies such as autonomy and reflective thinking. The intentional inclusion of SGs through Exploratory Teaching favored the students' learning, which allows us to agree that the teacher has a determining role in the inclusion of digital artifacts in the teaching and learning processes (Umboh et al., 2021). This teaching approach facilitated the integration of the SGs, in agreement with Freitas et al. (2024). The use of the SGs and TT's mediation during the task completion phase contributed to developing the students' mental arithmetic, as they were challenged to perform calculations quickly and flexibly (Buys, 2008). This allows us to agree with Narváez and Cañadas (2023), as the TT's mediation proved essential in guiding the student to complete the task. The moments of discussion were essential for exploring and sharing the different strategies used by the students, providing them with different ways of arriving at a result/solution (Buys, 2008). It was found that the TT used questioning during the orchestration of the discussion, as seen in Freitas et al. (2025). This questioning encouraged student participation and the development of mental arithmetic. The contribution of these knowledge-building moments is also visible in the study by Freitas et al. (2024). The study by Gomes et al. (2023) also shows the importance of this moment in sharing different resolution strategies and building knowledge. The qualitative results allow us to state that after the intervention, the students could apply various cognitive strategies (Hubber et al., 2014), with the freedom to use the ones they considered most effective while solving the tasks (Buys, 2008).

This exploratory teaching environment with SGs increased students' active participation and involvement, corroborating Maryana (2024), Mohammed et al. (2024), and Umboh et al. (2020). The phases of realization, discussion, and systematization of mathematical learning were essential for developing students' mental arithmetic, agreeing with the idea that these phases are important for constructing mathematical knowledge (Guerreiro et al., 2015; Ponte, 2017). The results show that, during the discussion, the students were challenged to distinguish between the parts and the sum of the addition operation. This enabled the students to make this distinction when solving the tasks in the post-intervention phase. TT's questioning was also essential in the learning systematization phase. This questioning allowed the students to develop their ability to perform addition using mental arithmetic. It should also be noted that the epistemic valorization carried out by TT in these moments may have contributed to the active participation of the students (Rodrigues et al., 2022), generating a good atmosphere of collaboration between students and teachers and between students and games (Hu, 2024; Umboh et al., 2020). This research is in line with international educational policies, particularly the Learning Compass 2030, as the results of this study relate to Transformative Competencies; Student Agency/Co-agency; Knowledge and Skills; and the Anticipation-Action-Reflection Cycle (OECD, 2018).

The results showed that the use of SGs as an epistemic tool (Costa et al., 2021; Lopes & Costa, 2019) helped the students of 1st year of primary school to think and reach the correct result using the decomposition strategy. The epistemic use of the SGs allows us to agree with Silva et al. (2021) since the students' learning improved. This improvement was evident in the statistically significant results on the students' GLK and GP and in the positive evolution of the strategies used by the students after the intervention.

6. Conclusion

The results showed that integrating PH SGs in the classroom, based on Exploratory Teaching, significantly contributed to students' mental arithmetic development students of 1st year of primary school. The integration of the SGs through this teaching approach made it possible to create a

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structured, motivating and challenging learning environment and allowed the students to actively construct their knowledge. These aspects are very much in line with the two guiding documents of the national education policy Student's Profile by the End of Compulsory Schooling and international education policy, through the Learning Compass. The active participation and involvement of the students of 1st year contributed to a significant improvement in the development of mental arithmetic. The instrumental orchestration carried out by the TT made it possible to select the SGs that best suited the intervention. The intentional use of SGs as an epistemic tool allowed students to explore different strategies and evolve in the type of representations used to solve the arithmetic operations of addition. The characteristic phases of Exploratory Teaching were essential for the construction of mathematical knowledge and for the active participation of the students, especially the moments of discussion and systematization of mathematical learning. The moments of discussion were fundamental for developing mental arithmetic and understanding the meanings of the arithmetic operation addition involved in the tasks. The TT's questioning during these moments encouraged the students' involvement and the development of mental arithmetic. The exploration guide encouraged students' autonomy. The use of rubrics made it possible to assess the students' knowledge and performance and to create a personalized action plan adapted to their difficulties.

The results show a statistically significant improvement in the students' LGP and GP. Thus, using the SGs of PH through Exploratory Teaching proved to be an effective way to promote the development of mental arithmetic in students in the 1st year of school of the 1st-grade primary school.

We acknowledge that the limitations of this study are the sample characteristics and the focus solely on 1st-grade students of the 1st-grade primary school. We suggest that subsequent studies investigate the integration of SGs at other grade levels, with the objective of identifying elements that could facilitate more effective integration across different grade levels. Additionally, a larger sample should be used in future studies to increase the likelihood of generalization.

7. Suggestion

We suggest for future researchers a larger sample, including other years of schooling, to identify aspects that contribute to the development of students' mental arithmetic through Serious Games in an Exploratory Teaching environment.

