

Research Article

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Classification of Abstract Thinking in Solving Mathematical Problems

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Abstract

Background/purpose. This research aims to classify and show the characteristics of the types of abstract thinking students use when solving mathematical problems.

Materials/methods. This descriptive qualitative research was conducted in a structured manner on students of the University of Muhammadiyah Surakarta, Faculty of Teacher Training and Education, Mathematics Education Study Program, semesters 3, 5, and 7. The instruments of this research include a two-variable linear equation (SELTV), story questions, and audio-visual recording tools. Data collection is carried out through tests. The data is analyzed through data reduction, presentation, and conclusion.

Results. This research finds a way to solve problems directly from the subject's thinking. Its findings are in the form of three classifications of abstract thinking: reflective abstraction, connected abstraction, and visual abstraction.

Conclusion. This research identifies and classifies the types of abstract thinking students use in solving mathematical problems, such as symbolic, logical, connective, and visual thinking. Every kind of abstract thinking plays an important role in solving various mathematical problems, including systems of linear equations in two variables.

1. Introduction

Mathematics is one of the sciences that is very important for everyday life. In mathematics, it is closely related to the processes of thinking and reasoning. Thinking and reasoning are related to each other (Alexander, 2016), especially in the process of solving problems. To effectively deal with various problems, one must have the ability to think and reason, according to (Wu & Yang, 2022; Nursoffina & Efendi, 2021; Medová et al., 2020). Thinking and reasoning can help solve mathematical problems. Therefore, thinking and reasoning can help come up with the right solution (Kholid et al., 2022). In mathematics, it is not only about the result of solving a problem, but also the process within it (Kholid et al., 2019). Therefore, it is necessary to develop these cognitive skills to support the level of understanding of interrelated mathematical concepts (Kholid et al., 2024). Cognition is the ability to acquire, process, and organize information, as well as to represent existing information (Kholid et al., 2020). So, this can be a leap for students to solve mathematical problems.

Problem-solving is the process of applying information that has been obtained from a problem to a new situation that was previously unknown (Kholid et al., 2021). Solving problems can use each individual's knowledge and experience (Sa'dijah et al., 2020). Solving this problem is very important because it can be used as a basis of reference in carrying out mathematics learning at any level. Problems in mathematics learning are given in the form of non-routine problems, such as story problems, descriptions of events, illustrations, and puzzles (Schoevers et al., 2020). Mathematical problem-solving is an effort that a person will make by using all their efforts to solve a problem that they will face and it requires a high level of mentality to improve problem-solving skills in mathematics learning (Pradana, 2024). Mathematical problem-solving is done by interpreting, symbolizing, manipulating, and solving problems using mathematical procedures or operations (Masduki et al., 2020). In this case, a concept of thinking is needed to solve the mathematical problem, namely in the form of abstract thinking. By thinking abstractly, one can find the relationships, concepts, and ideas needed to solve mathematical problems (Hakim & Nurlaelah, 2018).

Abstract thinking can be defined as a mental process that aims to deduce results and extract abstract meanings from things and relationships through hypothetical thinking through symbols, generalizations, and the ability to make assumptions and validate them (Aseeri, 2020). Using abstract thinking in problem-solving can facilitate the discovery of the right solution. Abstract thinking is very important because with this ability, it can construct mathematical concepts through meaningful learning, not directly transferred, or emphasized to memorize them only (Hong & Kim, 2016). The process of constructing a concept that occurs using prior experience or knowledge (Fitriani et al., 2018).

Abstract thinking is a means to build theoretical knowledge through the process of conceptualization, which involves a series of symbolic actions and thoughts that involve facts (Naranjo & Peña, 2016). Often in the process of abstract thinking, it is associated with real knowledge or experience that has happened. In problem-solving, when a person does abstract thinking, they focus on several aspects of the situation to make a decision and the process of the concrete situation to build relationships between elements (Yusepa et al., 2018).

The emphasis of abstract thinking is the effective use of concepts and symbols in dealing with a variety of specific situations in solving problems (Rubiyanti & Wijayanti, 2023; Kholid et al., 2021). This was also explained by Mukhtar (Anwar et al., 2022) that the abstraction involved in the mathematical content can start from a specific situation, and then mathematical ideas can be recognized from that situation. This ability to abstract also includes the ability to transform existing problems into mathematical models. Hamda (2018) states that abstract thinking skills are essential for mathematically representing problems, connecting concepts, and developing effective solutions. According to Benis-Sinaceur (2014), important aspects of abstract thinking are generalization,

symbolic representation, logical reasoning, correspondence between elements, and the creation of themes or abstractions of concrete problems.

Abstract thinking has an important role in math education, helping students develop deeper understanding and problem-solving skills (Melendy, 2014). This allows students to generalize concepts, engage in logical thinking, and solve complex mathematical problems (Kusmaryono et al., 2018). The next section outlines the importance of abstract thinking in various educational contexts. Context is sought in accordance with the content that appears relevant to their daily life (NG & TOH, 2023). The importance of abstract thinking in mathematics education is that with abstract thinking, students learn to generalize from specific examples, such as in understanding algebra and its advanced topics (Alam & Mohanty, 2024). Abstract thinking and generalization are very important to improve for students to understand mathematics in depth and improve their problem-solving skills (Trung, 2024).

From the explanation above, the research was conducted to classify the types of abstract thinking based on how students solve mathematical problems. In addition, this research also aims to demonstrate the characteristics of different types of abstract thinking. This research aims to answer the following questions:

- 1) What types of thinking can be identified in students when solving mathematical problems?
- 2) What are the characteristics of each type of thinking that students do?

2. Literature Review

In the abstract thinking process, there are several differences in students' answers so that classifications can be obtained that can be used as research material by researchers. To see the novelty of the research, the researcher conducted a review of the literature of previous research and divided it into three categories. The first category is the factors that affect abstract thinking in mathematics. The results of the study show that these factors are (1) self-efficacy and attitude (Özdemir et al., 2021); (2) learning style (Nurrahmah et al., 2021); (3) gender (Fitriani & Nurfauziah, 2019); (4) adversity quotient (Rubiyanti & Wijayanti, 2023); (5) numerical capabilities (Dewi et al., 2022).

