

Research Article

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Investigating Mathematical Knowledge for Teaching Early Childhood Education Teachers: A Starting Point for Designing a Professional Development Program

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Abstract

Background/purpose. The role of early childhood education (ECE) teachers is very important in building the foundation of mathematics, such as recognizing counting, patterns, and problem solving. However, many teachers still face challenges in understanding mathematical concepts in depth. This study aims to investigate the level of Mathematical Knowledge for Teaching (MKT) among early childhood education teachers and understand the factors influencing their MKT levels, serving as a foundation for designing an effective professional development programmed (PDP).

Materials/methods. This study uses an exploratory case study design because it allows researchers to understand the complexity of a problem and explore it in depth. This study involved 5 ECE teachers in West Java, Indonesia, with the following criteria: more than 10 years of teaching experience, BA in ECE, teaching in a public ECE school, and having more than 20 students per class. The data collection techniques include participant observation, tests, interviews, and documentation. The data analysis technique uses thematic analysis.

Results. Research findings show that the limited knowledge of MKT of ECE teachers, especially related to basic mathematics skills and children's mathematics development, is influenced by several factors. These factors include the lack of special training in mathematics teaching, limited appropriate learning resources, limited teaching experience, and a lack of focus on professional development in mathematics.

Conclusion. Improving ECE teachers' MKT requires targeted training, better resources, and focused professional development. The contribution of this research is that it can improve the quality of mathematics teaching in ECE and support children's mathematics achievements in the future.

1. Introduction

Early Childhood Education (ECE) is currently receiving attention from researchers worldwide, with mathematics education being one of the main focuses (Björklund & Ahlskog-Björkman, 2017; Papandreou & Tsiouli, 2022). This is because mathematics plays a crucial role in children's cognitive development, laying the foundation for logical thinking, problem-solving skills, and decision-making abilities in the future (Chen & McDunn, 2022; Drever et al., 2015; Hachey, 2013; Sudirman et al., 2021). Additionally, mathematical skills acquired early in life can support understanding of other subjects, such as science and technology (Kermani & Aldemir, 2015; Park et al., 2017; Sudirman et al., 2024). With an engaging and age-appropriate approach, mathematics learning can also help children develop a positive mindset towards challenges and new tasks.

Many studies highlight the importance of mathematics in early childhood. Watts et al. (2018) found that the mathematical abilities of children at 54 months significantly influence their mathematics achievement in school. Children's mathematical skills not only impact their future mathematical abilities but also other competencies (Efwan et al., 2024; Lino, 2016; Mellawaty et al., 2019; Noviyanti, 2020; Piasta et al., 2015; Whittaker et al., 2020). Furthermore, Lino (2016) stated that children who receive well-designed developmental activities between the ages of 0 and 6 tend to perform better academically in areas such as reading, writing, and mathematics. Additionally, such children are more capable of engaging in social interactions compared to their peers and adults (Lino, 2016). Therefore, high-quality, challenging, and accessible mathematics education for children aged 3 to 6 years is a critical foundation for future mathematics learning (Dunphy, 2009).

On the other hand, teachers play an important role in shaping mathematical skills (Fatihah et al., 2023; Parks & Wager, 2015; Sudirman et al., 2023; Thiel, 2010) and significantly improving student achievement with long-term impact (Kilday & Kinzie, 2009). A good teacher, including a mathematics teacher, must master both content depth and pedagogical knowledge (Gasteiger et al., 2020; Leong et al., 2015). Therefore, the concept of Mathematical Knowledge for Teaching (MKT) must be possessed by early childhood education teachers. According to Ball and Bass (2003), MKT is knowledge of content used to recognise, understand, and respond to problems and tasks encountered in teaching mathematics. Thus, mastering MKT is crucial for teachers as this knowledge can influence student mathematics achievement (Ekmekci et al., 2019).

However, there are several issues related to MKT faced by ECE teachers. According to Cerezci (2020) and Noviyanti and Suryadi (2019), most ECE teachers do not understand effective teaching strategies for connecting students' experiences with mathematics. As a result, many teachers use methods such as memorisation and practice in mathematics development activities (Gifford, 2004). Furthermore, Zhang et al. (2024) and Baumert et al. (2010) state that many ECE teachers lack an understanding of basic mathematics, students' mathematical learning, effective curricula, and teaching strategies for mathematics. In addition, high-quality mathematics learning development programmes are very rare compared to other fields such as reading, arts, or social skills (Clements et al., 2017). These issues highlight that there is minimal attention to mathematics in early childhood education, making professional development programmes for teachers essential (Clements & Sarama, 2011; Pela et al., 2023).

Although the importance of early childhood mathematics education and the role of teachers in developing mathematical knowledge have been widely discussed in various studies, there remains a lack of research that specifically examines the level of MKT among early childhood education teachers, particularly within the Indonesian context. Most previous studies have focused more on children's learning outcomes or general teaching strategies, without exploring in depth the extent of teachers' content and pedagogical knowledge in mathematics. In addition, there is limited research identifying the factors that influence variations in MKT levels among early childhood teachers. This

gap is crucial to address, as the development of effective PDP for teachers greatly depends on an accurate understanding of the actual conditions and needs of teachers in the field. Therefore, this study seeks to fill this gap by investigating the MKT levels of early childhood education teachers and the factors influencing them, in order to provide a solid foundation for designing a relevant and contextually appropriate PDP model.

Given the importance of MKT and the challenges faced by teachers in implementing mathematics development activities. The professional development programmed (PDP) can offer a solution to enhance MKT. PDP serve as a tool to improve and maintain the knowledge and skills relevant to a teacher's professional domain. To develop an effective PDP model, a thorough investigation into the mathematical knowledge of early childhood education teachers is required. Therefore, this study aims to investigate the level of MKT among early childhood education teachers and explore the factors influencing their MKT levels, providing a foundation for designing an effective PDP. The research questions are as follows:

- 1) What is the level of Early Childhood Education (ECE) teachers' Mathematical Knowledge for Teaching (MKT) in Indonesia?
- 2) What are the underlying factors contributing to the limited MKT among ECE teachers?
- 3) What type of Professional Development Program (PDP) is appropriate to improve ECE teachers' MKT?
- 4) To what extent can the proposed PDP—particularly the combination of summary strategy and Cognitively Guided Instruction (CGI) model—effectively address the limitations in ECE teachers' mathematical knowledge?

2. Literature Review

2.1. Mathematical Knowledge for Teaching (MKT)

MKT, as conceptualized by Ball et al. (2008), encompasses a multifaceted understanding essential for educators to effectively convey mathematical concepts. This framework comprises several interrelated domains, including Common Content Knowledge (CCK), which refers to fundamental mathematical principles, and Specialized Content Knowledge (SCK), which pertains to the unique mathematical understanding required for teaching (Ball & Bass, 2003). Additionally, it includes Knowledge of Content and Students (KCS), which focuses on anticipating students' misconceptions, Knowledge of Content and Teaching (KCT), which integrates instructional strategies with mathematical content, and Knowledge of Content and Curriculum (KCC), which involves aligning pedagogical approaches with educational standards (Gess-Newsome et al., 2019; Kuntze, 2012). Early childhood educators frequently encounter challenges in developing a profound comprehension of MKT, particularly within the domains of SCK and KCS. This difficulty stems from the predominant emphasis on general child development in teacher training programs rather than a targeted focus on mathematical instruction (Björklund et al., 2020; Björklund & Barendregt, 2016). As a result, many educators struggle with designing lessons that effectively bridge abstract mathematical theories with concrete, developmentally appropriate learning experiences (Kermani & Aldemir, 2015). Without a solid grasp of MKT, teachers may find it challenging to diagnose students' learning difficulties, employ adaptive teaching strategies, and foster mathematical thinking in a way that aligns with young learners' cognitive development (Zhang et al., 2024).

