

Research Article

Cite this article: Alabdulaziz, M. S., & Higgins, S. (2025). Faculty Members' Perspectives on Effective Instructional Methods for Teaching Calculus in UK and Saudi Universities. *Educational Process: International Journal*, 16, e2025223.
<https://doi.org/10.22521/edupij.2025.16.223>

Received February 22, 2025

Accepted May 1, 2025

Published Online May 12, 2025

Keywords: Faculty members' perspectives, calculus teaching, UK and Saudi Universities

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Faculty Members' Perspectives on Effective Instructional Methods for Teaching Calculus in the UK and Saudi Universities

Mansour Saleh Alabdulaziz , Steve Higgins

Abstract

Background/purpose. The purpose of this study was to elicit faculty members' perspectives on effective instructional methods for teaching calculus in the UK and Saudi Universities.

Materials/methods. An online questionnaire was administered to 150 UK faculty members and 156 Saudi faculty members specialising in curricula and methods of teaching mathematics and other related areas, such as pure mathematics.

Results. The results revealed that the instructional strategies viewed as most helpful in enabling students to develop a solid understanding of calculus in UK and Saudi Universities were the Flipped Classroom Strategy (FCS) and the GeoGebra program. In addition, a few participants mentioned other tools such as graphing calculators, Desmos, GeoGebra, MATLAB, Maple, WolframAlpha, and Mathematica (a computerised algebra system), along with strategies such as inquiry-based learning, project-based learning, team-based learning, and the Modified Moore method. Regarding the challenges and difficulties encountered when teaching calculus, faculty members stated that students move to university with incorrect beliefs about the best method for teaching the concept of the derivative and the concepts of limit and continuity. For instance, they considered teachers and textbooks to be the sole sources of knowledge. Finally, a common theme that emerged from the results was that all lessons are usually straightforward for students when working with calculus, with the exception of the concept of the derivative and the concept of limit and continuity.

Conclusion. This study concludes by presenting important recommendations for future practice.

1. Introduction

The global mathematics education community has recently expressed growing concern about the challenges that arise concerning the teaching and learning of calculus (Heim et al., 2015; Lang, 1999; Mkhathshwa, 2018, 2023a; Rasmussen et al., 2014). In part, this is because the concepts used in calculus are ‘inherently difficult’ (Tall, 1992, p. 14). El-khateeb (2015) highlighted the fact that students often performed poorly in calculus because they lacked the perceptual structure to understand the fundamental concepts required to learn new mathematical topics, concepts such as limit and continuity, and the concept of the derivative. Braun et al. (2017) and Hauk (2022) therefore called for teachers, universities, and colleges to identify better ways to teach mathematics in order to train the mathematicians of tomorrow.

Calculus has the power to elucidate central problems in mathematics, physical sciences, biological sciences, and engineering; hence, its concepts represent one of the most significant accomplishments of the human mind. Developed in the seventeenth century, calculus denoted a fundamental shift in numerous fields, not just mathematics. Unfortunately, calculus often serves as a filter which prevents a substantial number of students from pursuing a STEM (science, technology, engineering or mathematics) related career following their graduation from secondary school (Ellis et al., 2014; Dagley et al., 2018). Students’ ongoing interest in STEM careers and their success in learning calculus will therefore be enhanced by providing them with an improved learning experience (Ellis et al., 2014). Thus, to create a resource that calculus instructors can use to guide them on how best to teach calculus, a questionnaire was administered to experienced mathematics professors to elicit their opinions, grounded in their teaching experiences, on what they think are efficacious practices with respect to teaching calculus. The main aim of this study is to elicit faculty members’ perspectives on effective instructional methods for teaching calculus in the UK and Saudi Universities. The research questions addressed in this study were as follows:

1. What instructional strategies do calculus faculty members in the UK and Saudi Universities consider to be effective in enabling students to develop a solid understanding of calculus?
2. What aspects do they believe are easy or difficult for students when learning calculus?
3. What challenges and difficulties do they encounter when teaching calculus?

2. Literature Review

2.1. Traditional Calculus Classes

Bressoud et al. (2016) reported that in the traditional teaching and learning of calculus, students are given rules by teachers that they then utilise to solve routine problems without fully comprehending what they are aiming to achieve. A preliminary study on calculus classes in the Department of Mathematics, Tarbiyah Faculty of IAIN, Imam Bonjol Padang, indicated that the conventional strategy for teaching and learning calculus continues to predominate (Sepriyanti et al., 2017). However, in adopting this approach, students seldom question why a particular rule works and where it originates from. Skemp (1976) asserted that this strategy results in instrumental understanding, in that the rules are applied without any understanding as to why. This does not promote creative thinking and, moreover, prevents students from understanding calculus at a conceptual level.

Studies have reported that because calculus involves abstract and complex concepts, teaching and learning the subject can be extremely difficult (Gordon, 2004; Zachariades et al., 2007; Artigue et al., 2007). This is exacerbated by the fact that traditional calculus classes tend to focus on teaching computational procedures rather than ensuring students understand the concepts involved (Gordon, 2004). Numerous studies have found that the problems students face in understanding calculus are

underpinned by a poor comprehension of functions (Dubinsky & Schwingendorf, 1991; Tall & Vinner, 1981; Williams, 1991) and an inability to employ these to reason about and represent relationships (Carlson et al., 2002; Monk & Nemirovsky, 1994; Thompson, 1994).

Consequently, the instructional strategies employed in calculus have simply become a list of procedures that need to be followed, resulting in students simply applying the standard routines they perform in algebraic manipulations. As a result, less proficient students quickly become exasperated when trying to perform such manipulations in calculus (Tucker & Leitzel, 1995). Their expectation for common problems in calculus is that a lengthy algebraic solution will be available (Gordon, 2004). Furthermore, their understanding of the fundamental concepts that underpin calculus is weak (Zachariades et al., 2007). To address this, Gordon (2004) and Axtell (2006) recommended improving the calculus curriculum by balancing a conceptual understanding of calculus with the use of graphical, numerical, algebraic, and verbal representations. This is essential as visualisation and visual thinking will play a crucial role in enhancing strategies for teaching calculus. Hughes-Hallett (1991) also asserted that an understanding of calculus concepts will be enhanced by ensuring the correct balance is achieved between graphical, numerical, and analytical methods. Accordingly, Gordon (2004) proposed that to facilitate their learning of calculus, students should select appropriate tools such as graphing calculators and adopt a balanced approach to solving problems.

2.2. Students' Beliefs About Mathematics Teaching

Based on observations and conversations in which he participated throughout his lengthy career as a mathematics teacher, Garofalo (1989) identified a number of beliefs that are commonly held by secondary school mathematics students. First, formulas are more important than their derivations. Second, the only sources of authoritative knowledge are teachers and textbooks, and the majority of mathematical problems can be solved through direct application of the formulas, facts, theorems, rules, and procedures they provide. Third, only teachers/textbooks can impart knowledge, hence the beliefs on which they are predicated directly impact the ways in which students engage with mathematics, the approaches they adopt to solve mathematical problems, and their expectations as to what a mathematics classroom should be like (Bookman, 1993; Schoenfeld, 1995).

Smith and Moore (1990, 1991) asserted that school teachers should spend more time motivating students to take part in group tasks/activities and less time delivering lectures (see also Bookman & Blake, 2007). The majority of recent research studies aimed at exploring how to reform mathematics education to increase teaching efficiency focus either on calculus learning at the school and university level, or mathematics pedagogy at the elementary or secondary level (NCTM, 1989, 1991; Tucker, 1990). Consequently, those engaged in teaching mathematics now acknowledge the enormous value of calculus and hence are becoming more engaged in the process of reform (Young, 1987).