The second category is abstract thinking with other theories or approaches, namely (1) abstract algebra emphasizes understanding mathematics in two ways, curricular and cognitive, which connect with influential topics in the abstract process (Veith & Bitzenbauer, 2022); (2) Rostrolateral Prefrontal Cortex (RLPFC) has an important role in supporting the integration of abstract thinking, which is often self-generated (Dumontheil, 2014); (3) STEM learning can facilitate abstract thinking (Hanif et al., 2021); (4) Bilingualism is related to abstract thinking and mathematics learning (Zaripova et al., 2019); (5) Outdoor learning development reviewed abstract thinking skills (Sukisna & Suparwoto, 2020).

The third category is the dimension of abstract thinking, namely, (1) the level of abstract thinking (Darwish, 2014), (2) the abstract type according to problem-solving (Iswari et al., 2019), (3) aspects and levels of in abstract thinking (Benis-Sinaceur, 2014).

After reviewing the literature on abstract thinking, the researchers identified several research gaps. These gaps include the need for (1) classification of abstract thinking in solving mathematical problems, (2) characteristics of abstract thinking, (3) development of technology-based tools to improve abstract thinking skills, (4) determination of appropriate learning strategies to improve abstract thinking skills, and (5) learning evaluation to improve abstract thinking skills. This year, the research focused on gap point (1), namely, researchers classify abstract thinking in solving mathematical problems. The state of the art is presented in Figure 1.

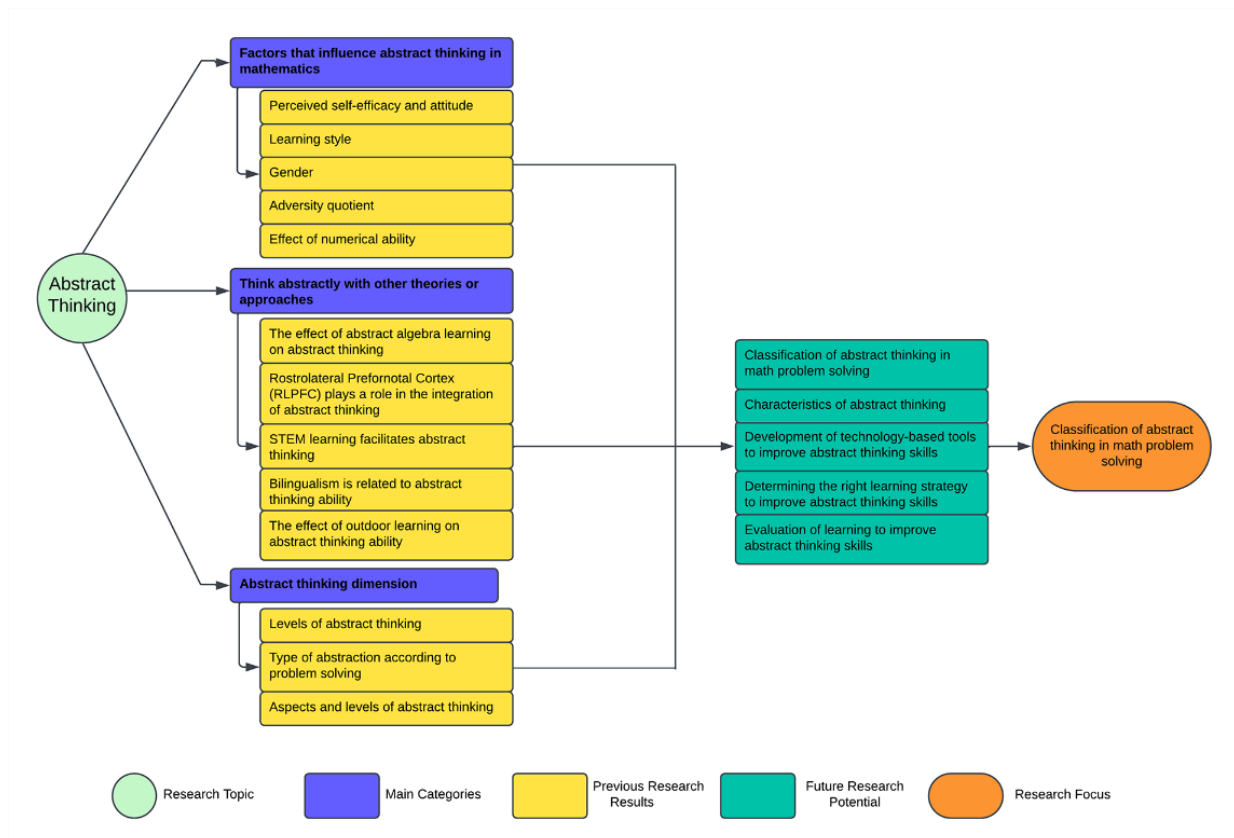


Figure 1. State of the art

Learning the classification of students' abstract thinking in solving mathematical problems is very important. The results of this research will contribute to a theoretical framework for future research that aims to improve students' abstract thinking skills. This research is in line with the focus of education research in Indonesia, which is guided by the National Research Master Plan (RIRN). The plan emphasizes themes related to social humanities, arts and culture, and education from 2017-2045, covering aspects of education and culture. The illustration of the focus of the research is presented in Figure 2.