2.2. Factors Influencing Mathematical Knowledge for Teaching (MKT) in Early Childhood Education (ECE) Teachers

The level of MKT among early childhood educators is significantly influenced by the availability and quality of professional training programs (Aguirre et al., 2013). Many teacher education curricula

prioritize general pedagogical approaches rather than specialized mathematical instruction, leading to a lack of structured training focused on essential mathematical concepts (Ball & Bass, 2003; Björklund & Barendregt, 2016). Without targeted professional development, teachers may struggle with conceptualizing mathematical ideas in ways that are developmentally appropriate for young learners (Noviyanti et al., 2020). Additionally, the absence of reflective practices within training programs hinders educators from critically assessing their instructional methods, thereby limiting their ability to adapt and refine their teaching strategies over time (Carpenter et al., 1996; Kim, 2018).

Furthermore, limited access to high-quality learning resources exacerbates the challenges faced by early childhood educators in strengthening their MKT. Many teachers rely on outdated or generic teaching materials that do not sufficiently address foundational mathematical concepts or align with the cognitive development of young children (Cerezci, 2020; Watts et al., 2018). The lack of emphasis on mathematics in professional development initiatives further contributes to this gap, as many programs focus primarily on social-emotional learning rather than fostering mathematical reasoning (Park et al., 2017). As a result, even experienced educators may struggle to effectively integrate mathematics into early childhood education, highlighting the urgent need for specialized training and enhanced resource accessibility (Ginsburg et al., 2008; Zhang et al., 2024).

2.3. Professional Development Program for Mathematical Knowledge for Teaching

Effective PDP for enhancing MKT among early childhood educators must incorporate practice-based approaches that foster both theoretical understanding and practical application (Sarama et al., 2003). Strategies such as lesson study, coaching, and collaborative learning enable teachers to refine their instructional techniques through continuous reflection and peer support (Kirkpatrick & Kirkpatrick, 2006). Integrating these approaches with child-centered mathematical learning theories ensures that teachers not only grasp fundamental mathematical concepts but also develop the ability to deliver them through engaging, developmentally appropriate activities (Piastra et al., 2015; Zaslow et al., 2010). Without a structured PDP that emphasizes both conceptual depth and pedagogical adaptability, teachers may struggle to effectively facilitate early mathematical learning (Sarama et al., 2003).

Moreover, an evidence-based PDP should include periodic evaluations to measure its effectiveness in improving teachers' instructional quality and students' mathematical development (Falletta, 1998). Ongoing assessments allow educators to identify specific gaps in their understanding and adjust their teaching strategies accordingly (Moore & Cuevas, 2022; Rini et al., 2023). Additionally, aligning professional development initiatives with real classroom challenges ensures that training is not merely theoretical but directly applicable to everyday teaching practices (Zaslow et al., 2010). By continuously refining these programs based on empirical data and participant feedback, the impact of PDPs on early childhood mathematics education can be maximized, ultimately leading to improved student outcomes and a more mathematically competent teaching workforce (Piastra et al., 2015; Sarama et al., 2003).

3. Methodology

3.1. Study Design

This study adopts an exploratory case study design. This design was chosen because it allows the researcher to understand the complexity of a problem and explore it in depth (Sarwanto et al., 2020; Yaniawati et al., 2023). Additionally, this design enables the researcher to investigate the level of MKT and delve deeper into the factors that influence the MKT level among ECE teachers, as well as provide a broader insight into the existing context. With this approach, the researcher can formulate questions such as "What is the level of MKT among ECE teachers in Indonesia?" and "Why is the level of MKT among ECE teachers as such?" which are highly relevant to achieving the aims of this research.

3.2. Setting

The participants involved in this study are five ECE teachers from one city in West Java, Indonesia, selected through purposive sampling. The selection process began with the researcher visiting the Head of the West Java Department of Education, specifically the Early Childhood Education Division, to obtain an overview of the ECE schools in the area. In this study, West Java was chosen as the case study location because it represents the general state of education in Indonesia. As the most populous province, West Java has a diverse range of schools and socio-economic backgrounds, making it relevant for studying the challenges in early childhood education. Additionally, West Java is actively involved in various educational reforms, particularly in mathematics education. Therefore, this study is expected to provide insights that will not only be beneficial for West Java but can also be applied to other regions with similar conditions. Thus, the choice of West Java is highly relevant to the purpose of this research.

After being provided with a list of ECE schools in the region, the researcher contacted public ECE schools to obtain permission and identify teachers who met the required criteria. There are five criteria for selecting the sample: (a) experience teaching at a kindergarten (with children aged 5-6 years), (b) approximately 10 years of teaching experience, (c) holding a bachelor's degree in early childhood education, (d) teaching in a public ECE school (not a private school), and (e) having at least 20 students in the class. These criteria were designed to ensure that the research participants had sufficient teaching experience and a deep understanding of the ECE context in Indonesia. The selection was limited to teachers from public ECE schools due to differences in policies and resources available in public versus private schools.

After these criteria were established, five schools met the requirements. The researcher then arranged meetings with the school principals and the recommended teachers. Before the implementation of the study, the researcher sought approval by asking the five selected teachers to participate voluntarily. The researcher explained the purpose, procedures, and expected benefits of the study to the teachers, ensuring that they clearly understood what would be involved during the study. Furthermore, the researcher assured the teachers that their participation was voluntary and that they were free to withdraw at any time without any negative consequences. Once the teachers had received sufficient information, they were asked to provide written consent as confirmation of their willingness to participate in the study. Subsequently, three teachers from the five schools met the required criteria. The criteria regarding the teachers' gender, age, and school origin can be seen in Table 1. In this research, the participants are given pseudonyms, namely Ani, Popon, Santi, Siti, and Srimaya.

Table 1. The criteria regarding the teachers' gender, age, and school origin

No	School	Total	Gender	Age
1	State ECE School 1	1	All Female	25-45 years
2	State ECE School 2	1	All Female	30-47 years
3	State ECE School 3	1	All Female	28-43 years
4	State ECE School 4	1	All Female	24-40 years
5	State ECE School 5	1	All Female	28-45 years

3.3. Data Collection Techniques

Because this research aims to find about Mathematical Knowledge for Teaching (MKT), the activities carried out referenced the theory and concept of MKT. Therefore, the data collection

method was adjusted to the aspects contained in the Pedagogical Content Knowledge (PCK) and Subject Matter Knowledge (SMK). Following are the stages of the data collection.

3.3.1 Analysis of Learning Cases

In this activity, teacher participants were presented with articles on cases of classroom learning. This activity aims to find out the teacher's ability to analyse problems that occur in development activities and identify the strengths and weaknesses/shortcomings of the cases presented. The activity lasted for 45 minutes.

3.3.2 Teacher's Knowledge of Children's Mathematical Development

In this activity, the researcher examined the extent of the teachers' knowledge of the processes and stages of children's mathematical development. The questions given were related to verbal counting sequences, counting, ordinal number of words, addition/subtraction, divisions of sets, written number symbols, and words that refer to the Knowledge of Mathematical Development instrument developed by Kim (2018). The following are examples of questions given for 45 minutes

Read the following example of a problem

Amri is five years old. His Mom, Ida, asked him to count the number of buttons divided into the following groups (presenting the buttons as shown by the picture)



When asked "How many are there in total?". What will Amri do? Explain.

3.3.3 Teacher's Knowledge About Basic Mathematics

The participants were given 45 minutes to solve 30 questions about basic mathematics developed by Briggs (2018). The following are examples of the questions given:

Fill in the blanks of the following number sequence.