For instance, learners may understand the concepts underpinning certain functions in a manner that serves them well in particular scenarios, but this level of comprehension does not generally support the development of robust inferences of derivatives and illustrations related to this (Tall, 1992; Monk & Nemirovsky, 1994; Ferrini-Mundy & Graham, 1994; Williams, 1991; White & Mitchelmore, 1996). A study by Kuh et al. (2011) revealed that engaging students in thought-provoking and intellectually stimulating creative work is essential to advance their learning and promote collegiate excellence.

Praslon (1999) explored the excogitation of derivatives in a case study on the discontinuities present in the transition from school to university. The findings illuminate the efforts made by students to adapt mathematically robust learning aids to scenarios that are complex and puzzling. However, students' limited realm of experience means that such efforts often result in oversimplification.

2.3. Teaching the Concept of the Derivative

The concept of the derivative not only lies at the heart of undergraduate-level differential calculus but also has multiple applications in several other domains of study, such as mathematics, economics, physics, and engineering (Berry & Nyman, 2003; Jones, 2017; Mkhathshwa, 2018, 2023a; Siyepu, 2013). Given its importance in these fields, numerous studies have examined the problems students encounter in applying the concept of the derivative to solve problems based on calculus. For instance, a number of researchers have investigated students' lack of expertise in using calculus rules to calculate derivatives (Clark et al., 1997; Engelke, 2004; Mkhathshwa, 2020; Piccolo & Code, 2013). Other researchers have reported that students find it difficult to construct and/or interpret graphs of derivatives (Habre & Abboud, 2006; García-García & Dolores-Flores, 2021; Hacıomeroglu et al., 2010; Ubuz, 2007). Finally, Ibrahim and Rebello (2012) reported that although students are frequently able to understand and employ derivatives in relation to kinematics, they find it more difficult to do so in contexts unrelated to kinematics, such as in economics.

Faced with these extensive problems, it is useful to speculate about the degree to which potentially useful pedagogical approaches in mathematics education have been used to teach calculus topics such as the derivative. These include methods and approaches such as the flipped classroom, inquiry-based learning, project-based learning, team-based learning, and the Modified Moore method (Aryal, 2022; Cronhjort et al., 2018; McLoughlin, 2009; Mkhathshwa, 2021; Peter et al., 2020; Viirman & Pettersson, 2022; Wu & Li, 2017). McLoughlin (2009) identified several reasons why an inquiry-based approach is challenging to adopt when teaching calculus and suggested using the Modified Moore method as a viable alternative. We contend that, apart from the flipped classroom, the teaching approaches listed above have been insufficiently explored with respect to the teaching of calculus. In fact, research evidence indicates that the traditional lecture method continues to be the go-to method when teaching mathematics, even for undergraduate-level calculus (Ashraf, 2020; Code et al., 2014; Speer et al., 2010).

Findings derived from another strand of research suggest that to help students develop a strong comprehension of different calculus concepts, including the derivative, various educational technologies should be integrated into the teaching of calculus, including graphing calculators, Desmos, GeoGebra MATLAB, Maple, and WolframAlpha (Leng, 2011; Wu & Li, 2017; Zbarsky et al., 2021). To address the paucity of research directly examining effective pedagogical approaches for teaching calculus, the current study elicited the views of veteran calculus teachers. Related to this, a recent study by Mkhathshwa (2023b) explored the views of experts concerning effective approaches to teaching rate problems in calculus. The findings revealed that, when addressing this type of problem, students' reasoning regarding rate quantities that can be represented by derivatives could be assisted by the use of diagrams during classroom instruction.

2.4. Using the Flipped Classroom Strategy to Help Students Acquire the Concept of the Derivative

An approach that has become increasingly common in recent years is interactive teaching, whereby students actively engage in discussions, solving problems, and other tasks. An interactive-teaching model that has been successfully introduced in university mathematics is the flipped classroom (Jungic et al., 2015; Maciejewski, 2015; Murphy et al., 2015). The rationale underpinning this approach is that the imparting of information and instruction regarding content occurs outside of class time, leaving the latter free for interactions and conceptual development. This could include activities such as peer-to-peer discussions, problem solving, and the application of conceptual knowledge (DeLozier & Rhodes, 2016).

Numerous studies have assessed the effect of the Flipped Classroom Strategy (FCS). For instance, Renfro (2014) found that the FCS positively impacted students' achievement and their mathematical

reasoning in calculus. More broadly, it was a comprehensive pedagogical strategy that enabled students to become lifelong learners. By contrast, Ramaglia (2015) reported that study group averages in achievement did not differ as a result of using the FCS, and neither did students' perceptions of their mathematical abilities improve, which would have motivated them to pursue mathematics courses in the future. With regard to university students, levels of achievement were positively impacted as they preferred to watch educational videos rather than read texts when the FCS was applied (Sahin et al., 2015).

The aims of the FCS extend beyond covering the scientific content required to help students achieve good results in mathematics, particularly on courses involving calculus, which students find especially difficult. Tall (2008) asserted that students new to calculus must transform their thinking into a formal structure comprising intuitive thinking and mathematical proof. However, Dane et al. (2016) pointed out that because students typically learn by mastering the procedural knowledge associated with calculus concepts, and hence use algorithms without linking them to their conceptual knowledge, they find it difficult to conceptualise the hierarchical structure of these concepts. Zengin (2017) thus suggested that mathematics students' conceptual understanding would be enhanced by using the FCS. This is because the FCS will enable them to acquire calculus concepts more easily and form clear mental images of them, enhancing their understanding and increasing their capacity.

Although a substantial proportion of learning in the classroom is devoted to acquiring concepts, for such understanding to be meaningful, students must actively engage with the material. This is important as failure to master the fundamental concepts will create numerous problems for students learning new topics at advanced levels (Abbas & Al-Absi, 2007; Muzangwa & Chifamba, 2012; El-khateeb, 2015). Gordon and Gordon (2006) emphasised the importance of using authentic contextual examples and problems throughout the process of teaching and learning calculus, and of compelling the students to think rather than simply manipulate symbols. Dane et al. (2016) stipulated that students should be provided with tasks by their teachers that will enable them to link their conceptual and procedural knowledge, and thus acquire the calculus concepts they have learnt.

The mechanism through which conceptualisation in calculus occurs is encapsulated in Tall's theory of the Three Worlds of Mathematics (TWM) (Tall, 2008). This mechanism consists of three stages. The first stage is conceptual-embodied, where mental conceptions are built according to the characteristics things possess. Contemporary technology has a significant role to play in this stage, as constructing a clear and accurate representation of the concept facilitates the creation of drawings that align with the characteristics of the functions used in calculus, offering the potential for implementing changes that depict examples of the concept of the derivative. From the very beginning, a structural image is used as a tool to function with the assistance of computer programs. The second stage is proceptual-symbolic. In its symbolic form, the concept is employed through the application of 'process' and the thinking of 'conceptual'. It connects to the previous stage, where the association between the symbolic and graphical assists in forming a clear understanding of calculus concepts. Thus, symbols of derivation are used to deal with the image of the concept and elucidate its characteristics. The third stage is formal-axiomatic. Here, the sequence in which meaningful concepts are constructed formally is predicated on the use of proven definitions of abstract theory and the definitions of known objects. To define the formal concept of the derivative in this stage, the limit is used as a priori knowledge (Tall, 2008).