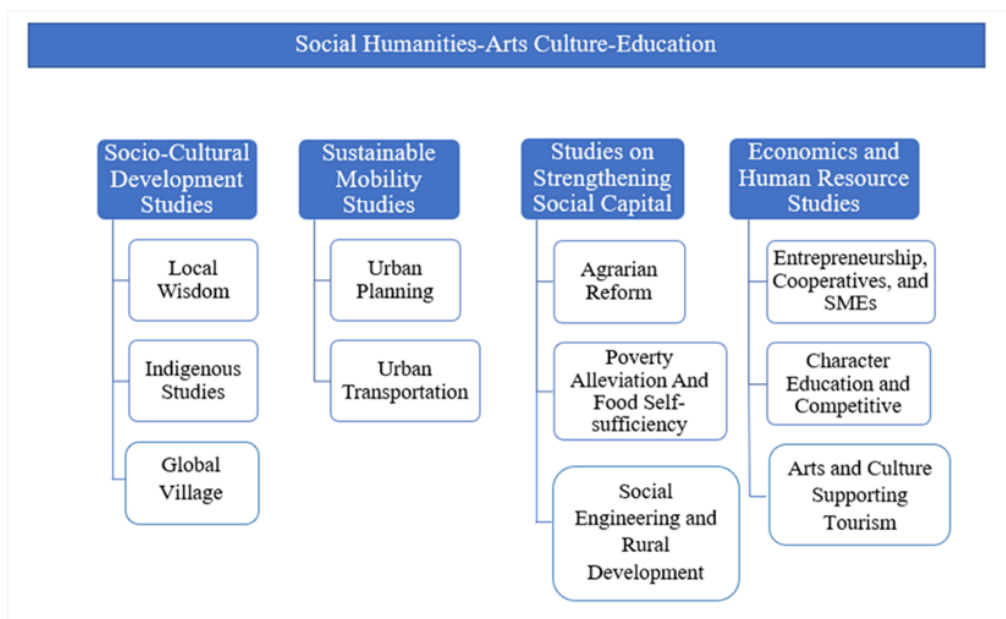


Figure 2. Research Focus

3. Methodology

3.1. Research Design

The methods used in this study are qualitative, exploratory, and descriptive research. Qualitative descriptive research provides a systematic and accurate picture of data based on facts, phenomena, and social processes (Creswell & Cresswell, 2014), following the facts on the ground without manipulation (Sagala et al., 2019).

3.2. Participants

This research involves 47 students of the Mathematics Education—University of Muhammadiyah Surakarta, Indonesia. Table 1 presents the distribution of the participants.

Table 1. Distribution of Participants

Semester	Gender	Number	Sum
3	M	6	13
	F	7	
5	M	7	16
	F	9	
7	M	10	18
	F	8	
Grand Sum			47

The subjects chosen are those who represent abstract thinking when solving the problem of a Two-Variable Linear System (SELTV). The stages carried out are recognizing problems, building, and constructing (Pangaribuan, 2018). The researcher excluded students who (1) did not think abstractly when solving mathematical problems and (2) experienced abstract thinking but did not match the core of the expected answer. The selection of subjects is carried out until the researcher obtains representative data.

3.3. Research Tools

The main instrument of this research is the researcher himself, who plays a role in planning and collecting data, analyzing and summarizing data and reporting research results. The accompanying instruments for this research are (1) mathematical problems in the content of the Two-Variable Linear Equation System (SELTV) and (2) audio-visual recording devices.

The instrument is validated by experts in two fields, namely the field of qualitative research in mathematics education and mathematics education research. Feedback from experts included the need to simplify the structure of the text to make it easier for problem solvers to understand and the need to include additional images and illustrations to provide more context to the problem. Subjects do not receive special treatment to create natural data. Such treatment can be in the form of (1) informing the subject about the subject of the test material, (2) educating the subject, or (3) other similar matters. Figure 3 presents a mathematical problem.

Question

To celebrate the 79th Indonesian Independence Day, Mrs. Ana and Mr. Budi prepared a children's competition in their respective villages. Mrs. Ana and Mr. Budi prepared several gifts that will be given to the children. The gift was purchased at the same store. Due to Independence Day, the store provides several economical packages, namely package one consisting of 2 packs of books and 1 pack of pencils and package two consisting of 3 packs of books. If Mrs. Ana buys 3 packages of one at a price of Rp288,000.00 and Mr. Budi buys 2 packages of one and 1 package of two at a price of Rp300,000.00, then:

- Decide on the price of package one and package two!
- Set the price of 1 pack of books and 1 pack of pencils!
- Mrs. Ana bought 3 more packages one and 1 package two, how much does Mrs. Ana have to pay?



Figure 1. Contextual questions to test abstract thinking skills

3.4. Data Collection Procedure

The data collection procedure is carried out as follows: (1) The researcher prepares research instruments such as mathematical problems, subjects, and recording tools; (2) The subject analyzes mathematical problems; (3) Subjects are asked to solve problems using creative solutions; (4) The researcher reviews the results of solving student problems; (5) Researchers transcribe the subject's writing as they solve the problem; (6) The researcher writes the results of the subject's answers; (7) The researcher will document the results of the subjects' responses and the results of the interview. The data collection procedure is illustrated in Figure 4.

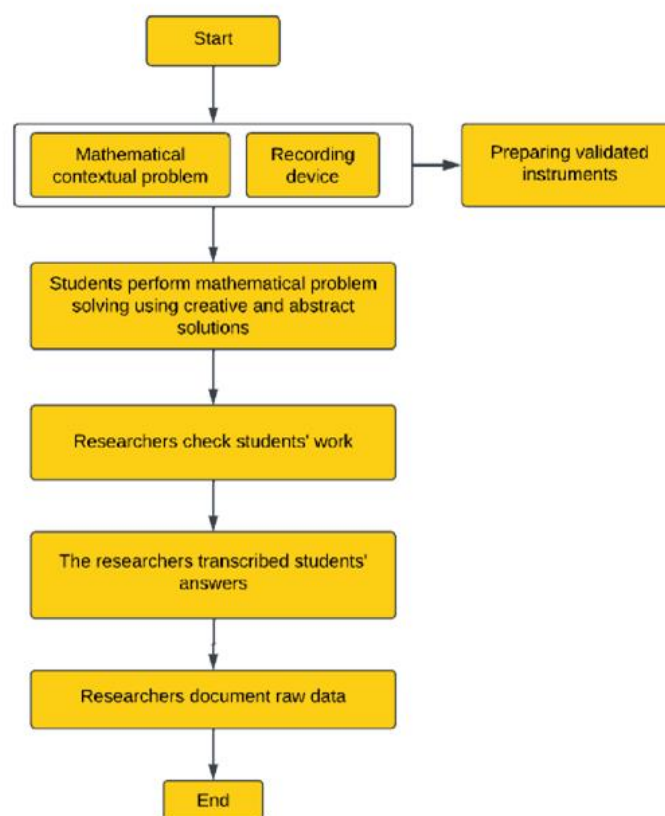


Figure 2. Data collection procedure

3.5. Data Analysis

The researcher used a constant comparison procedure for data collection and analysis, as described by Memon et al. (2017). This method of data analysis involves a continuous comparison process that gradually develops categories of information by comparing data with each other, ranging from specific observations to general conclusions.