1) *__, __, __, 39, __, 59, __, 79*

2) *What do you know about circles? Explain their difference from the ball?*

3) *Are there rational or irrational numbers?*

4) *The distance between Jakarta and Bandung is 200 km. Mr. Halim departed from Jakarta at 09.00 and arrived at Bandung at 11:30. The average speed of Mr. Halim's vehicle is ... km / hour*

3.3.4 Classroom Observation

Observations were conducted based on agreement with the participants. The timing of the observations was determined by when the participants carried out mathematical development activities in their classrooms according to the design they had created. During each session, the researcher recorded all development activities, both inside and outside the classroom. These activities were captured using three video cameras placed at the front, on the left/right side, and at the back of the classroom. To ensure comprehensive data collection, a semi-structured observation tool was used, providing a flexible yet sufficient framework for recording and analyzing the activities. The video recordings obtained were then analyzed using content analysis to identify and understand the interactions and developments occurring during the learning activities.

3.3.5 Documentary Study

In this activity, lesson plans and evaluation documents were analysed using a rubric validated by experts. For lesson plans, the aspects analysed included conformity to competency aspects,

indicators, learning objectives, teaching materials, learning resources, learning media, learning models, learning scenarios, and learning assessment designs. As for evaluation documents, the aspects analysed include aspects of ability (growth and development), evaluation models/techniques, goals, and timing of activities.

3.3.6 Interview

The type of interview in this research was the unstructured one. Interview was carried out when the researcher would like to follow up on the teachers' answers to a previous problem or question. In this case, the interview was carried out based on the results obtained by the respondents in the analysis of learning cases, tests on basic mathematical questions and children's mathematical development, and observation. All interviews were recorded to get maximum results.

3.4. Data Analysis Techniques

Various analysis methods were used to analyze the data collected in this study, depending on the type of data obtained. Thematic analysis was employed to analyze qualitative data from interviews and observations. Thematic analysis aims to identify patterns or themes in the phenomenon being studied, following the steps outlined by Krippendorff (2022), which include: (a) Preparing and interpreting data; (b) Exploring and coding databases; (c) Describing and shaping themes; (d) Representing and reporting findings; and (e) Interpreting the meanings of the findings. On the other hand, content analysis was applied to analyze data obtained from video recordings, allowing the researcher to understand the interactions that took place during the teaching activities. The stages of content analysis include: (a) Raw data collection, (b) Familiarization with the data, (c) Open coding, (d) Axial coding, (e) Theme refinement, and (f) Reporting themes. The processes of thematic analysis and content analysis are illustrated in Figure 1, which shows how the data were analyzed to generate findings that can be understood and utilized in this study.

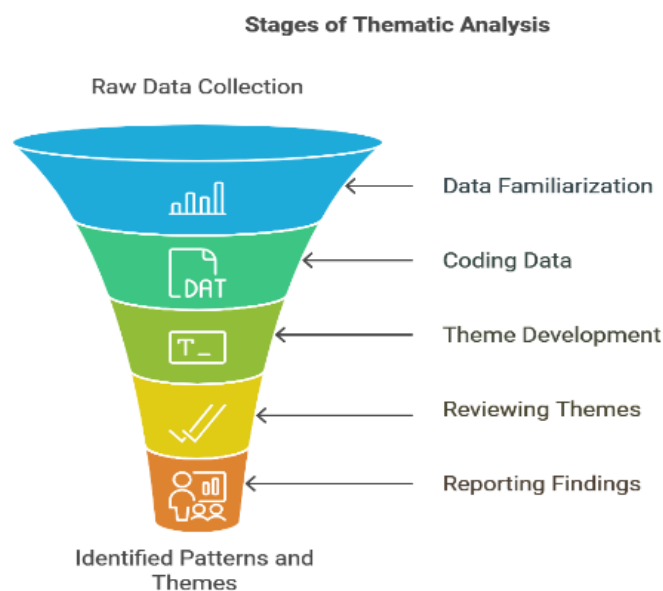


Figure 1. The diagram illustrating the process of thematic analysis

3.5. Ethical Considerations

To ensure ethical transparency, this study obtained formal ethical approval from the Research Ethics Committee of Universitas Terbuka on August 20, 2024. The researcher also adhered to ethical guidelines by preparing a written informed consent form signed by both participants and the researcher prior to data collection. The consent form explained the purpose of the study, the voluntary nature of participation, and the measures taken to protect participants' privacy and rights. In addition, the researcher obtained permission from the authorized school authorities before

conducting the study. All collected data were kept confidential—real names were replaced with pseudonyms in all reports and publications, and participants' faces were not shown in any photos or videos included in the research findings. These measures were taken to ensure participants' identities remained protected and in accordance with ethical standards for research involving human subjects.

4. Results

The data obtained from the analysis of tests, observations, and interviews were analysed using a content analysis approach. In the Open Coding stage, various key elements were identified, such as teaching experience, professional training, creative teaching strategies, classroom management, and challenges such as limited resources and difficulties in motivating students. The Axial Coding stage then linked these elements, establishing logical relationships between the categories identified. Finally, in the Theme Refinement stage, key themes such as Knowledge Sources, Subject Matter Knowledge, Pedagogical Content Knowledge, and Limitations were formulated to provide in-depth insights into the aspects influencing learning. The results can be seen in Figure 2.