The FCS is used in all stages of the TWM to facilitate its practical application (Bram, 2013; Strayer et al., 2016). First, the mental conception is built outside of the classroom and is effectively supported by the use of technology, a process which does not require a high degree of technical aptitude (Bram, 2013). Strayer et al. (2016) stressed the importance of assignments in this stage. Collectively, the different stages stimulate students to consider the characteristics of different concepts and to search and investigate the image of the concept. In the second stage, students engage in tasks that involve

building relationships between the symbols and the wording of the term or the rules, and then describe the dynamics of the concept. This enables them to understand the different ways in which the concept can be expressed (Tall, 2008). The application of these tasks is based on Higher Order Thinking Skills (HOTS), enabling students to follow the assumptions in the task to acquire known results from the concept as the classification between validity of the outcome or error in relation to the facts given, and to concentrate on understanding and making sense of mathematical definitions. Such tasks will also enable students to construct new knowledge, which they can then implement in the future (Maharaj & Wagh, 2016; Strayer et al., 2016). The third stage focuses on the relationship between assignments and tasks, allowing students to formally construct meaningful concepts according to definitions of known objects and to depict the conceptual structure (Tall, 2008). Comprehension tests are used to assess the degree to which students have acquired the calculus concepts. This involves the teacher asking step-by-step questions in order to identify the conceptual errors students are making whilst simultaneously making sure that the concept has been correctly defined (Akgün et al., 2010). In addition, tests focusing on procedural knowledge will demonstrate whether students simply memorise concepts without thinking about or understanding them (Dane et al., 2016; Tall, 2008).

Our study found that among first-year university students, the degree to which they acquired the derivative concepts of a calculus course was somewhat imbalanced (Denbel, 2015; Vincent, 2016). A similar imbalance was identified in the capacity to employ learnt thinking skills (Al-Bajalani, 2019; Alzahrani, 2016). Moreover, despite the growing number of studies confirming that use of the FCS has had a substantial impact on students' learning, the researchers' experience of teaching indicates that university-level teaching of calculus continues to be heavily reliant on traditional lectures (Buch & Warren, 2017; Hart et al., 2017). The researchers believe it is essential to modernise university teaching strategies so that they align with existing developments by incorporating technology into the educational curriculum. Accordingly, they utilised the FCS whilst teaching the calculus course, which was based on applying critical thinking skills (CTS). They assessed the degree to which this novel compound strategy was effective in teaching undergraduate students.

2.5. The Role of Technology in the Teaching of Derivatives

Although the current study primarily focuses on faculty members' views on effective pedagogical methods for teaching calculus, it is essential to review extant literature on the specific role played by technology in this process. Indeed, one of the questionnaire items in the current study asked faculty members about the technologies they use and how they use them in teaching calculus. Most of their responses concentrated on using technology to teach students about the concept of the derivative. Hence, this issue is important to consider. Notably, numerous studies have yielded positive findings with respect to integrating technology into the teaching of derivatives (e.g. Berry & Nyman, 2003; Borji et al., 2018; Haciomeroglu & Andreasen, 2013; Hiyam et al., 2019; Illanes et al., 2022; Rosly et al., 2020; Sari et al., 2018; Tall et al., 2008). For instance, in a study examining the way students thought about the geometric relationship between a function and its derivatives, Berry and Nyman (2003) reported that students' understanding was productively supported by the use of two educational technologies: a TI-83plus graphing calculator and a calculator-based ranger. This finding aligns with that of Hiyam et al. (2019), who enhanced students' ability to form connections between graphs of functions and graphs of their derivatives using Mathematica, a computerised algebra system.

In another study, Sari et al. (2018) presented the results of a teaching experiment in which GeoGebra, a dynamic mathematics software, was used to increase the ability of students to visualise the concept of the derivative. The researchers concluded that using GeoGebra in this way can potentially enhance students' understanding of the derivative in several function representations. Haciomeroglu and Andreasen (2013) also used GeoGebra to increase students' understanding of the

concept of the derivative and reported similar results. To increase students' calculational knowledge of derivatives in differential calculus, Rosly et al. (2020) reported on the use of Wolfram Alpha, a web-based computational knowledge engine. Finally, Borji et al. (2018) found Maple to be an efficacious tool for enabling students to visualise the concept of the derivative.

Even though advocates have provided an inordinate amount of support for the use of technology in mathematics teaching at all levels, research evidence indicates that the integration of technology into mathematics education has, to date, been rather limited (Agyei et al., 2022; Kamau, 2014; Marshall, 2012). Indeed, critics' views on using technology in the teaching of mathematics, including in calculus, have been reported in a number of studies (e.g. Hegedus et al., 2015; Nabb, 2010; Sammartino, 2023; Serkan, 2013). Such critics contend that the integration of technology into mathematics classrooms has resulted in the diminution of mathematical skills among students, particularly algebraic skills, even though such skills are vital for advanced mathematics (Nabb, 2010; Serkan, 2013). Related to this, Sevimli (2016) reported the findings of a study examining students' attitudes towards the use of technology in calculus. During this research, Sevimli observed that more importance was placed on procedural skills by students receiving traditional instruction, whereas those using technology afforded greater importance to the development of conceptual skills.

2.6. Teaching the Concepts of Limit and Continuity and the Use of Technology

Calculus is an extremely important branch of mathematics, one which furnishes students with a sound foundation for learning more advanced mathematical concepts. Moreover, calculus is widely used in everyday life. For instance, it can be used to analyse and combine the functions of costs, demands, and revenues to obtain a function of total production and total profit. This equation can then be differentiated to derive a new equation that can be used to calculate profit optimisation values (output) based on the number of items produced (input) (Marsitin, 2019). Nevertheless, calculus has 'inherently difficult concepts' (Tall, 1992, p. 14), such as the limit of a function. Learning the concept of limit provides the requisite platform through which to access almost every other topic in calculus. Concepts derived using the concept of limit are continuity, convergence, derivatives, and integrals. Thus, learning calculus can be considered the study of limits. Yet despite its immense value, learners typically encounter cognitive difficulties in understanding the abstract content of calculus. For instance, prior research suggests that students' conceptual understanding of the concept of limit is incomplete and lacks structure (Cornu, 1991; Davis & Vinner, 1986; Juter, 2005; Roh & Lee, 2011; Tall, 1992; Tall & Vinner, 1981). Consequently, an increasing amount of research has been conducted over several decades to investigate these challenges. For example, students frequently encounter difficulties in comprehending the definition of convergence (Roh & Lee, 2011) and the strict definition of the limit of a function (Tall, 1992). Furthermore, students often exhibit misunderstandings that are confused with the concept of limit (e.g. Fernández-Plaza et al., 2013; Moru, 2009; Sebsibe & Feza, 2020; Williams, 1991), a factor which may be attributable to linguistic ambiguities, over-generalisation, and inadequate knowledge of the content on the part of the instructor.

A substantial amount of attention has been paid to the concepts of 'limit and continuity' by researchers, as these are essential for learning advanced mathematical concepts (Mastorides & Zachariades, 2004). Limits are fundamental calculus concepts that describe how a function behaves near a specific input (Bressoud et al., 2017). Studies have revealed the different strategies adopted by students when thinking about limits, ranging from the theoretical to the practical (Areaya & Sidelil, 2012). To understand limits and continuity where observations are not generalised in pragmatic rationality, three types of rationality need to be understood by students: only specific cases are examined; to deduce generalisations, empirical rationality is applied; theories, properties, lemma and corollary are established according to theories which describe theoretical rationality (Branchetti et al., 2020).

The dynamic nature of calculus often makes it difficult for mathematics teachers to demonstrate on a blackboard. Furthermore, according to Thompson and Harel (2021), students found the image of a function at a point to be confusing and had confusion about piecewise functions by solving inequalities, and were unable to identify the limit of a function from a graph. They also found it challenging to sketch trigonometric, piecewise, exponential, and logarithmic functions, and to identify their domains as essential calculus concepts.