In this study, we applied it to transform the raw data from the students' answer sheets into abstract thinking stages. Data is collected from various sources, including individuals. Researchers classify data based on abstract thinking methods such as reflective abstraction, connected abstraction, and visual abstraction. During this process, researchers continuously compare stages and categories to minimize errors and strengthen the evidence for each category. The following constant comparison procedure used in this study is illustrated in Figure 5.

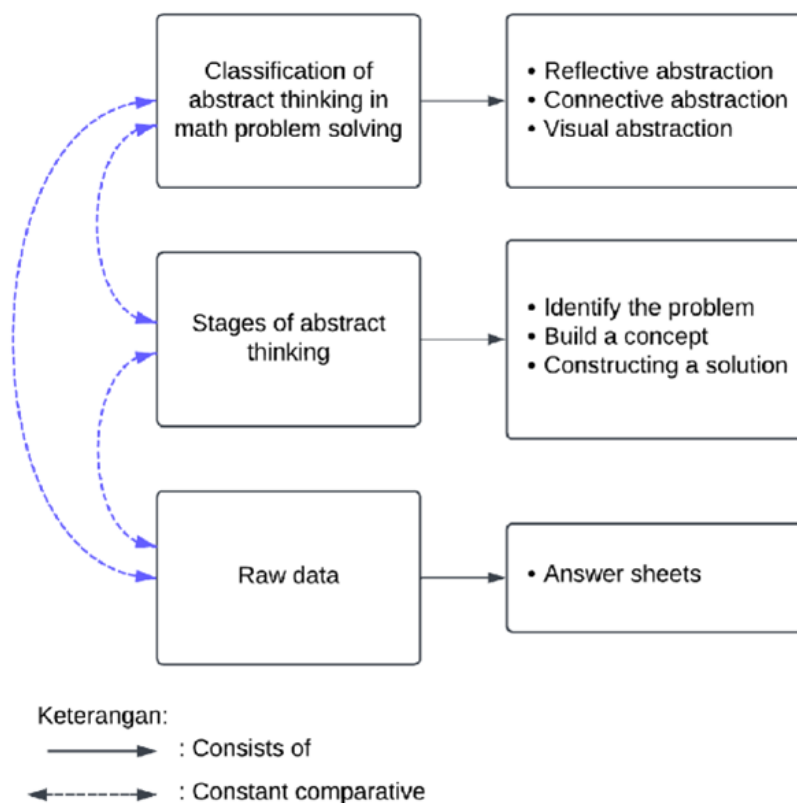


Figure 3. Implementation of CCP in data analysis

4. Results

The results of the study show that the abstract thinking process of each student is different. This can be seen from the results of the contextual mathematics problem-solving that has been carried out. Based on the results of these answers, there are three classifications of students' abstract thinking in solving contextual mathematical problems. Subject 1 (S-1) shows reflective abstraction. Subject 2 (S-2) shows a connected abstraction, and Subject 3 (S-3) shows a visual abstraction. Data from all three subjects showed early signs of data credibility.

4.1. Classification 1: Reflective Abstraction

Figures 6, 7, and 8 are the S-1 answers to the classification of abstract thinking type 1 in solving mathematical problems.

Original Version:

Diketahui : . paket satu (x)
 . paket dua (y)
 . buku (a) per pak
 . pensil (b) per pak
 yang dibeli
 $3x = 288.000$
 $2x + 1y = 300.000$

$x = 2a + 1b$
 $y = 3a$

Translate Version:

Known: • package one (x)
 • package two (y)
 • books (a) per pack
 • pencils (b) per pack

$x = 2a + 1b$
 $y = 3a$

yang dibeli
 $3x = 288.000$
 $2x + 1y = 300.000$

Figure 4. Answer S-1

Judging from Figure 6, before solving the problems that have been given, S-1 identifies the problem and builds a concept. Here, S-1 knows that there are four elements in the problem, and each of these elements is symbolized by a variable, namely package one (x), package two (y), book per pack (a), and pencil per pack (b). From what is known, s-1 gets a problem into four linear equations of two variables.

Original Version:

Ditanyakan :
 a) Tentukan harga paket satu dan paket dua.
 • Jika $3x = 288.000$, mk. $x = \frac{288.000}{3} = 96.000$
 maka paket satu berharga 96.000,
 • Jika $x = 96.000$, maka $2x + y = 300.000$
 $2(96.000) + y = 300.000$
 $192.000 + y = 300.000$
 $y = 108.000$,
 Jadi paket dua seharga 108.000

Translate Version:

Ask:
 a) Find the price of package one and package two
 • If $3x = 288.000$, then $x = \frac{288.000}{3} = 96.000$
 Then package one costs 96.000
 • If $x = 96.000$, then $2x + y = 300.000$
 $2(96.000) + y = 300.000$
 $192.000 + y = 300.000$
 $y = 108.000$
 So package two costs 108.000

Figure 5. Answer S-1

Original Version:

b) Tentukan harga 1 pak buku dan 1 pak pensil.

• Jika $y = 108.000 = 3a$
 $\frac{108.000}{3} = a$
 $36.000 = a$
 maka harga 1 pak buku adalah 36.000

• Jika $2a + b = 96.000$
 $2(36.000) + b = 96.000$
 $72.000 + b = 96.000$
 $b = 96.000 - 72.000$
 $b = 24.000$
 maka harga 1 pak pensil adalah 24.000

c) Jika Bu Ana membeli 3 paket satu ($3x$) dan 1 paket dua (y)
 berapa harga yang harus ditanyakan?
 $x = 96.000$ $y = 108.000$
 $3x = (96.000)3$
 $= 288.000$

• Jika $3x = 288.000$ dan $y = 108.000$
 dan $3x + y = 396.000$ maka Bu Ana harus
 membayar senilai 396.000 untuk mendapatkan
 3 paket satu dan 1 paket dua.