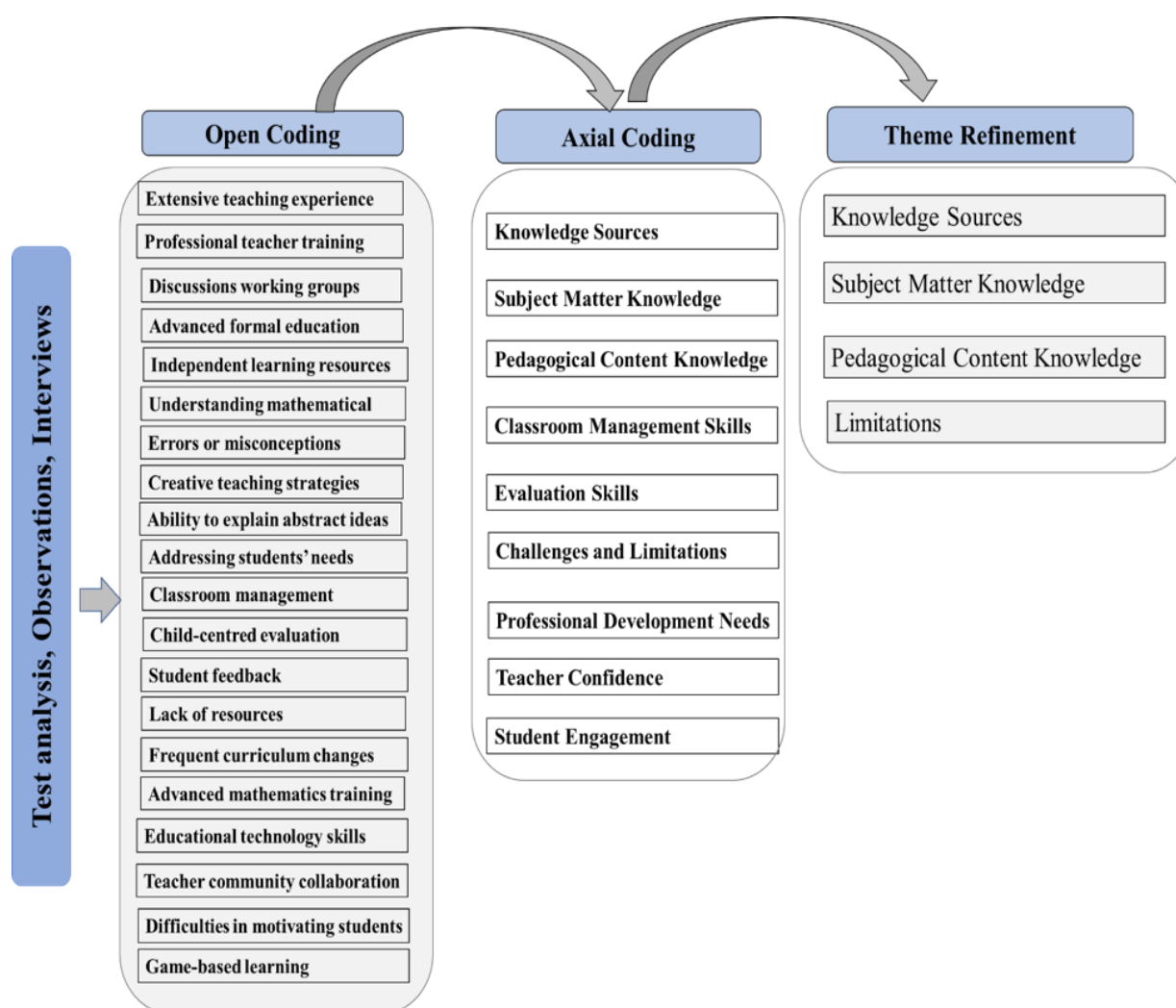


Figure 2. Results of the thematic analysis

Based on the results of the thematic analysis, the themes generated in this analysis are a) Knowledge Sources; b) Subject Matter Knowledge (SMK); c) Pedagogical Content Knowledge (PCK); and d) Limitations. The interrelationship among the themes is presented in Figure 3.

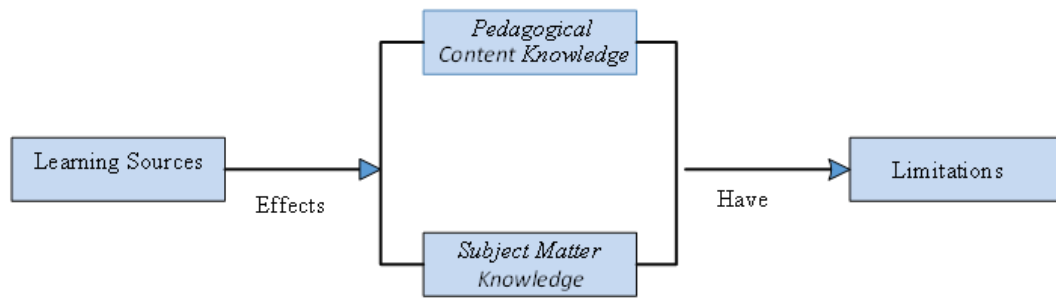


Figure 3. Interrelationship among themes

4.1. Teacher's Knowledge Sources

All participants agreed that experience is the most frequently used source of knowledge. During more than 10 years of teaching, participants recognised various children's characteristics and teaching situations. Siti and Popon revealed that through experience, they became skilled at making learning plans, managing classes, and conducting evaluations, while Srimaya revealed that knowledge, when answering questions, is obtained through experience.

Apart from experience, some participants stated that lectures and training/workshops are another source of knowledge. Santi, for example, participated in a professional teacher training program. One of the materials taught is mathematics teaching media. *"So, I learned how to teach using a 'smart tube' that can be rotated and some replicas of eggs. The eggs were then written with numbers 1-10, and the children rotated the tube until the eggs were released from the tube while counting the eggs," Santi said.* In addition, all participants have participated in the 2013 curriculum implementation workshop organized by the Education Office.

The next source of knowledge is college and school. All participants agreed that teacher's knowledge, especially about mathematical content and learning strategies, can be obtained from their studies in schools and college. In addition to the sources described, Santi added that she is very actively involved in the Teacher Work Group. In this group, teachers design instruction together with the group members. The teachers also discuss lesson plans and the curriculum implemented in their schools that is guided by the 2013 curriculum mandated by the government and adjusted to the characteristics of the school. Meanwhile, Ani explained that *"I design the instruction myself, of course guided by the prevailing curriculum. The design changes almost every year, because I always base it on evaluation from previous years' learning."*

4.2. Subject Matter Knowledge (SMK)

Subject Matter Knowledge (SMK) is the level of teachers' knowledge in mastering mathematics, especially the material taught (Ball et al., 2008; Ball & Bass, 2003). As stated earlier, the test items on basic mathematics developed by Briggs (2018) were used as a reference to measure SMK. The results show that participants answered the questions well. All participants could correctly answer questions on sequence of numbers, addition, and some geometry and measurement questions. However, there were two sets of questions that participants answered incorrectly because they did not know about the materials. Examples of such questions are: *"What is the difference between number and numeral?"*, *"Is π a rational or irrational number?"*, and *"What do you know about circles? Explain their difference from the ball?"*

In addition to test items, SMK was also measured through observation. As described in the method section, observations were made based on agreement with participants according to the learning plans they had made. Based on the plans, each participant agreed to be observed for 4-5 meetings. The findings show that four participants carried out development activities properly and

there were no misconceptions in teaching. However, it was not the same for Ani, who made a mistake when inviting children to play *Sondah* (hopscotch). Ani told the children several times to jump in a square, while the shape she made was actually rectangular. When interviewed, Ani stated that she was still confused by the concepts of numbers and geometry. So, at the end of the meeting, we discussed the basic mathematical concepts. As for other participants, they did not make any mistake in teaching concepts.

As discussed in the Subject Matter Knowledge section, the participants have limitations on the concepts of number, solid figure, and plane figure because they never applied them to children. To follow up on this finding, the researcher (R) interviewed them. The following is an example of an interview with Srimaya (S).

R: What do you know about the square?

S: The square is quadrilateral with the same sides. But, the children sometimes call it a “box”.

R: What is the shape of this object, ma'am? (showing her a square object)

S: Square (with a smile)

Q: Ok then, what shape is this paper? (showing her a rectangular shape)

S: Square

Q: What is the difference between a square, a rectangle, and a quadrilateral then?

S: Sorry ma'am, I'm confused (laughing) ...

Meanwhile, the answers from other participants to questions on quadrilateral figure are provided in Table 2.

Table 2. Participants' answers to basic mathematics questions

Participants	Answer
Popon	“The difference between a quadrilateral and a square lie in the lengths of the sides. A quadrilateral's sides are of similar lengths, while a rectangle has long(er) sides. So, there are many kinds of quadrilaterals like a rectangle, a kite, a trapezoid.”
Santi	“A quadrilateral is a four-angle line. Well, there's a rectangle and a square, so in my opinion a quadrilateral is the one that is square-shaped. Like a box, yes, like that. If a child gives an example of a box, well that is a rectangle. So, the square is one that has four sides.”
Siti	“Rectangles have four sides. They are just the same.”
Popon	“There are many kinds of square. There is a quadrilateral, which is the one with the same sides; there is also a kite, a parallelogram; so, if asked (my answer is) that a square has four sides, and there are many (types of it). As for this one, well this is still a quadrilateral.”

Based on the interview, several participants revealed that they already knew about two-dimensional (plane) and three-dimensional (solid) figures, from their schools and college years. However, because they never applied these concepts to children, all participants stated that they forgot about the concepts.

4.3. Pedagogical Content Knowledge (PCK)

PCK in this context refers to the ability to teach mathematics effectively (Ball et al., 2008; Ball & Bass, 2003). It comprises of Knowledge of Content and Students (KCS), Knowledge of Content and Teaching (KCT), and Knowledge of Curriculum (KC). To find about their Knowledge of Curriculum, the researcher conducted an analysis of learning and evaluation plans based on the rubric previously validated by experts. Regarding the learning design, all participants have designed their instruction very well. Each teacher has planned play activities in annual, semester, weekly and daily programs. Based on documentary analysis, the designs made by the teachers are in accordance with the curriculum. Aspects of competency, indicators, learning objectives, teaching material, learning resources, learning media, learning models, learning scenarios and learning assessment designs are appropriately made and interrelated with each other.