Hohenwarter et al. (2008) explained that GeoGebra can assist in reducing some of the difficulties faced regarding derivatives, the limit of a function at points, the domain of a function through a dynamic approach, sketching a function given in polar or parametric functions, and trigonometric functions. Although GeoGebra cannot provide students with instantaneous feedback when the class is of a certain size, students are able to assess their own work (Wassie & Zergaw, 2019). For instance, GeoGebra can simplify the process of creating instructive practice sheets or applets that include stimulating questions. Indeed, both geometric and algebraic study tools are now included in mathematics education.

The increased access students have to software such as GeoGebra means they are provided with stimulating and productive learning environments that promote engagement and success (Yerizon et al., 2021). For example, the dynamic geometry software (DGS) makes it easier for students to understand that the graph represents a continuous function. The multiple representations supported in GeoGebra, including geometric, algebraic, and calculus commands, mean that, using visualisation, students are motivated to engage in the class and assist in clarifying the theorem (Zulnadi et al., 2019). Through the discovery of mathematical truths and relationships, students enhance their ability to think creatively (Uwurukundo et al., 2022). The DGS also illuminates the dynamic relationships between the concepts of limits, derivatives, and continuity of functions. To construct a relationship between geometry and algebra, both CAS and DGS use GeoGebra (Körtesi et al., 2022; Zengin et al., 2012).

Having reviewed previous studies, the following points were noted: Most of the studies that deal with calculus teaching indicate the modernity of the topic and its importance. In addition, the researchers benefited from the literature by drawing on the pedagogical literature and literature reviews, adopting scientific methodology to form the theoretical framework used in this research, and formulating their research methodology. Moreover, what distinguished this study is that it dealt with determining the faculty members' perspectives on effective instructional methods for teaching calculus in UK and Saudi universities, which had not been studied before.

3. Methodology

3.1. Methodology Research Approach

This qualitative study adopted an online questionnaire to elicit experts' (calculus faculty members) views on the teaching of calculus. The study was conducted in Saudi Arabia and the United Kingdom (UK) universities. All participants were faculty members who specialised in curricula and methods of teaching mathematics and other related areas, such as pure mathematics, including a total of 150 UK faculty members and 156 Saudi faculty members. The study was conducted in the second term (starting date 25.01.2024).

3.2. Questionnaire Design

The questions on the questionnaire can vary from closed-ended to open-ended depending on the nature of the data to be collected (Wilkinson & Birmingham, 2003; Cohen et al, 2007; Bryman, 2008). The questionnaire was designed in two parts. The first part (close-ended questions) contains background data about the participants, followed by nine statements aimed at obtaining the

perceptions of faculty members on effective instructional methods in the teaching of calculus in the UK and Saudi Universities.

3.3. Translation of the Questions

The questionnaire was written in Arabic and later translated into English, a process that was followed by asking four people to produce a translation to compare and ensure accuracy. For further reliability, two competent translators from the translation department in three Saudi universities were also consulted. Because the respondents' language in Saudi Arabia was Arabic, the questionnaire was read by five specialists in the Arabic language, who were consulted separately to receive their comments on the possibility of any ambiguous wording.

3.4. Questionnaire Pilot

A further process, which is of assistance in evaluating the validity and reliability of the questionnaire, is to test the research instrument (Aldridge & Levine, 2001; Bell, 2005; Bryman, 2008; Cohen et al., 2007; Gay & Airasian, 2003; Wilkinson & Birmingham, 2003). Moreover, Cohen et al. (2007) emphasise the significance of piloting the questionnaire as well as editing its contents.

The questionnaire was piloted meticulously in order to guarantee an acceptable standard of validity. Subsequently, the following groups participated in piloting it:

- Experts, as indicated by Cohen et al. (2007).
- Colleagues, as indicated by Wilkinson and Birmingham (2003).
- A participants' sample in the main study as indicated by Cohen et al. (2007), Wilkinson and Birmingham (2003) and Bryman (2008).

Sixteen Saudi colleagues, who are experienced in this area completed the questionnaire. Furthermore, they were requested to mention their individual experiences regarding ethical matters, and to determine the relevance surrounding the questionnaire questions. Twenty-three faculty members from UK and Saudi, who were representative of those who participated in the main study, completed the questionnaire.

According to Bryman (2008), this process has a potential impact on future representations within the sample. This is particularly the case where the selected group is representative of part of the main sample; consequently, the 21 faculty members who completed the questionnaire were not included. Since colleagues are unrepresentative of the target population, Aldridge and Levine (2001) are critical of the dependence on them in evaluating the questionnaire. In contrast, Cohen et al. (2007) propose that experts on questionnaires are consulted, such process being applied in this paper.

It is advocated by Gay and Airasian (2003) that, in order to verify the validity of the questionnaire, those who review it should answer it themselves. Following the completion of the questionnaire in this pilot study, each participant in every group responded to questions according to Bell (2005), which included:

- Did you fail to answer any of the questions?
- Were the instructions ambiguous in any way?
- Was the questionnaire too short, the correct length, or excessively lengthy?
- What part or parts of the questionnaire did you find unclear?
- Please suggest any topics that you think could be added.
- Was the questionnaire design appealing and acceptable?

- Do you wish to add any comments?

With regard to the level of the scope and clarity of the questionnaire, the answers were useful.

3.5. The Benefits Gained from Piloting the Questionnaires

The participants' comments at the pilot stage led to numerous advantages. Firstly, the pilot sample was clear when the Arabic and English versions of the questionnaire were compared. Because the first language of the UK participants, is English, they preferred the English version, and the first language of the Saudi participants, is Arabic, they preferred the Arabic version; moreover, Gass and Mackey (2007) advocated this. Furthermore, the participants emphasised the significance of the letter that accompanied the questionnaire, the purpose of such letter being to clarify the study. Moreover, they recommended that the participants read this letter in the main study. The researchers also used this feedback to improve the questionnaire by reducing its length and deleting repetitive statements.

3.6. Validity and Reliability of the Questionnaire

Although the questionnaire's validity and reliability may be tested by applying several methods, the principal aspect of the questionnaire is validity, of which reliability is a subset. Validity indicates the questionnaire to be comprehensive and that, in a fair manner, it addresses the matter under investigation (Gass & Mackey, 2007). Fraenkel and Wallen (2008, p. 153) define the validity as 'the appropriateness, meaningfulness and usefulness of the specific inferences researchers make based on the data they collect'. Subsequently, it is possible to verify the questionnaire's external validity by considering the opinions of academics regarding its framework and content (Bryman, 2008; Cohen et al., 2007; Cowl, 1996; Gay & Airasian 2003; Gass & Mackey, 2007). In order to attain this objective, five members of mathematics education department at Saudi Arabian universities, and six members of mathematics education department at UK universities were consulted for discussion of the questions asked in the questionnaire.

Therefore, in this study, the researcher adopted face-validity and content-validity. Oppenheim (2001) stated that 'content-validity seeks to establish that the items or questions are a well-balanced sample of the content domain to be measured' (p. 162). There are actually numerous methods of attaining content-validity, ranging from dependable research construction by submitting the plan to experts, to analysing the validity and reliability statistically. There are formulae; for example, Cronbach's Alpha coefficient which ranges from zero to one (unreliability-reliability) (Gay & Airasian, 2000).

Having established the ideas of validity and reliability, the following section will consider the face validity of this study.