Translate Version:

b) Find the price of 1 pack of books and 1 pack of pencils

• If $y = 108.000 = 3a$
 $\frac{108.000}{3} = a$
 $36.000 = a$
 then the price of 1 pack of books is 36.000

• If $2a + b = 96.000$
 $2(36.000) + b = 96.000$
 $72.000 + b = 96.000$
 $b = 96.000 - 72.000$
 $b = 24.000$
 then the price of 1 pack of pencils is 24.000

c) If Mrs. Ana buys 3 packages of one ($3x$) and 1 package of two (y), what price should be asked?

• $x = 96.000$ • $y = 108.000$
 $3x = (96.000)3$
 $= 288.000$

If $3x = 288.000$ and $y = 108.000$ and $3x + y = 396.000$ then Mrs. Ana has to pay 396.000 to get 3 packages one and 1 package two.

Figure 6. Answer S-1

From Figures 7 and 8, it can be seen that S-1 gradually solves the problems of points (a), (b), and (c). For point (a), S-1 solves the question by dividing 288,000 by 3 so that the value x or the price of package one is found, which is Rp96,000.00, then the result of the value of x is substituted to the equation $2x + 1y = 300,000$ so that the value of y or the price of package two is obtained which is Rp108,000.00. For point (b) S-1, the result of point (a) is used to find variable a , which is the price of 1 pack of books, and variable b , which is the price of 1 pack of pencils. S-1 substitutes the value y to the equation $y = 3a$ so that the value a is obtained, which is Rp36,000.00 then subject one substitutes the values x and a to the equation $x = 2a + 1b$ and obtains the value b , which is Rp24,000.00. For point (c), S-1 substitutes the values of x and y , then the additional amount of money that Mrs. Ana must pay is Rp396,000.00.

In this case, S-1 solves the problem with previously known knowledge. S-1 assumes every known element of the problem and then converts it into an algebraic equation. Then, with the previously known knowledge, S-1 solves it with a logical and structured mathematical method, the substitution method. During the problem-solving process, S-1 carries out all the stages of abstract thinking, namely identifying problems, building concepts, and constructing solutions. This approach shows that S-1 utilizes abstract thinking with previously known knowledge.

4.2. Classification 2: Connected Abstraction

Figures 9, 10, and 11 are the answers to S-2 on the classification of type 2 abstract thinking in solving mathematical problems.

<u>Original Version:</u>	
Diketahui : misal x = 1 pak buku dan y = 1 pak pensil. maka Paket (1) = $2x + y$ Paket (2) = $3x$ Pembelian 3 paket 1 seharga Rp 288.000 2 paket 1 + 1 paket 2 seharga Rp 300.000 Ditanya : (a), (b), dan (c) (tertera pada soal)	
<u>Translate Version:</u>	
Known: e.g. x = 1 pack of books and y = 1 pack of pencils then Package (1) = $2x + y$ Package (2) = $3x$ Purchase 3 packages 1 at a price of Rp288.000 2 packages 1 + 1 package 2 at a price of Rp300.000 Asked (a), (b), and (c) (stated in the question)	

Figure 7. Answer S-2

Judging from Figure 9, before solving the problems that have been given, S-2 identifies the problem and builds a concept. Here, S-2 writes what is known in the problem, namely one pack of books symbolized by the x variable and one pack of pencils symbolized by the y variable so that package (1) = $2x + y$ and package (2) = $3x$. Then, the purchase in question is three packages: 1 for IDR 288,000.00 and 2 packages: 1 + 1 package 2 for IDR 300,000.00. Then the things asked in points (a), (b), and (c).

Original Version:

Penyelesaian:

(a) misal paket 1 = p dan paket 2 = q. maka

$$3p = 288.000$$

$$2p + 1q = 300.000$$

dikonversi jadi matriks koefisien

$$\begin{bmatrix} 3 & 0 & : & 288.000 \\ 2 & 1 & : & 300.000 \end{bmatrix}$$

akan dilakukan operasi baris elementer, direduksi hingga menjadi matriks eselon tereduksi.

$$\begin{bmatrix} 3 & 0 & : & 288.000 \\ 2 & 1 & : & 300.000 \end{bmatrix} B_{1(\frac{1}{3})} \begin{bmatrix} 1 & 0 & : & 96.000 \\ 2 & 1 & : & 300.000 \end{bmatrix}$$

$$B_{21(-2)} \begin{bmatrix} 1 & 0 & : & 96.000 \\ 0 & 1 & : & 108.000 \end{bmatrix}$$

dan matriks tersebut kita dapatkan

➤ harga paket satu (1) Rp. 96.000

➤ harga paket dua (2) Rp. 108.000

Translate Version:

Settlement:

(a) Suppose package 1 = p and package 2 = q, then

$$3p = 288.000$$

$$2p + 1q = 300.000$$

converted to a coefficient matrix

$$\begin{bmatrix} 3 & 0 & : & 288.000 \\ 2 & 1 & : & 300.000 \end{bmatrix}$$

will be performed elementary row operations, reduced echelon matrix

$$\begin{bmatrix} 3 & 0 & : & 288.000 \\ 2 & 1 & : & 300.000 \end{bmatrix} B_{1(\frac{1}{3})} \begin{bmatrix} 1 & 0 & : & 96.000 \\ 2 & 1 & : & 300.000 \end{bmatrix} B_{21(-2)} \begin{bmatrix} 1 & 0 & : & 96.000 \\ 0 & 1 & : & 108.000 \end{bmatrix}$$