Based on the documents analysed, all participants report the progress of the children after each lesson to the stakeholders daily. Ani always gives evaluation to each child, saying "Every child is given questions; every activity is recorded." Especially for the number concept, Siti gave an example of evaluating whether a child has mastered the concept of numbers or not. This evaluation is very useful for the teacher. Through this evaluation, the teacher can find out the child's development, as well as what strategies to be deployed for the next meeting.

4.4. Knowledge of Students (KCS)

To measure Knowledge of Students (KCS), the questions given were drawn from Kim (2018) Knowledge of Mathematical Development instrument. The findings show that the participants had limitations in identifying the development of children's knowledge. The five participants mostly answered "do not know" to the items in the instrument. The researcher conducted an interview related to the results obtained by the participants. They reasoned that they never learned by paying attention to the stages of a child's ability as questioned in the test. Santi had some knowledge about the stages of a child development from the attachment to the Regulation of the Ministry of Education and Culture No. 137, but she did not know about it elaborately. In the results of Pedagogical Content Knowledge (PCK) test, it is found that all participants had limited Knowledge of Students (KCS), specifically about children's mathematical development. All participants responded "do not know" to most of the questions given. These results are reinforced by the following examples of observations.

During classroom teaching and learning, Siti invited children to sing at the beginning of learning. Next, Siti (S) threw a question on the number of children present in the class and that of children (C) who were absent, and then she counted them.

S: Okay, let's count how many boys do we have in the class?

C: One, two, three four, five ... there are five (while shouting)

S: How many girls?

C: one ... two ... three ... four ... there are four ...

S: So how many male and female students are there? (the teacher asks the child to add $5 + 4$) ...

Five in the mouth (pointing to the mouth), four on your fingers (while raising four fingers)

Follow me, okay ... five ... six ... seven ... eight ... nine (the children following her) ... So there are nine ...

When confirmed about the statement "five in the mouth, four on the fingers," Siti explained that this was done to facilitate teachers in teaching addition. In this case, Siti ignored the different stages of a child's abilities, and asked all children to do the addition in her way.

4.5. Knowledge of Teaching (KCT)

To find out the Knowledge of Teaching (KCT) of the participants, the researcher gave questions on the analysis of articles on learning cases. One of the articles presented to them was concerned with the development activities carried out by Ibu Putri to introduce the concept of numbers one to three to children in kindergarten. All participants properly identified the strengths and weaknesses of Ibu Putri's teaching activities. The results of their analysis are presented in Table 3.

Table 3. Teachers' identification of the case's strengths and weaknesses

Participant	Strengths	Weaknesses
Popon	The teacher teaches the concept of number three in each activity.	The teacher asks the child to immediately write number three at the beginning of the meeting.
Santi	The teacher introduces numbers to children in the form of activities that are not abstract.	The teacher does not invite children to be involved in fine motor skills before teaching writing number 3.
Siti	The teacher introduces numbers to children in the form of activities that are not abstract.	The teacher continuously assists children, which is considered inappropriate.
Ani	The teacher makes some reflection.	The teacher lacks mastery of the class.
Srimaya	The teacher makes some reflection.	The teacher does not use a specific strategy in teaching number 3.

Based on the interview, Popon emphasized that in the article presented to the participants, the teacher's action to ask the child to immediately write number three at the beginning of the meeting is an inappropriate step. Special strategies and exercises for children are needed before teaching writing number three. For example, children can be invited to do fine motor skill activities, such as tearing paper or learning number one first. Other participants considered it inappropriate for the teacher to continuously help the children and advised the teacher to gradually reduce her assistance.

In addition to deploying a case study, KCT was also measured through classroom observation and interviews to follow up on findings. Here, all participants properly carried out mathematical development activities in accordance with the design they had made. All respondents chose learning media from the immediate environment around children, such as clothespins, rocks, seeds, buttons, bottle caps, beams, Lego blocks, sticks, candles, paper and the like. Siti added that children like colouring paper the most. Children will tear the coloured paper into several pieces as they desire. In addition to learning numbers, children's motor skills will also be trained. Santi chose a different medium, "Songs can also be a medium to introduce various themes of numbers, like the 'My Five Balloons' song. Or we can be creative by making simple songs by ourselves while demonstrating our fingers as a tool for counting, so that children will gradually catch up with abstract concepts in numbers," Santi said.

Communicating effectively, empathically and politely with young children is a must for teachers. During observations, all participants had very good communication with children. They were very good at attracting attention so that children focused on the lessons. When children lost their concentration, the teachers invited them to sing or persuaded them by giving their favourite toys in class. Playing is indeed the main activity in the process of early childhood learning. All participants are skilled at designing and conducting play activities in introducing mathematics. The following are the types of games performed in the participants' class and their benefits.

Table 4. Types of games and their benefits

Participant	Type of Game	Explanation of Its Benefits
Popon	Snake and Ladders	This is a game for kids played by two or more players. Children will move up or down the ladders according to the number shown on the dice that is rolled in turn. If they happen to step on a ladder, they will climb the ladder; on the other hand, if they step on a snake's mouth, they must go down the tail. In this activity, the children will learn to count, add, and subtract numbers.
Srimaya	Jump rope	In this game, children will learn to count the number of jumps made. In other words, they will learn sequence counting.
Siti	Hen and its chicks	In this game, one child will play the role of a wolf who is going to eat the chicks. The mother of the chicks will protect her children. In the game, students will learn to count the number of chicks that get caught by the wolf and the number of chicks the mother hen has successfully protected. The game is also accompanied by songs, so that children will feel happy, and at the same time, they will learn to count and add up numbers.
Ani	Hopscotch	In this game, children will jump from one square to the next. In this game, children will have to avoid jumping onto a square containing fragments of tile roof. A player loses when s/he steps on or outside the line. Through this activity, children learn to count.
Srimaya	Running race	This game involves several balls and three pairs of buckets. To play this game, children will have to run and move the balls from one bucket to another. Children should move several balls according to the teacher's instruction. In this game, children learn to count.

5. Discussion

Based on the findings, it was found that early childhood teachers have a limited understanding of mathematical content. This is evident in their difficulty in linking fundamental mathematical concepts with contextual and relevant learning activities for young children. Moreover, their responses during interviews and observations revealed gaps in their mastery of basic mathematical principles. These findings are in line with those of Sarama et al. (2003) who reported that teachers tend to rely on text for context, not emphasizing interactive discourse that supports the task. Most ECE teachers identify this discipline as an area of their weakest concentration (Kilday & Kinzie, 2009); they are often uncomfortable with mathematics (Dunphy, 2009; Noviyanti & Suryadi, 2019). As professional educators, teachers are not only required to carry out their duties professionally, but they also must have professional knowledge and abilities. Knowledge of content influences: (a) student mathematics achievement (Baumert et al., 2010; Gess-Newsome et al., 2019); b) how the teacher composes and teaches the material (Mishra et al., 2009); c) the teacher's ability to connect concepts and skills during teaching (Clements et al., 2017); and d) specific decisions on pedagogical content (Kuntze, 2012).

The next finding shows that participants' knowledge about children's mathematical development is also still limited. This can be seen from the participants' answers to questions on a child's development problems. In addition, the results of observations on one of the participants show that

the participant tend to guide children to learn without paying attention to individual child development. The finding of this study corresponds to that of Kim et al. (2015), revealing that teachers have limited knowledge of child development. Meanwhile, teachers' knowledge of mathematical development determines their ability to give significant impacts on children's development of mathematical concepts and goals (Björklund et al., 2020; Björklund & Ahlskog-Björkman, 2017; Björklund & Barendregt, 2016). If teachers know what conceptions and misconceptions children might face as they develop their mathematical content, teachers will be better prepared to face the challenges (Mapolelo & Akinsola, 2015). Feeling motivated to apply knowledge of student thinking in a more meaningful way will help teachers build an in-depth understanding because the teachers base their instruction on students' own thinking strategies (Fives & Buehl, 2008). Teachers as the More Knowledgeable Other (MKO) need to have the ability to anticipate how students think through an assignment from a development activity (Noviyanti, 2020).