3.6.1. Face-Validity

In order to attain the optimal degree of face-validity in this research, particular actions were taken prior to the final dissemination of the questionnaire. Such actions may be summarized as follows: submitting the questionnaire to a proofreader in order to improve clarity; and submitting it to experts and specialists in this area of research and study. The final step was to submit the questionnaire to a committee comprising referees, in order to assess for every aspect including clarity, language, duplication, and contradiction. The Arabic and the English versions were presented to this body, accompanied by a cover letter. Furthermore, the questionnaire was submitted to a statistician for the purpose of assessment, as well as to receive help in selecting the correct measurement (Cohen et al., 2004; Gay & Airasian, 2000; Oppenheim, 2001).

3.7. Face-Validity Sampling Procedures

Various methods may be applied in conducting this questionnaire, for instance, personal interviews, email, or mail (Aldridge & Levine, 2001; Bell, 2005; Cohen et al., 2007; Gay & Airasian, 2003; Wilkinson & Birmingham, 2003). In this study emails inviting faculty members who were specializing in mathematics education and other related areas to participate in the study along with a link to the online questionnaire were distributed. A reminder email was sent three weeks after the initial invitation to encourage participation. The questionnaire was attached with a letter explaining it as recommended by Cohen et al. (2007). Therefore, a cover letter was included with the distributed questionnaire in order to guarantee the largest possible number of participants. The letter indicated the research's principal topic and the means of responding to the questions and asked the respondents to be cooperative by replying quickly; it also provided details on how the questionnaire should be returned. Furthermore, strict confidentiality would be guaranteed with the answers, which we used exclusively for research purposes. Finally, the participants were thanked in advance for their participation. Cohen et al. (2004) stated the purpose of the covering letter was 'to indicate the aim of the survey, to convey to respondents its importance, to assure them of confidentiality, and to encourage their replies' (p.97).

3.8. Data Analysis

Strength with this study was that open questions were also used in Part 2. It consisted of nine open-ended questions, which were analysed by using a qualitative method. Thematic analysis was chosen since it is a flexible and diverse approach to analyse answers (Braun and Clarke, 2006). This type of analysis is not tied to any theoretical framework and rich descriptions of the data could be achieved by using this method. Open-ended questions were used to make the participants provide the response categories instead of being forced to choose from preselected categories selected by the researcher. Thus, insight into the participants' own perceptions about the problem is possible by using open-ended questions and the perceptions on the subjects are not closely tied to the researcher's own perceptions.

It was necessary to supply the following three ethical requirements. Firstly, all participants were informed that they were volunteers in this study and had the right to ask for any of the responses they had given previously to be removed. Secondly, the confidentiality of participants' identities and personal details was guaranteed, meaning that their names would not be included in the course of the translation procedure. The third involved providing the participants comprehensive details regarding the purposes of the research.

4. Results and Discussion

This section presents the results from the questionnaire in two sections which correspond to the two different parts of the questionnaire.

4.1. Part 1 – Demographic Analysis

The first part of the questionnaire elicited demographic information from the respondents regarding gender; age; class size; programme of studies in which the discipline is taught; teaching experience; academic status; and the sections of calculus taught. Descriptions of the respondents in terms of demographics are presented in this section.

Of the 306 respondents, 200 (65.3%) were male (110 out of the 200 from UK, and 90 out of the 200 from Saudi), and 106 (34.7%) were female (56 out of the 106 from UK, and 50 out of the 106 from Saudi). In terms of age, 70 UK faculty members were aged between 44–51, 50 were aged between 36–43, and 30 were aged between 28–35. Among Saudi faculty members, 30 respondents

were aged between 44–51, 40 were aged between 36–43, and 86 were aged between 28–35. All participants taught classes ≤ 30 students, and all taught bachelor's programmes.

The majority of faculty members (148 Saudi faculty members and 150 UK faculty members) had more than 5 years of teaching experience.

In terms of academic status, among UK faculty members, 70 respondents were professors, 50 were associate professors, 20 were assistant professors, and 10 were lecturers. Among Saudi faculty members, 30 respondents were professors, 40 were associate professors, 70 were assistant professors, and 16 were lecturers. In terms of the number of sections of calculus taught, all participants taught 11 to 15 sections in one course.

4.2. Part 2 – Open-Ended Questions

In this section, a qualitative analysis is conducted on open-ended questions. Thematic analysis is carried out where the answers are analysed according to patterns or themes (Braun and Clarke, 2006).

This section discusses the findings of the questionnaire in relation to the literature. It does so according to three dimensions: (A) effective instructional strategies for teaching calculus topics; (B) students' difficulties in learning calculus, and (C) challenges of teaching calculus.

4.2.1. Effective Instructional Strategies for Teaching Calculus Topics

All participants (except for two from the UK) agreed that the single most effective instructional strategies they have used to teach the concept of the derivative and the concepts of limit and continuity are the FCS and GeoGebra. In addition, two UK participants believed that inquiry-based learning, project-based learning, team-based learning, and the Modified Moore method, could be effective in enabling students to develop a solid understanding of calculus in UK universities (Aryal, 2022; Cronhjort et al., 2018; McLoughlin, 2009; Mkhathswa, 2021; Peter et al., 2020; Viirman & Pettersson, 2022; Wu & Li, 2017).

4.2.1.1. The Advantages of the Flipped Classroom Strategy (FCS)

Overall, 58 UK faculty members and 45 Saudi faculty members stated that the FCS enabled students to become lifelong learners, a finding in alignment with that of Renfro (2014). Furthermore, 30 UK faculty members and 25 Saudi faculty members stated that when the FCS strategy was applied, their students expressed a preference for watching educational videos rather than reading texts, and this had a positive impact on levels of achievement, especially among university students (Sahin et al., 2015).

Furthermore, 62 UK faculty members and 86 Saudi faculty members expressed the view that their students' conceptual understanding of calculus was enhanced by using the FCS, which is in line with the findings of Zengin (2017). This was evidenced in their superior acquisition of concepts, their capacity to form clear mental images of concepts, their increased ability to retain information, and the fact that understanding became much easier. This is a crucial outcome given that students who fail to master the basic concepts find it extremely challenging to learn new topics at advanced levels (Abbas & Al-Absi, 2007; Muzangwa & Chifamba, 2012; El-khateeb, 2015; Dane et al., 2016).

Overall, 60 UK faculty members and 86 Saudi faculty members reported that when using the FCS, the mental conception is created outside of the classroom, whereas within the classroom, students participate in tasks aimed at constructing relationships between the symbols and the wording of the term or the rules, and describing the dynamics of the concept. This enables them to recognise the various ways in which the concept can be expressed. Moreover, the FCS focuses on the relationship between assignments and tasks, allowing students to construct essential concepts formally according

to definitions of known objects, and to display the conceptual structure. These phases align with Tall's theory of the Three Worlds of Mathematics (TWM), the first stage of which is conceptual-embodied, where the mental conception is constructed according to the characteristics of things, the second stage is proceptual-symbolic, and the third stage is formal-axiomatic.

Regarding the use of technology, most participants (300) expressed a need for more support, with just six answering that they did not wish to use such technology (4 from the UK and two from Saudi Arabia). When asked to explain why they decided to use technology, a number of reasons were given. From this, two main themes were identified where at least three responses contained the following ideas:

Theme 1: Technology Facilitates the Robust Understanding of an Array of Calculus Concepts

10 UK participants and 4 Saudi participants stated that integrating educational technologies such as graphing calculators, Desmos, GeoGebra MATLAB, Maple, and WolframAlpha into the teaching of calculus enables students to develop a strong comprehension of different calculus concepts such as the derivative, a finding which aligns with previous studies (e.g. Leng, 2011; Wu & Li, 2017; Zbarsky et al., 2021).

A total of 5 UK participants and 10 Saudi participants expressed the belief that utilising technology such as a calculator-based ranger and a TI-83plus graphing calculator to teach derivatives in calculus enhances students' comprehension of the geometric relationship between a function and its derivatives, a finding that is in agreement with that of Berry and Nyman (2003).