To practitioners and decision-makers, we suggest integrating Serious Games through Exploratory Teaching so that the integration is structured, and students develop essential skills and improve their learning.

Declarations

Author Contributions. Conceptualization: B.G., F.M.; Data Curation: B.G., R.P., V.R., F.M.; Formal analysis: Y.F., B.G., S.G., F.M.; Funding acquisition: F.M.; Investigation: Y.F., B.G., R.P., E.P., V.R., S.G., F.M.; Methodology: B.G., R.P., V.R., F.M.; Project Administration: B.G., R.P., V.R., F.M.; Resources: B.G., F.M.; Software: Y.F.; Supervision: S.G., F.M.; Validation: Y.F., B.G., R.P., E.P., V.R., S.G., F.M.; Visualization: Y.F., B.G., R.P., E.P., V.R., S.G., F.M.; Writing—original draft preparation: Y.F., F.M.; Writing—review and editing: Y.F., S.G., F.M. All authors have read and agreed to the published version of the manuscript.

Conflicts of Interest. Competing interest: The authors have no competing interests to declare that are relevant to the content of this article.

- Data availability: If requested
- Materials availability: Not applicable
- Code availability: Not applicable

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Ethical Approval. The anonymity of all involved participants was maintained, in strict compliance with the Declaration of Helsinki and approved by the Ethics Committee of the Polytechnic Institute of Coimbra (reference: 101 CEIPC/2022 approved on June 24, 2022).

Data Availability Statement. To review the data from this study, contact the primary author for more discussion about the request.

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About the Contributor(s)

Yelitza Freitas was a student of Polytechnic University of Coimbra, Coimbra, Portugal. Her main research interests include teacher professional development and the use of digital artifacts in teaching. She has published in academic and international journals on education.
Email: yelitzamafreitas@gmail.com
ORCID: https://orcid.org/0000-0002-9394-7724

Beatriz Guiomar was a student at the Polytechnic Institute of Coimbra. His main research interests include promoting mental arithmetic and mental calculation with natural numbers, rational numbers and decimal numbers.
Email: bia.guiomar@gmail.com
ORCID: https://orcid.org/0009-0003-5175-4558

Sofia Margarida Correia Gonçalves has a professional trajectory in the field of education, combining teaching experience, pedagogical leadership, and scientific research. She holds a PhD in Educational Sciences, specializing in Teaching Organization, Learning, and Teacher Training. She is a researcher in European research projects, a Professor at the Polytechnic Institute of Coimbra – School of Education, and the Director of the Specialization Course in School Administration and Organization. She is the author of training programs on innovative educational practices.
Email: sofiagoncalves@esec.pt
ORCID: https://orcid.org/0000-0002-9893-7875

Ricardo Pinto holds a Ph.D. in Educational Technology and serves as a Mathematics teacher for primary and secondary education. He is currently the coordinator of the Hypatiamat Project and a trainer in the fields of mathematics education and educational technology. He has published academic articles and newspaper pieces on general education and mathematics.
Email: rmnpslb@gmail.com
ORCID: https://orcid.org/0000-0002-2483-1250

Elisabete Pinto is a primary school teacher. She is interested in beginning reading and mental calculation. She is also interested in developing curiosity, critical thinking, and a love of discovery and learning. She values differentiated strategies to work towards inclusion and developing essential skills in students. Her research interests focus on using new technologies in education and digital literacy.
Email: elisabete.aersi@gmail.com
ORCID: https://orcid.org/0009-0004-1294-0769

Virgílio Rato is a professor at the Polytechnic Institute of Coimbra, Portugal. His main research interests include the professional development of teachers, pedagogical supervision in the school context, professional competencies in teaching today, and educational assessment.
Email: virgilior@esec.pt
ORCID: https://orcid.org/0000-0002-2412-4302

Fernando Martins completed his Aggregate Degree in Child Studies (Childhood, Development, and Learning [Elementary Mathematics]) in 2021/04/09 from the University of Minho, a Ph.D. in Mathematics in 2007 from the University of Beira Interior, a Master's Degree in Mathematics in 2003 from the University of Beira Interior and a Degree in Mathematics Teaching in 2001 from the University of Beira Interior. He has been Full Professor (Professor Coordenador Principal) at the Polytechnic University of Coimbra - Coimbra Education School since 12/2023. Researcher at the inED - Centre for Research and Innovation in Education (UIDB/05198/2020, and DOI identifier https://doi.org/10.54499/UIDB/05198/2020), Instituto de Telecomunicações (UIDB/50008/2020, and DOI identifier https://doi.org/10.54499/UIDB/50008/2020) and Research Group in Training, Education, and Intervention. He works in the areas of Exact Sciences with an emphasis on Mathematical Modelling, Elementary Mathematics and Elementary Mathematics Didactics, and Social Sciences with an emphasis on Educational Sciences.
Email: fmlmartins@esec.pt
ORCID: https://orcid.org/0000-0002-1812-2300