The limited knowledge of ECE teachers is not something new. Some researchers have reported that most colleges and universities do not offer much information about the topic of mathematics education for ECE, or supplement teacher education with adequate skills to teach mathematics to children (Ginsburg et al., 2008; Hachey, 2013; Thiel, 2010). Education for early childhood teachers offers some or no requirements for mathematics education (Aguirre et al., 2013; Ginsburg et al., 2008). Some early childhood programs do not focus on high-quality math instruction, although much research supports the theory that early mathematical experience promotes children's readiness and can influence the outcome of later mathematics education (Björklund et al., 2020; Clements et al., 2017).

In this regard, the findings of this research contribute to the design of PDP. Since the teachers under research cited the inability to recall the mathematical concepts (forgetfulness) as their main reason for having these limitations, giving a summary of the concepts is an appropriate strategy to overcome these limitations. This strategy is very important to recall the main ideas of the material to be presented, to prevent the occurrence of forgetfulness and reduce the difficulties experienced by students in remembering all the material (Cockcroft, 2015; Noviyanti et al., 2020). Giving a summary is one model of design in teaching to conduct a review of the material presented (Noviyanti, 2020; Zaslow et al., 2010), and useful to prevent the occurrence of forgetfulness in the newly taught material (Cockcroft, 2015; Noviyanti et al., 2020). In other words, it is suggested that summary is a useful strategy for reviewing what is learned (Noviyanti & Suryadi, 2019). Because the limitations of basic mathematics are only found in concepts (not dominant), it is recommended that the summary is only given in one session at the beginning of the meeting.

The second limitation of the participants is in terms of children's mathematical development. This limitation is more dominant, because all participants could not properly answer most questions on this topic. Based on this finding, the researcher recommends a Cognitively Guided Instruction (CGI) model. CGI is an approach in teaching mathematics that is built on children's natural problem-solving strategies (Carpenter et al., 1996). The CGI program, as described by Carpenter et al. (1996) identifies specific strategies that help teachers to understand how children think so that teachers can direct children to mathematical understanding. SGI is an attempt to make teachers adjust themselves to students' mathematical understanding. This model stands in contrasts with the more traditional approach to teaching symbolic counting first and then trying to make students apply this knowledge in problem solving situations.

In practice, the CGI model uses videos as a tool to invoke teachers' knowledge. The purpose of watching video recordings is to make teachers master typical mathematical concepts at the class level they teach (Carpenter et al., 1996; Noviyanti, 2020). In addition to providing videos, it is also recommended to provide articles in the form of stories that describe classroom activities. In this case, participants can discuss the patterns and development of children's mathematics through the

intervention of the videos and articles. According to Carpenter et al. (1996), there are several things that might be discussed in the CGI model, namely: (1) how the child sees a problem; (2) what approach is taken by the child to solve the problem and what it means in terms of conceptual abilities of the child; (3) why the solution works and how the solution is different from other solutions; (4) how this child solves other types of problems, and (5) how this child's solution can indicate the child's overall understanding of different concepts.

This research has a positive impact on teacher education, especially in Indonesia, because it provides a general description of ECE teachers' Mathematical Knowledge for Teaching and their limitations and offers solutions to overcome these limitations. The PDP design developed is expected to improve the quality of teachers, especially those who already hold a bachelor's degree in ECE education and become classroom teachers in Indonesia. Through the CGI model, teachers can utilize their experiences in the classroom to improve their knowledge of children's development. This is in line with the statement of Noviyanti (2020) that PDP is prioritized because the education community has realized that school is a place for teachers to learn.

This study also has a positive impact on mathematics education, considering the importance of early childhood mathematics on children's mathematical achievements in the future. For this reason, the improved quality of ECE teachers in Indonesia can affect mathematics education in general. This is in line with the National Association for the Education of Young Children and National Council of Teachers of Mathematics (2000) statement that to promote the development of high-quality mathematics education, program developers and policy makers must: (a) make more effective early childhood teacher preparation and continue professional development in mathematics; (b) use a collaborative process to develop systems that are in accordance with high quality standards, math curriculum, and assessment; (c) design institutional structures and policies that support teacher mathematics learning, team work, and planning; and (d) provide the necessary resources that will help overcome obstacles to improving the mathematical abilities of children in the classroom and society.

6. Conclusion

This study aims to determine the MKT of ECE teachers and explore the reasons behind the results. The findings of this study provide a basis for designing a PDP. The results show that ECE teachers have limited knowledge related to basic mathematical abilities and children's mathematical development. Based on these findings, the researcher suggests the use of a combination of a summary strategy and the CGI model in the PDP. In its implementation, the PDP should be more focused on the CGI model, given that the teachers' limited knowledge of children's mathematical development is more dominant. This research has a positive impact on improving the quality of ECE teachers in Indonesia as it provides an overview of the limitations in ECE teachers' Mathematical Knowledge for Teaching and offers solutions to address these limitations. Additionally, this research contributes to mathematics education, aligning with several studies that highlight the importance of early childhood mathematics for children's future mathematical achievements. The proposed PDP is expected to improve teacher quality and the quality of education in Indonesia, especially early childhood mathematics education.

However, this study has several limitations. First, the sample size was small, which may limit the generalizability of the findings, as it may not fully represent all ECE teachers in Indonesia. The research was also conducted in specific regions, which may not reflect the diversity of teaching practices across the country. Additionally, this study focused primarily on MKT and did not consider other factors, such as cultural or social influences, that might affect teacher development. The study also assessed teachers' knowledge at one point in time and did not evaluate the long-term impact of the PDP. Lastly, the proposed PDP model may not address all aspects of teacher development, such

as emotional or social factors, suggesting the need for further improvement in future programs. These limitations point to areas for further research and refinement.

7. Suggestion

This study suggests that the PDP for ECE teachers should focus on the CGI model to help teachers understand children's mathematical development. Additionally, a combined approach with summarisation strategies is recommended. The PDP should be followed by continuous evaluation to ensure improvements in teaching quality. Strengthening collaboration between educational institutions and professional training bodies is also essential to ensure the PDP's relevance. The findings of this study are expected to support educational policies that enhance the quality of mathematics education in early childhood, ultimately impacting children's future mathematical achievements.

Declarations

Author Contributions. M.N.: Literature review, conceptualization. M.N. and S.: methodology, data analysis. M.N. and C.A.R.: review-editing and writing, original manuscript preparation.

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Data Availability Statement. The data supporting the findings of this study are available upon reasonable request. For access to the datasets used and analysed during this research, please contact the corresponding author at meryn@ecampus.ut.ac.id. All data have been anonymized to protect the privacy of participants.