Notably, most participants used GeoGebra when teaching the concept of the derivative and the concepts of limit and continuity. In explaining its advantages, 20 UK participants and 34 Saudi participants stated that GeoGebra made it easier for students to visualise the concept of the derivative, and 18 UK participants and 20 Saudi participants expressed their belief that GeoGebra could potentially enhance students' comprehension of the derivative in multiple function representations. These findings align with those of Sari et al. (2018).

Furthermore, in agreement with the results of Hohenwarter et al. (2008), 9 UK participants and 20 Saudi participants stated that GeoGebra can assist in reducing the difficulties associated with, *inter alia*, derivatives, the limit of function at points, the domain of function using a dynamic approach, sketching a function given in polar or parametric functions, trigonometric functions. This is crucial as the dynamic nature of calculus means it is an extremely difficult topic to demonstrate visually. Moreover, students found the image of a function at a point to be confusing and its limit confusion about piecewise functions by solving inequalities, and were unable to identify the limit of a function from a graph. They also found it challenging to sketch trigonometric, piecewise, exponential, and logarithmic functions, and to identify the domains of these functions as basic calculus concepts (Thompson & Harel, 2021).

In addition, 9 UK participants and 21 Saudi participants believed that student engagement and success would be increased by the fact that GeoGebra provides them with access to more stimulating and effective learning scenarios. Consequently, they understood that the graph denotes a continuous function through the DGS. Moreover, visualisation motivated students to engage in the class and assisted in clarifying the theorem. Through the discovery of mathematical truths and relationships, students' creative thinking skills developed substantially, a finding consistent with several other studies (e.g. Yerizon et al., 2021; Uwurukundo et al., 2022). As well as illuminating the concepts of limits, derivatives, and continuity of functions, the DGS depicts the dynamic relationships between these concepts. GeoGebra is used by both CAS and DGS to form links between geometry and algebra (Körtesi et al., 2022; Zengin et al., 2012). When adopting this approach to clarify the theorem, the

multiple representations supported in GeoGebra, including geometric, algebraic, and calculus commands, therefore encourage students to participate in the lesson (Zulnaidi et al., 2019).

Theme 2: Technology Can Enable Students to Make Connections Between Concepts

In line with Hiyam et al. (2019), 6 UK participants and 5 Saudi participants pointed out that Mathematica (a computerised algebra system) increased the capacity of students to, inter alia, identify links between graphs of functions and graphs of their derivatives,

Moreover, as identified in several previous studies (Körtési et al., 2022; Zengin et al., 2012; Zulnaidi et al., 2019; Uwurukundo et al., 2022), 69 UK participants and 40 Saudi participants perceived GeoGebra as useful for identifying the relationships between the concepts of limits, derivatives, and continuity of functions.

Notably, the 6 participants who opted not to use technology when teaching calculus explained that they believed this would lead to an impoverishment of the mathematical skills, particularly algebraic skills, that students need to learn advanced mathematics. Strikingly, although extremely strong support has been expressed for using technology in mathematics teaching at all levels of university, research indicates that the integration of such technology into mathematics education has, to date, been rather limited (Agyei et al., 2022; Kamau, 2014; Marshall, 2012). Given the perceived impoverishment of mathematics skills mentioned by the participants in the current study, it could be argued that such critics attribute as much value to a procedural understanding of mathematical ideas as they do to a conceptual understanding. Related to this, Sevimli (2016) found that students taught in a traditional manner gave greater importance to procedural skills, whereas students taught using technology afforded more importance to conceptual skills.

4.2.2. Students' Difficulties in Learning Calculus

Regarding the second question, questionnaire participants generally believed that lessons are usually easy for students to grasp when working with related calculus, except for the concepts of the derivative and of limit and continuity. This suggests to faculty members that a suitable teaching method needs to be employed to assist undergraduate students in understanding these concepts. This is unsurprising as multiple studies have found that students find it difficult to comprehend the concept of the derivative when faced with calculus-based problems (e.g. Clark et al., 1997; Engelke, 2004; Mkhathshwa, 2020; Piccolo & Code, 2013) and, in addition, also display a partial and unstructured conceptual understanding of the concept of limit (e.g. Cornu, 1991; Davis & Vinner, 1986; Juter, 2005; Roh & Lee, 2011; Tall, 1992; Tall & Vinner, 1981).

In the current study, most participants (130 UK faculty members and 126 Saudi faculty members) stated that students found the derivative and of limit and continuity especially challenging when solving calculus.

Regarding the concept of the derivative, one UK faculty member stated that '*students typically display an inability to apply calculus rules when calculating derivatives*'. Another UK participant stated that '*students find it difficult to construct or interpret graphs of derivatives*', a finding highlighted in previous research (García-García & Dolores-Flores, 2021; Haciomeroglu et al., 2010).

One Saudi faculty member described in detail the reasons as to why students experienced such difficulties: '*I found that students were reluctant to engage in visual reasoning in the classroom. I believe that in classroom settings, a large number of students continue to rely on using algorithms and designated procedures to solve mathematical problems. This prevents them from using visuals as a tool for reasoning and devising solutions. Furthermore, most students view visual representations, like graphs, as part of the procedure they need to follow – they do not link them to the process of*

logical reasoning or offer any rationale for the steps they take in their solution. As a result, they do not possess the skills in critical thinking and reasoning needed to move on to higher levels of education.

A UK faculty member identified another difficulty students faced: *'Although students are usually able to understand derivatives when applied to kinematics, they find this much more challenging in scenarios that do not relate to kinematics, like economics'*, an observation that aligns with the work of Ibrahim and Rebello (2012).

Another Saudi faculty member opined that *'the derivative is a concept that has a vital role to play in economics. An example of this would be a marginal analysis conducted to investigate the impact of minimal changes on the existing state of a production. Students need to be able to make sense of the derivative if they wish to know how it is used in contexts related to economics. However, they often find this challenging'*.

Regarding the difficulties students face in understanding the concepts of limit and continuity, one UK faculty member stated that *'the concepts used in calculus are innately challenging; consequently, students often display a fragmented and loosely structured understanding of the concept of limit'*. Similarly, one Saudi faculty member observed that *'in my classes, students often find the strict definition of the limit of a function hard to comprehend'*. A UK faculty member concurred, stating that *'students exhibit an array of misunderstandings regarding the concept of limit'*. Such observations support the findings of previous studies (e.g. Tall, 1992; Cornu, 1991; Davis & Vinner, 1986; Juter, 2005; Roh & Lee, 2011; Tall & Vinner, 1981). For example, students frequently find it challenging to comprehend the definition of convergence (Roh & Lee, 2011), the rigorous definition of a limit of a function (Tall, 1992), and display a variety of misunderstandings concerning the notion of limit (e.g. Fernández-Plaza et al., 2013; Moru, 2009; Sebsibe & Feza, 2020; Williams, 1991).

4.2.3. Student Challenges of Teaching Calculus

Concerning the third question, all participants agreed that teaching calculus at a university is associated with several challenges. For instance, based on observations and the conversations in which they participated throughout their lengthy careers, 230 faculty members (80 from the UK and 170 from Saudi) stated that they often faced difficulties when teaching the concepts of the derivative and of limit and continuity, and that this was evident among students who had just enrolled at university. This suggests that students arrive at university with false preconceptions regarding the way in which calculus should be taught. These include beliefs developed whilst at secondary school, including the view that teachers and textbooks are the principal arbiters of knowledge and that most mathematical problems can be solved by directly applying the formulas, facts, theorems, rules, and procedures imparted by teachers and contained in textbooks. Such findings corroborate several earlier studies' findings (e.g., Garofalo, 1989; Bookman, 1993; Schoenfeld, 1995). Such information is crucial, as most of the faculty members taking part in this study had at least 5 years of teaching experience in various academic positions, such as professor, associate professor, assistant professor, and (less commonly) lecturer.