from the matrix we get

➤ price of package one (1) Rp96.000

➤ price of package two (2) Rp108.000

Figure 8. Answer S-2

Original Version:	
(b) Paket 1 = $2x + y$ Paket 2 = $3x$ $\Rightarrow 2x + y = 96.000$ $3x = 108.000$ diketahui menjadi matrik koefisien $\begin{bmatrix} 2 & 1 & 96.000 \\ 3 & 0 & 108.000 \end{bmatrix}$ dan di ORE hingga menjadi matrik eselon tereduksi $\begin{bmatrix} 2 & 1 & 96.000 \\ 3 & 0 & 108.000 \end{bmatrix} \xrightarrow{B_2 \times (-1)} \begin{bmatrix} 2 & 1 & 96.000 \\ 1 & 0 & 36.000 \end{bmatrix} \xrightarrow{B_1 \times (-1)} \begin{bmatrix} 1 & 0 & 36.000 \\ 2 & 1 & 96.000 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 36.000 \\ 2 & 1 & 96.000 \end{bmatrix} \xrightarrow{B_2 \times (-2)} \begin{bmatrix} 1 & 0 & 36.000 \\ 0 & 1 & 24.000 \end{bmatrix}$ dan matriks tersebut kita dapatkan $\Rightarrow$ 1 pak buku sebagai Rp 36.000 $\Rightarrow$ 1 pak buku sebagai Rp 24.000	(c) Jika bu Ana membeli lagi 3 paket satu dan 1 paket 2. berapa harga yang harus dibayar Paket satu = Rp. 96.000 Paket dua = Rp. 108.000 3. paket satu + 1 paket 2 dua $= 3(96.000) + 1(108.000)$ $= 288.000 + 108.000$ $= 396.000$ Kesimpulan : Jadi uang yang harus dibelikan bu Ana untuk membeli 3 paket satu dan 1 paket 2 adalah Rp. 396.000 Total uang yang dibelikan bu Ana di toko tersebut adalah Rp. 288.000 + Rp 396.000 = Rp. 684.000.
Translate Version:	
(b) Package 1 = $2x + y$ Package 2 = $3x$ $\Rightarrow 2x + y = 96.000$ $3x = 108.000$ converted to a coefficient matrix $\begin{bmatrix} 2 & 1 & 96.000 \\ 3 & 0 & 108.000 \end{bmatrix}$ will be performed elementary row operations, reduced echelon matrix $\begin{bmatrix} 3 & 0 & 288.000 \\ 2 & 1 & 300.000 \end{bmatrix} \xrightarrow{B_2 \times \left(\frac{1}{3}\right)} \begin{bmatrix} 2 & 1 & 96.000 \\ 1 & 0 & 36.000 \end{bmatrix} \xrightarrow{B_1 \times (-1)} \begin{bmatrix} 0 & 1 & 24.000 \\ 1 & 0 & 36.000 \end{bmatrix} \xrightarrow{B_1 \times (-2)} \begin{bmatrix} 1 & 0 & 36.000 \\ 0 & 1 & 24.000 \end{bmatrix}$ from the matrix we get $\Rightarrow$ 1 pack of books for Rp36.000 $\Rightarrow$ 1 pack of pencils for Rp24.000	(c) If Mrs. Ana buys another 3 packages of one and 1 package of two how much be paid Package one = Rp96.000 Package two = Rp108.000 3 package one + 1 package two $= 3(96.000) + 1(108.000)$ $= 288.000 + 108.000$ $= 396.000$ Conclusion So the money Mrs. Ana has to spend to buy 3 packages of one and 1 package of two is Rp396.000 The total money spent by Mrs. Ana at the store is Rp288.000 + Rp396.000 = Rp684.000

Figure 9. Answer S-2

Figure 10 and Figure 11 is the result of solving the problem that has been carried out by S-2. For point (a) that is asked is the price of each package, the first step taken is to assume package 1 with the p variable and package 2 with the q variable so that two linear equations are obtained. Then S-2 converts the equation into a matrix form and operates it with an elementary row operation so that the value of each variable or price of each package is obtained, namely for package 1 (p) Rp96,000.00 and package 2 (q) Rp108,000.00. For point (b), the question is the price of 1 pack of books, supposedly with the x variable, and one pack of pencils, supposedly with the y variable. Here S-2 substitutes the price of each package so that two linear equations are obtained. Then S-2 solves it the same as point (a) by converting the equation into a matrix form and solves it using elementary row operations so that the price of 1 pack of books (x) is Rp36,000.00 and 1 pack of pencils (y) is Rp24,000.00. For point (c) S-2, substitute the price of each package so that you can get the price that must be paid by Mrs. Ana in its entirety.

In this case, S-2 solves the problem using a combination of the concepts of algebraic equations with matrices. S-2 identifies the problem and then equates the elements contained in it with certain variables. Furthermore, S-2 gets an equation and then solves it with the concept of a matrix, namely elementary row operations. During the problem-solving process, S-2 carries out all the stages of abstract thinking, namely identifying problems, building concepts, and constructing solutions. This approach shows that S-2 makes use of abstract thinking by applying a combined solution between two mathematical concepts to solve a problem.

4.3. Classification 3: Visual Abstraction

Figures 12, 13, and 14 are S-3 answers to the classification of abstract thinking type 3 in solving mathematical problems.

Before solving the problems that have been given, S-3 identifies problems and builds concepts. Here, S-3 knows that there are four elements in the question, namely one pack of books, one pack of pencils, package 1, and package 2. Then, S-3 develops the concept by reasoning elements with variables "x" for the price of 1 pack of books and "y" for the price of 1 pack of pencils so that an algebraic equation for package 1 is $2x + y$ and package two is $3x$. Then, each package was substituted into Mrs Ana and Mr Budi's purchase, and a two-variable linear equation was obtained, as shown in Figure 12.