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References

- Aguirre, J. M., Turner, E. E., Bartell, T. G., Kalinec-Craig, C., Foote, M. Q., Roth McDuffie, A., & Drake, C. (2013). Making connections in practice: how prospective elementary teachers connect to children's mathematical thinking and community funds of knowledge in mathematics instruction. *Journal of Teacher Education*, 64(2), 178-192. <https://doi.org/10.1177/0022487112466900>
- Ali, C. A., Asemanni, E., & Tangkur, M. (2024). Pupils' didactical Mileu in an inclusive discourse in basic triangle geometry. *International Journal of Didactic Mathematics in Distance Education*, 1(2), 70-82. <https://doi.org/10.33830/ijdmde.v1i2.9523>

- Ball, D. L., & Bass, H. (2003). Toward a practice-based theory of mathematical knowledge for teaching. *Annual Meeting of the Canadian Mathematics Education Study Group*.
- Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? In *Journal of Teacher Education*, 59(5), 389-407. <https://doi.org/10.1177/0022487108324554>
- Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., Klusmann, U., Krauss, S., Neubrand, M., & Tsai, Y. M. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal*, 47(1), 133-180. <https://doi.org/10.3102/0002831209345157>
- Björklund, C., & Ahlskog-Björkman, E. (2017). Approaches to teaching in thematic work: early childhood teachers' integration of mathematics and art. *International Journal of Early Years Education*, 25(2), 1-14. <https://doi.org/10.1080/09669760.2017.1287061>
- Björklund, C., & Barendregt, W. (2016). Teachers' pedagogical mathematical awareness in Swedish early childhood education. *Scandinavian Journal of Educational Research*, 60(3), 359-377. <https://doi.org/10.1080/00313831.2015.1066426>
- Björklund, C., van den Heuvel-Panhuizen, M., & Kullberg, A. (2020). Research on early childhood mathematics teaching and learning. *ZDM - Mathematics Education*, 52(4), 607-619. <https://doi.org/10.1007/s11858-020-01177-3>
- Briggs, M. (2018). Mathematics in the early years: a bolder start. *FORUM*, 60(3), 317-326. <https://doi.org/10.15730/forum.2018.60.3.317>
- Carpenter, T. P., Fennema, E., & Franke, M. L. (1996). Cognitively guided instruction: a knowledge base for reform in primary mathematics instruction. *Elementary School Journal*, 97(1), 3-20. <https://doi.org/10.1086/461846>
- Cerezci, B. (2020). Measuring the quality of early mathematics instruction: A review of six measures. *Early Childhood Education Journal*, 48(4), 507-520. <https://doi.org/10.1007/s10643-019-01013-8>
- Chen, S., & McDunn, B. A. (2022). Metacognition: History, measurements, and the role in early childhood development and education. *Learning and Motivation*, 78, 101786. <https://doi.org/10.1016/j.lmot.2022.101786>
- Clements, D. H., Fuson, K. C., & Sarama, J. (2017). What is developmentally appropriate teaching? *Teaching Children Mathematics*, 24(3), 178-188. <https://doi.org/10.5951/teacchilmath.24.3.0178>
- Clements, D. H., & Sarama, J. (2011). Early childhood teacher education: The case of geometry. *Journal of Mathematics Teacher Education*, 14(2), 133-148. <https://doi.org/10.1007/s10857-011-9173-0>
- Cockcroft, K. (2015). The role of working memory in childhood education: Five questions and answers. *South African Journal of Childhood Education*, 5(1), 1-18. <https://doi.org/10.4102/sajce.v5i1.347>
- Drever, A. I., Odders-White, E., Kalish, C. W., Else-Quest, N. M., Hoagland, E. M., & Nelms, E. N. (2015). Foundations of financial well-being: Insights into the role of executive function, financial socialization, and experience-based learning in childhood and youth. *Journal of Consumer Affairs*, 49(1), 13-38. <https://doi.org/10.1111/joca.12068>

- Dunphy, E. (2009). Early childhood mathematics teaching: challenges, difficulties and priorities of teachers of young children in primary schools in Ireland. *International Journal of Early Years Education*, 17(1), 3-16. <https://doi.org/10.1080/09669760802699829>
- Efwan, N. S., Afriansyah, E. A., Luritawaty, I. P., Arwadi, F., & Yadav, D. K. (2024). The Level of students' mathematical creative thinking skills as measured by their self-confidence. *International Journal of Didactic Mathematics in Distance Education*, 1(2), 125–136. <https://doi.org/10.33830/ijdmde.v1i2.9355>
- Ekmekci, A., Corkin, D. M., & Fan, W. (2019). A multilevel analysis of the impact of teachers' beliefs and mathematical knowledge for teaching on students' mathematics achievement. *Australian Journal of Teacher Education*, 44(12), 57-80. <https://doi.org/10.14221/ajte.2019v44n12.4>
- Falletta, S. V. (1998). Book review: evaluating training programs: the four levels. *American Journal of Evaluation*, 19(2), 259 – 279. <https://doi.org/10.1177/109821409801900215>
- Fatihah, J. J., Sudirman, S., & Mellawaty, M. (2023). Improving geometric thinking skills through learning cycles assisted by interactive geometry books. *International Journal of Mathematics and Sciences Education*, 1(2), 81–85. <https://doi.org/10.59965/ijmsed.v1i2.74>
- Fives, H., & Buehl, M. M. (2008). What do teachers believe? Developing a framework for examining beliefs about teachers' knowledge and ability. *Contemporary Educational Psychology*, 33(2), 134–176. <https://doi.org/10.1016/j.cedpsych.2008.01.001>
- Gasteiger, H., Bruns, J., Benz, C., Brunner, E., & Sprenger, P. (2020). Mathematical pedagogical content knowledge of early childhood teachers: a standardized situation-related measurement approach. *ZDM - Mathematics Education*, 52(2), 193–205. <https://doi.org/10.1007/s11858-019-01103-2>
- Gess-Newsome, J., Taylor, J. A., Carlson, J., Gardner, A. L., Wilson, C. D., & Stuhlsatz, M. A. M. (2019). Teacher pedagogical content knowledge, practice, and student achievement. *International Journal of Science Education*, 41(7), 944-963. <https://doi.org/10.1080/09500693.2016.1265158>
- Gifford, S. (2004). A new mathematics pedagogy for the early years: In search of principles for practice. *International Journal of Early Years Education*, 12(2), 99-115. <https://doi.org/10.1080/0966976042000225507>
- Ginsburg, H. P., Lee, J. S., & Boyd, J. S. (2008). Mathematics education for young children: what it is and how to promote it. *Social Policy Report*, 22(1), 1-23. <https://doi.org/10.1002/j.2379-3988.2008.tb00054.x>
- Hachey, A. C. (2013). The early childhood mathematics education revolution. *Early Education and Development*, 24(4), 419-430. <https://doi.org/10.1080/10409289.2012.756223>
- Kermani, H., & Aldemir, J. (2015). Preparing children for success: integrating science, math, and technology in early childhood classroom. *Early Child Development and Care*, 185(9), 1504-1527. <https://doi.org/10.1080/03004430.2015.1007371>
- Kilday, C. R., & Kinzie, M. B. (2009). An analysis of instruments that measure the quality of mathematics teaching in early childhood. *Early Childhood Education Journal*, 36(4), 365–372. <https://doi.org/10.1007/s10643-008-0286-8>
- Kim, S. (2018). Technological, pedagogical, and content knowledge (TPACK) And beliefs of preservice secondary mathematics teachers: examining the relationships. *Eurasia Journal of Mathematics, Science and Technology Education*, 14(10), em1590. <https://doi.org/10.29333/ejmste/93179>