The participants noted that they faced such challenges at the outset when they strived to use alternative methods such as GeoGebra or the FCS to teach these concepts. The principal problem was that students were unfamiliar with these tools as their high school teachers did not use them. Nevertheless, students feel that these tools are more effective in enabling them to understand the basic concepts of calculus than traditional methods of teaching, examples of which have been presented in a number of studies (e.g. Tall, 1992; Monk & Nemirovsky, 1994; Ferrini-Mundy & Graham, 1994; Williams, 1991; White & Mitchelmore, 1996).

In the current study, UK faculty members stated that students may conceptualise functions in a manner that works well in a small number of highly specific scenarios, but is generally not consonant

with or supports the development of a robust inference of derivatives, a finding also reported by Kuh et al. (2011). One of these participants asserted that intellectually stimulating, creative work is essential to enhance pupil learning and achieve collegiate excellence.

Overall, 50 UK participants argued that school teachers should focus more on motivating students to partake in group tasks/activities and less on delivering lectures, a recommendation also made by Smith and Moore (1990, 1991). Most of the leading researchers currently working on mathematics pedagogy at the elementary or secondary level, or on calculus learning at the school and university level, have long highlighted the need for a process of reform that increases the overall efficiency of teaching (NCTM, 1989, 1991; Tucker, 1990). Consequently, mathematical instructors have come to recognise the significant role played by calculus and hence are becoming more engaged in this process of reform (Young, 1987).

Thus, given that educators are cognisant that calculus is integral to a multitude of fields, and vital for solving complex problems and making informed decisions in a world that is rapidly changing, it is vital for those who teach calculus to scrutinise the instructional methods they employ and determine which are the most effective. This is why the researchers incorporated the following two questions into their questionnaire: 'What can you learn from calculus teaching methods around the world?' and 'How is calculus used in everyday life?'

All participants stated that the mathematics education community across the world has become ever more engaged with issues concerning the teaching and learning of calculus. Despite this, they believed that most instructors used traditional methods to teach calculus. Three UK participants asserted that applying rules without understanding the reasons as to why results in instrumental understanding, which precludes the development of creative thinking. Thus, students seldom ask why a rule works or where it comes from, as they lack the ability to understand calculus at a conceptual level (Skemp, 1976).

Nevertheless, with respect to the second question, there was a consensus among all participants that, from physical laws governing the universe to the intricacies of biological processes and the complexities of economic systems, practical applications of calculus pervade almost every aspect of our lives. Accordingly, we will acquire a more profound understanding of our world by leveraging the power of derivatives and integrals, facilitating innovation, discovery, and progress across a multitude of disparate fields. By exploring the limitless applications of calculus, new domains of understanding will be accessed, creating the foundation for a future sculpted by mathematical insight and inventiveness. For example, in differential calculus at the undergraduate level, the concept of the derivative is not only central but also has myriad applications in fields such as economics, physics, and engineering (see Berry & Nyman, 2003; Jones, 2017; Mkhathshwa, 2018, 2023a; Siyepu, 2013). For instance, calculus can be employed to analyse and combine functions of costs, demands, and revenues to obtain a function of total production and total profit; this equation can then be differentiated to generate a new equation that, based on the number of items produced (input), calculates profit optimisation values (output) (Marsitin, 2019).

5. The Research Implications and Limitations

Different practical and theoretical implications can be drawn from this research. From the theoretical perspective, the current study contributes by resolving the deficiency in the literature with respect to faculty members' perspectives on effective instructional methods for teaching calculus in UK and Saudi universities. The findings can benefit educators by offering insights so that they can make more informed decisions regarding calculus teaching. From a practical perspective, due to the use of the previous instructional strategies of education, the importance of these strategies has been shed on enabling students to develop a solid understanding of calculus in the UK and Saudi Universities.

Despite its strengths, there are a number of limitations with the research that need to be addressed. First, this study focused only on UK and Saudi faculty members. This was driven by the previous experience of the first author, who studied in the UK and was originally from Saudi Arabia, and the second author, who was originally from the UK. This decision enabled the project to be conducted efficiently. Second, the sample comprised faculty members only, as students were excluded owing to the challenges associated with eliciting approval to extend the survey to a larger audience. Third, in any research, there is a possibility that the answers may contain unfamiliar or unanticipated findings. This can occur irrespective of meticulous planning; for example, piloting the questionnaire as required. This was evident in the unanticipated responses to Question 9 on the questionnaire (second section), which was thought to have been caused by a misinterpretation of the question design (semantic differential). Fourth, although surveys demonstrate associations and permit description, they provide no evidence of specific causal relationships, unlike experimental research techniques. This includes the surveys conducted in this research. Creswell (2012, p. 376) explains that, 'survey designs differ from experimental research in that they do not involve a treatment given by the researcher. Because survey researchers do not experimentally manipulate the conditions, they cannot explain cause and effect as well as experimental researchers can...survey researchers often correlate variables. Casual connections could, however, be demonstrated in any future research by the application of intervention techniques.'

6. Conclusion and Recommendations

The results revealed that all participants (except for two from the UK) agreed that the instructional strategies could be effective in enabling students to develop a solid understanding of calculus. The FCS and GeoGebra programs are used in the UK and Saudi universities. In addition, a small number of participants mentioned other tools such as graphing calculators, Desmos, GeoGebra, MATLAB, team-based learning, Maple, WolframAlpha, and Mathematica (a computerised algebra system), along with strategies such as inquiry-based learning, project-based learning, team-based learning, and the Modified Moore method. Regarding the challenges and difficulties encountered when teaching calculus, most participants (130 UK faculty members and 126 Saudi faculty members) stated that students move to university with incorrect beliefs about the method for teaching the concepts of the derivative, and of limit and continuity, the principal belief being that teachers and textbooks are the sole authority of knowledge. Finally, with regard to what was easy or difficult for students when solving calculus, a common theme that emerged was that lessons are often straightforward for students when working with related calculus, the notable exceptions being the concept of the derivative and the concepts of limit and continuity.

In view of the findings, the researchers recommend the following: Firstly, adopting the FCS as an effective method for teaching the concept of the derivative and the concepts of limit and continuity to undergraduate students, thereby increasing their level of achievement in calculus. Secondly, adopting the GeoGebra program as an effective method for teaching the concept of the derivative and the concepts of limit and continuity to undergraduate students, thereby increasing their level of achievement in calculus. Thirdly, the GeoGebra programme should be included in mathematics in general, but primarily for teaching the concept of the derivative and the concepts of limit and continuity. Fourthly, mathematics teachers should deliver fewer lectures and make greater use of technologies such as GeoGebra, graphing calculators, Desmos, GeoGebra MATLAB, Maple, WolframAlpha, and Mathematica (a computerised algebra system) when teaching calculus. Fifthly,

Mathematics teachers should make greater use of strategies such as inquiry-based learning, project-based learning, team-based learning, and the Modified Moore method when teaching calculus. Sixthly, courses and training workshops should be held for secondary school mathematics teachers on how to use learning models, including the FCS and GeoGebra programs, in the mathematics (and particularly the calculus) curriculum. Seventhly, the concept of the derivative has

numerous applications in multiple fields of study; therefore, school teachers and faculty members should simplify the concept to reduce its difficulty for students. Finally, most students also faced problems with limits and continuity, yet the concept of limits provides the necessary foundation upon which to access almost all other topics in calculus. Therefore, educators should concentrate on making this concept easier for students to understand.