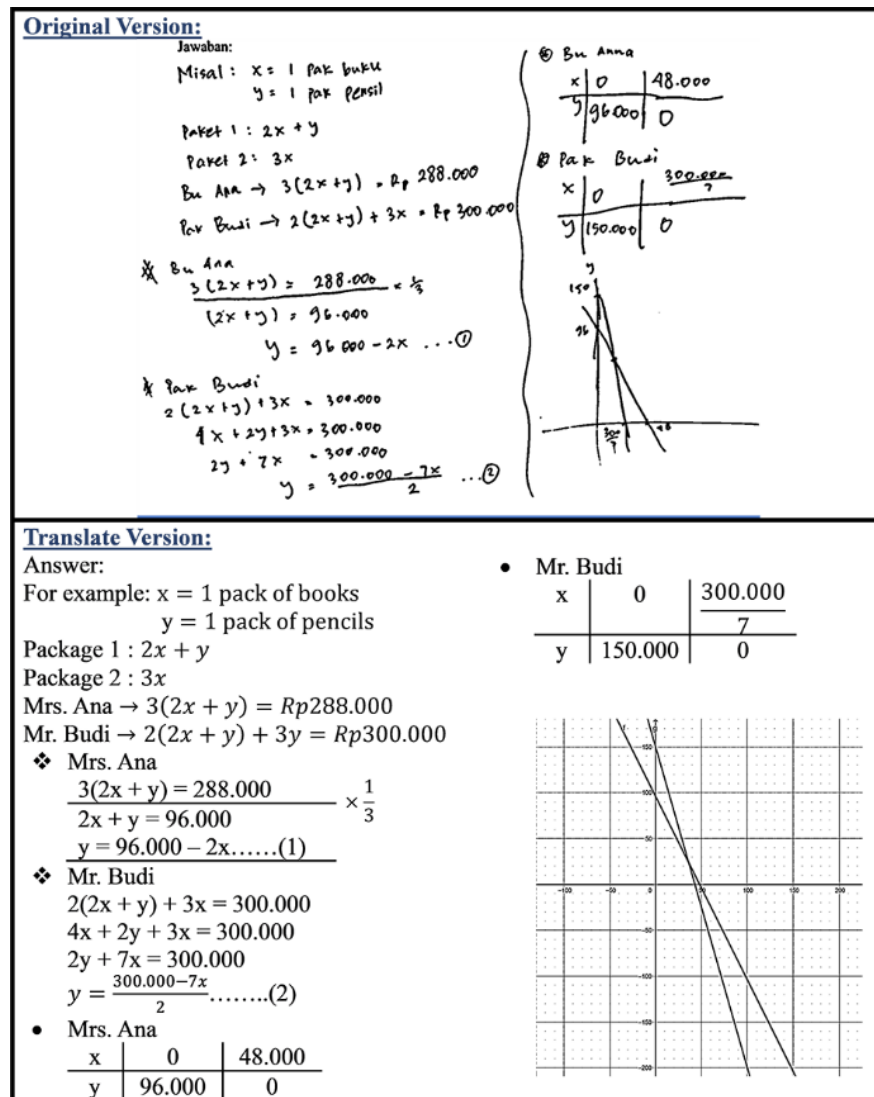


Figure 10. Answer S-3

To solve the problem, S-3 visualizes a linear equation of two variables that is a representation of the existing mathematical problem in the form of a graph. Before that, S-3 looked for the intersection points of each equation's x and y axes so that two linear lines and the intersection points of the two lines were obtained. Furthermore, S-3 calculates the coordinates of the cut-off point using algebraic properties, as shown in figure 13. The results of the calculation of the intersection point of the two linear lines can be used to solve the problems at points (a), (b), and (c), as shown in Figure 14.

Original Version:

✓ Persamaan 1 = Pers. 2

$$96.000 - 2x = \frac{300.000 - 7x}{2}$$

$$192.000 - 4x = 300.000 - 7x$$

$$-4x + 7x = 300.000 - 192.000$$

$$3x = 108.000$$

$$x = 36.000$$

X Substitusi nilai x ke Pers. ①

$$y = 96.000 - 2x$$

$$= 96.000 - 2(36.000)$$

$$= 96.000 - 72.000$$

$$y = 24.000$$

Translate Version:

❖ Equation 1 = eq. 2

$$96.000 - 2x = \frac{300.000 - 7x}{2}$$

$$192.000 - 4x = 300.000 - 7x$$

$$-4x + 7x = 300.000 - 96.000$$

$$3x = 108.000$$

$$x = 36.000$$

❖ Substitution of x value to eq. (1)

$$y = 96.000 - 2x$$

$$= 96.000 - 2(36.000)$$

$$= 96.000 - 72.000$$

$$y = 24.000$$

Figure 11. Answer S-3

Original Version:

a) Harga paket 1 $\rightarrow 2x + y = 2(36.000) + 24.000$

$$= 72.000 + 24.000$$

$$= 96.000$$

Harga paket 2 $\rightarrow 3x = 3(36.000)$

$$= 108.000$$

$\therefore$ Jadi Harga paket 1 adalah Rp 96.000
+ Paket 2 adalah Rp 108.000 ✓

b) Harga 1 Pak buk adalah Rp 36.000
Harga 1 pak pensil adalah Rp 24.000 ✓

c) Pembayaran Bu Ana

$$3(2x + y) + 3(2x + y) + 3x = 6x + 3y + 6x + 3y + 3x$$

$$= 15x + 6y$$

$$= 15(36.000) + 6(24.000)$$

$$= 540.000 + 144.000$$

$$= \text{Rp } 684.000 \checkmark$$

Translate Version:

a) Package 1 price $\rightarrow 2x + y = 2(36.000) + 24.000$

$$= 72.000 + 24.000$$

$$= 96.000$$

Package 1 price $\rightarrow 3x = 3(36.000)$

$$= 108.000$$

❖ So, the price of package 1 is Rp96.000. The price of package 2 is Rp108.000

b) The price of 1 pack of books is Rp36.000. The price of 1 pack of pencils is Rp24.000

c) Bu Ana Payment:

$$3(2x + y) + 3(2x + y) + 3x$$

$$= 6x + 3y + 6x + 3y + 3x$$

$$= 15x + 6y$$

$$= 15(36.000) + 6(24.000)$$

$$= 540.000 + 144.000$$

$$= \text{Rp}684.000$$

Figure 12. Answer S-3

From Figure 13, the cut-off point of two linear lines is obtained, namely for $x = 36,000$ and for $y = 24,000$, which means that the price of 1 pack of books is Rp36,000.00 and the price of 1 pack of pencils is Rp24,000.00 to answer the problem in point (b). Then, S-3 substitutes x and y into a two-variable linear equation, which is a representation of the problem in point (a), so that the price of package 1 is Rp96,000.00 and the price of package 2 is Rp108,000.00. For point (c), S-3 adds up the total amount of money that must be paid by Mrs. Ana so that a total of Rp684,000.00 is obtained.

In this case, S-3 solves the problem by visualizing the problem in the form of a graph. The graph illustrates the relationship between the price of 1 pack of books and one pack of pencils. Next, S-3 gets a two-variable linear equation and then finds the cut-off point. During the problem-solving process, S-3 carries out all the stages of abstract thinking, namely identifying problems, building concepts, and constructing solutions. This approach shows that S-3 makes use of abstract thinking by applying visualization of problems through graphs.