- Kim, Y., Sherraden, M., Huang, J., & Clancy, M. (2015). Child development accounts and parental educational expectations for young children: Early evidence from a statewide social experiment. *Social Service Review*, 89(1), 99–137. <https://doi.org/10.1086/680014>
- Kirkpatrick, D. L., & Kirkpatrick, J. D. (2006). *Evaluating Training Programs: The Four Levels* (3rd ed.). San Francisco, CA: Berrett-Koehler Publishers, Inc.
- Krippendorff, K. (2018). *Content analysis: An introduction to its methodology*. Sage publications. <https://doi.org/10.4135/9781071878781>
- Kuntze, S. (2012). Pedagogical content beliefs: Global, content domain-related and situation-specific components. *Educational Studies in Mathematics*, 79(2), 273-292. <https://doi.org/10.1007/s10649-011-9347-9>
- Leong, K. E., Meng, C. C., & Abdul Rahim, S. S. (2015). Understanding Malaysian pre-service teachers mathematical content knowledge and pedagogical content knowledge. *Eurasia Journal of Mathematics, Science and Technology Education*, 11(2), 363-370. <https://doi.org/10.12973/eurasia.2015.1346a>
- Lino, D. (2016). Early childhood education: Key competences in teacher education. *Educația Plus*, 14(3), 7-15.
- Mapolelo, D. C., & Akinsola, M. K. (2015). Preparation of mathematics teachers: lessons from review of literature on teachers' knowledge, beliefs, and teacher education. *American Journal of Educational Research*, 3(4), 1-12. <https://doi.org/10.12691/education-3-4-18>
- Mellawaty, M., Sudirman, S., Waluya, S. B., & Rochmad, R. (2019). Creative thinking ability on the integrating mathematical habits of mind in missouri mathematics project learning. In *Journal of Physics: Conference Series* (Vol. 1315, No. 1, p. 012083). IOP Publishing.
- Mishra, P., Koehler, M., & Harris, J. (2009). Teachers' technological pedagogical content knowledge and learning activity types: curriculum-based technology integration reframed. *Journal of Research on Technology in Education*, 41(4), 393-416. <https://doi.org/10.1080/15391523.2009.10782536>
- Moore, S., & Cuevas, J. A. (2022). The effects of cognitively guided phonetic instruction on achievement and self-efficacy in elementary students in a response to intervention program. *Journal of Pedagogical Research*, 6(4), 35-49. <https://doi.org/10.33902/JPR.202215063>
- National Council of Teachers of Mathematics (NCTM). (2000). *Principles and Standards for School Mathematics*. USA: NCTM
- Noviyanti, M. (2020). Teachers' knowledge of children's mathematical development. *Journal of Education and Learning*, 14(3), 315-321. <https://doi.org/10.11591/edulearn.v14i3.15872>
- Noviyanti, M. E. R. Y., & Suryadi, D. I. D. I. (2019). Basic mathematics knowledge of early childhood teachers. *Journal of Engineering Science and Technology*, 1, 19-27.
- Noviyanti, M., Suryadi, D., Fatimah, S., & Adriany, V. (2020). Mathematics and children's construction in early childhood education. In *Early Childhood Education in the 21st Century*. <https://doi.org/10.1201/9780429434914-13>
- Papandreou, M., & Tsiouli, M. (2022). Noticing and understanding children's everyday mathematics during play in early childhood classrooms. *International Journal of Early Years Education*, 30(4), 730-747. <https://doi.org/10.1080/09669760.2020.1742673>

- Park, M. H., Dimitrov, D. M., Patterson, L. G., & Park, D. Y. (2017). Early childhood teachers' beliefs about readiness for teaching science, technology, engineering, and mathematics. *Journal of Early Childhood Research*, 15(3), 730-747. <https://doi.org/10.1177/1476718X15614040>
- Parks, A. N., & Wager, A. A. (2015). What knowledge is shaping teacher preparation in early childhood mathematics? *Journal of Early Childhood Teacher Education*, 36(2), 124-141. <https://doi.org/10.1080/10901027.2015.1030520>
- Pela, S. O., Le, N. N., Kaboro, P. G., & Nurjamil, A. (2023). Innovation of physics e-module: utilizing local wisdom of lampung's handwritten batik in teaching heat and temperature material to foster students' scientific attitude. *Schrödinger: Journal of Physics Education*, 4(4), 132-138. <https://doi.org/10.37251/sjpe.v4i4.924>
- Piasta, S. B., Logan, J. A. R., Pelatti, C. Y., Capps, J. L., & Petrill, S. A. (2015). Professional development for early childhood educators: Efforts to improve math and science learning opportunities in early childhood classrooms. *Journal of Educational Psychology*, 107(2), 404-422. <https://doi.org/10.1037/a0037621>
- Rini, R. Y., Mutaqin, M. F. T., & Fajari, L. E. W. (2023, December). Implementation of Learning with the Science Technology Engineering Art and Mathematics (STEAM) Approach in Bringing Up the Higher Order Thinking Skills (HOTS) Profile in Early Childhood. In Proceedings of the 6th International Conference on Learning Innovation and Quality Education (ICLIQE 2022) (Vol. 767, p. 80). Springer Nature. https://doi.org/10.2991/978-2-38476-114-2_8
- Sarama, J., DiBiase, A. M., Clements, D. H., & Spitler, M. E. (2003). The professional development challenge in preschool mathematics. In *Engaging young children in mathematics* (pp. 415-446). Routledge.
- Sarwanto, S., Fajari, L. E. W., & Chumdari, C. (2020). Open-ended questions to assess critical-thinking skills in Indonesian elementary school. *International Journal of Instruction*, 14(1), 615-630. <https://doi.org/10.29333/IJI.2021.14137A>
- Sudirman, S., Mellawaty, M., Yaniawati, P., & Indrawan, R. (2021). Augmented reality application: What are the constraints and perceptions of the students during the covid 19 pandemic's 3D geometry learning process?. In *Journal of Physics: Conference Series* (Vol. 1783, No. 1, p. 012007). IOP Publishing. <https://doi.org/10.1088/1742-6596/1783/1/012007>
- Sudirman, S., Andrés Rodríguez-Nieto, C., Bongani Dhlamini, Z., Singh Chauhan, A., Baltaeva, U., Abubakar, A., O. Dejarlo, J., & Andriani, M. (2023). Ways of thinking 3D geometry: exploratory case study in junior high school students. *Polyhedron International Journal in Mathematics Education*, 1(1), 15-34. <https://doi.org/10.59965/pijme.v1i1.5>
- Sudirman, S., García-García, J., Rodríguez-Nieto, C. A., & Son, A. L. (2024). Exploring junior high school students' geometry self-efficacy in solving 3D geometry problems through 5E instructional model intervention: A grounded theory study. *Infinity Journal*, 13(1), 215-232. <https://doi.org/10.22460/infinity.v13i1.p215-232>
- Thiel, O. (2010). Teachers' attitudes towards mathematics in early childhood education. *European Early Childhood Education Research Journal*, 18(1), 105-115. <https://doi.org/10.1080/13502930903520090>
- Watts, T. W., Duncan, G. J., Clements, D. H., & Sarama, J. (2018). What is the long-run impact of learning mathematics during preschool? *Child Development*, 89(2), 539-555. <https://doi.org/10.1111/cdev.12713>

- Whittaker, J. V., Kinzie, M. B., Vitiello, V., DeCoster, J., Mulcahy, C., & Barton, E. A. (2020). Impacts of an early childhood mathematics and science intervention on teaching practices and child outcomes. *Journal of Research on Educational Effectiveness*, 13(2), 177-212. <https://doi.org/10.1080/19345747.2019.1710884>
- Yaniawati, P., Sudirman, S., Mellawaty, M., Indrawan, R., & Mubarika, M. P. (2023). The potential of mobile augmented reality as a didactic and pedagogical source in learning geometry 3D. *Journal of Technology and Science Education*, 13(1), 4-22. <https://doi.org/10.3926/jotse.1661>
- Zaslow, M., Tout, K., Halle, T., Whittaker, J. V., & Lavelle, B. (2010). Toward the identification of features of effective professional development for early childhood educators. literature review. Office of Planning, Evaluation and Policy Development, US Department of Education.
- Zhang, W., Li, L., & Disney, L. (2024). Chinese early childhood teachers' perspectives on mathematics education in play-based contexts. *Early Years*, 44(3-4), 765-780. <https://doi.org/10.1080/09575146.2023.2237205>

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