5. Discussion

Referring to the results of data analysis, all subjects tend to use all stages of abstract thinking even though there are stages that have not been realized optimally. Of the many subjects, only a few apply abstract thinking because, according to Nurrahmah (2021) is that some students still have difficulties in solving questions with abstract concepts, so there is a need for knowledge that can lead students to understand the abstract concepts (Hakim & Nurlaelah, 2018). In this case, conceptual knowledge and procedural knowledge can be used (Hakim & Nurlaelah, 2018). Moreover, both types of knowledge can be improved by employing technology in mathematics classes (Kholid et al., 2023).

After they understand the abstract concept, they need to gain the use of ideas and experience to solve problems. Previously, Fitriani et al. (2018) stated that the concept construction process that occurs in abstract thinking can use experience or prior knowledge to solve problems. In the results of the research, the subject solves the problem by processing ideas that are not directly related to physical objects or concrete situations but are related to concepts, principles, and broader relationships. This was also explained by Yusepa et al., who stated that their abstract thinking focused on several aspects of the situation to make a decision and the process of concrete situations to build relationships between elements (Yusepa et al., 2018). This means that the subject uses an abstract way of thinking to solve problems. By thinking abstractly, the subject can identify problems and then connect the relevant elements of a problem, as well as reinforce the concept (Naranjo & Peña, 2016). Moreover, mathematical problem-solving demands appropriate steps (Sutama et al., 2022) and critical strategies (Sutama & Anisti, 2024).

This study identifies three different classifications of students' abstract thinking in solving mathematical problems:

- 1) Classification I: Students solve problems with previously known knowledge.
- 2) Classification II: Students solve problems using a combination of algebraic equation concepts and matrices because they know that they are interconnected.
- 3) Classification III: Students solve problems by visualizing existing problems in the form of graphs.

This classification reflects the various strategies that students use in abstract thinking to solve mathematical problems, each of which indicates a different method of solving.

In Classification I, it is a reflective abstraction. This abstraction is characterized by students solving problems by involving previous knowledge that is already known and then implemented in new existing situations. Using previously known knowledge can be a strategy for solving mathematical problems or can improve the problem-solving ability of each student. This is in line with

previous research: (1) in mathematical problem solving, reflective abstraction involves reflecting on concepts that have been learned and applying them to new situations. All of this has an impact on students' ability to solve problems well (Balila et al., 2023); (2) emphasizes that prior knowledge of reflective abstraction is essential for integrating relevant information in mathematical problem-solving (Ulia et al., 2024); (3) reflective abstraction helps build mathematical logic and effectively solve problems (Nurrahmah et al., 2023); (4) reflective abstraction allows building conceptual knowledge, developing new strategies, and effectively organizing and solving problems (Kariadinata, 2021). Thus, reflective abstraction can solve problems effectively and can improve problem-solving skills.

In Classification II, it is a connected abstraction. This abstraction is characterized by solving problems by connecting two mathematical concepts directly. This is done by students because they see a connection between concepts so that they can find solutions to existing problems. This is in line with previous research: (1) connecting two mathematical concepts means discovering how they relate to each other to create key concepts, which serve as the basis for solving problems; this approach allows them to look at complex problems as a whole, which allows them to find effective and efficient solutions (Hamda, 2018); (2) connecting two mathematical concepts can improve their understanding and ability to solve mathematical problems successfully (Pambudi et al., 2020). By connecting two mathematical concepts, key concepts that serve as the basis for problem-solving are obtained.

In Classification III, it is a visual abstraction. The classification of visual abstraction is characterized by students visualizing the problem at hand, namely by describing the graph. Visualizing these problems can be a strategy for solving mathematical problems. This is in line with some previous research: (1) Visualization in mathematics can be used for problem-solving and facilitating understanding (Khatin-Zadeh et al., 2023); (2) tools such as Geogebra facilitate the visualization of abstract mathematical concepts so that they can improve understanding (Do'stov et al., 2022); (3) Visualization is helpful in the abstraction process, such as aiding comprehension, building between concepts, and facilitating the internalization of mathematical ideas so that it can improve understanding and problem-solving skills (Yilmaz & Argun, 2018); (4) Visualization in mathematics helps in understanding and creating ideas, challenging the idea that drawing is just an illustration. It improves understanding of evidence and concepts at all levels (Lehning, 2015); (5) How visual forms can relate abstract logic to sensory experience, enriching understanding of complex theories (Cirafici, 2018). By visualizing the problem, the subject can find relationships from the information included and understand what is being asked in the question to achieve generalizations (Rahayuningsih et al., 2020). Thus, problem visualization can aid in problem-solving by providing a clear and intuitive understanding of the problem, making it a practical strategy.

6. Conclusion

This study finds a new classification of students' abstract thinking in solving mathematical problems. First, the classification of reflective abstraction is problem-solving with abstract thinking stages that are characterized by students considering previously known knowledge. Second, is connected abstraction, which is problem-solving with abstract thinking stages characterized by students connecting two mathematical concepts directly. Third, visual abstraction, which is problem-solving with abstract thinking stages characterized by students visualizing problems in the form of graphs before solving existing problems.

7. Suggestion

The limitation of this study is the number of participants. If this study involves more and wider participants, there is a possibility that the results of the study will be different. However, this research makes a theoretically important contribution to future research and field practice. In addition, this

research can be applied to the secondary school level and teachers can apply mathematical education with differentiated abstract thinking. Beyond the research roadmap, future research can focus on the role of learning models to improve abstract thinking skills and the development of teaching materials that are differentiated in abstract thinking classification.

8. Suggestion

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Declarations

Author Contributions. All authors have read and approved the published version of the article.

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Ethical Approval. This study discovered how students' abstract thinking for mathematical problem-solving. They voluntarily agreed to participate by signing informed consent forms. Their participation does not put them in a dangerous condition and is physically or mentally harmful. The researchers ensured the confidentiality of the participants' identities throughout the data collection and presentation process in accordance with ethical considerations and guidelines set by The Committee on Publication Ethics (COPE). This study was approved by the Ethics Committee of Universitas Muhammadiyah Surakarta in 2025.

Data Availability Statement. The data supporting this study's findings are confidential, as bound by the human ethics application.

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