7. Implications for Further Studies

The researchers suggest conducting further research in the following areas: Conduct similar studies to identify the impact of other strategies on teaching calculus. In addition, similar studies should be conducted with other school grades to further identify the impact of using FCS and GeoGebra to teach mathematics. Moreover, further studies should be conducted to identify the difficulties and obstacles hindering the use of the FCS and GeoGebra in teaching mathematics. Furthermore, future studies could be extended to other areas of the education field. Finally, researchers could investigate the students' perspectives on the difficulties with learning limits and the continuity of functions.

Declarations

Author Contributions. The author has read and approved the published version of the article.

Conflicts of Interest. The authors declare no conflict of interest.

Funding. The author received no financial support for this research.

Ethical Approval. Permission to conduct the research was granted through the Research Ethics Committee of IAU (IRB-2024-15-061)

Data Availability Statement. Data is available by the corresponding author upon request.

References

- Abbas, M., & al-Absi, M. (2007). *Curricula and methods of teaching mathematics for the lower basic stage*. Jordan, Dar Al-Masirah
- Agyei, E., Darko Agyei, D., & Benning, I. (2022). In-service mathematics teachers' preparedness, knowledge, skills, and self-efficacy beliefs of using technology in lesson delivery. *Cogent Education*, 9(1), 2135851. <https://doi.org/10.1080/2331186X.2022.2135851>
- Akgün, L., Isleyen, T., Tatar, E., Soylu, Y., & Duru, A. (2010). Comprehension test in calculus course. *Procedia Social and Behavioural Sciences*. 2, 1527–1531. <https://doi.org/10.1016/j.sbspro.2010.03.229>
- Al-Bajalani, F. (2019). Teaching "Academic Debate and Critical Thinking" to Undergraduates in Kurdistan, Iraq. *Conference Paper: Challenges and Prospects, At University of Salford, Manchester*. <https://www.researchgate.net/publication/331481669>
- Aldridge, A., & Levine, K. (2001). *Surveying the social world: Principles and practice in survey research*. Open University Press.
- Alzahrani, G. (2016). The Availability of Some Critical Thinking Skills and Their Relationship with Some Variables Concerning The Preparatory Year Students for The Faculty of Sciences and Arts at Makhwah - Al Baha University, KSA. *International Journal for Talent Development*. 7(13), 155-176. <https://doi.org/10.20428/ijtd.7.2.8>
- Areaya, S., & Sidelil, A. (2012). Students' difficulties and misconceptions in learning concepts of limit, continuity and derivative. *The Ethiopian Journal of Education*, 32(2), 1-37.

- Artigue, M., Batanero, C., & Kent, P. (2007). Thinking and learning at post-secondary level. In F. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 1011-1049). Information Age Publishing.
- Aryal, H. P. (2022). *The effect of inquiry-based learning on Calculus I students' math anxiety* [Doctoral dissertation, Ohio University].
- Ashraf, A. L. A. M. (2020). Challenges and possibilities in teaching and learning of calculus: A case study of India. *Journal for the Education of Gifted Young Scientists*, 8(1), 407-433. <https://doi.org/10.17478/jegys.660201>
- Axtell, M. (2006). A two-semester precalculus/calculus sequence: A case study. *Mathematics and Computer Education*, 40(2), 130-137.
- Bell, J. (2005). *Doing your research project: A guide for first-time education, health and social science researchers*. Open University Press.
- Berry, J. S., & Nyman, M. A. (2003). Promoting students' graphical understanding of the calculus. *The Journal of Mathematical Behavior*, 22(4), 479-495. <https://doi.org/10.1016/j.jmathb.2003.09.006>
- Bookman, J. (1993). An expert novice study of metacognitive behavior in four types of mathematics problems. *Problems, Resources, and Issues in Mathematics Undergraduate Studies*, 3(3), 284-314. <https://doi.org/10.1080/10511979308965710>
- Bookman, J., & Blake, L. (2007). Seven years of Project CALC at Duke University approaching steady state? *Problems, Resources, and Issues in Mathematics Undergraduate Studies*, 6(3), 221-234. <https://doi.org/10.1080/10511979608965825>
- Borji, V., Alamolhodaei, H., & Radmehr, F. (2018). Application of the APOS-ACE theory to improve students' graphical understanding of derivative. *Eurasia Journal of Mathematics, Science and Technology Education*, 14(7), 2947-2967. <https://doi.org/10.29333/ejmste/91451>
- Brame, C. (2013). *Flipping the classroom*. Center for Teaching, Vanderbilt University. Retrieved Sep. 25, 2024 from: <http://cft.vanderbilt.edu/guides-sub-pages/flipping-the-classroom>
- Branchetti, L., Calza, G., Martani, S., & Saracco, A. (2020). Continuity of real functions in high school: a teaching sequence based on limits and topology. INDRUM.
- Braun, B., Bremser, P., Duval, A., Lockwood, E., & White, D. (2017). What Does Active Learning Mean For Mathematicians?. *Notices of the AMS*, 64(2), 124-129. <http://dx.doi.org/10.1090/noti1472>
- Bressoud, D., Ghedamsi, I., Martinez-Luaces, V., & Törner, G. (2016). *Teaching and Learning of Calculus*. https://doi.org/10.1007/978-3-319-32975-8_1
- Bressoud, D., Martinez-Luaces, V., Ghedamsi, I., & Törner, G. (2017). Topic Study Group No. 16: Teaching and Learning of Calculus. In *Proceedings of the 13th International Congress on Mathematical Education* (pp. 447- 452). https://doi.org/10.1007/978-3-319-62597-3_43
- Bryman, A. (2008). *Social research methods*. Oxford University Press.
- Buch, G., & Warren, C. B. (2017). The Flipped Classroom: Implementing Technology To Aid In College Mathematics Student's Success. *Contemporary Issues in Education Research*, 10(2), 109-116. <https://doi.org/10.19030/cier.v10i2.9921>
- Carlson, M., Jacobs, S., Coe, E., Larsen, S., & Hsu, E. (2002). Applying covariational reasoning while modeling dynamic events: A framework and a study. *Journal for Research in Mathematics Education*, 33(5), 352-378. <https://doi.org/10.2307/4149958>

- Clark, J. M., Cordero, F., Cottrill, J., Czamocha, B., Devries, D. J., St. John, D., Tolias, G., & Vidakovic, D. (1997). Constructing a schema: The case of the chain rule? *Journal of Mathematical Behavior*, 16(4), 345-364. [https://doi.org/10.1016/S0732-3123\(97\)90012-2](https://doi.org/10.1016/S0732-3123(97)90012-2)
- Code, W., Piccolo, C., Kohler, D., & MacLean, M. (2014). Teaching methods comparison in a large calculus class. *ZDM-Mathematics Education*, 46, 589-601. <https://doi.org/10.1007/s11858-014-0582-2>
- Cohen, L., Manion, L., & Morrison, K. (2004) *Research methods in education* (6th ed.). Routledge.
- Cohen, L., Manion, L., & Morrison, K. (2007). *Research methods in education* (7th ed.). RoutledgeFalmer. <https://doi.org/10.4324/9780203029053>
- Cornu, B. (1992). Limits. In D. Tall (Ed.), *Advanced mathematical thinking* (pp. 153-166). Dodrecht, The Netherlands: Kluwer Academic.
- Cronhjort, M., Filipsson, L., & Weurlander, M. (2018). Improved engagement and learning in flippedclassroom calculus. *Teaching Mathematics and its Applications: An International Journal of the IMA*, 37(3), 113. <https://doi.org/10.1093/teamat/hrx007>
- Crowl, T. K. (1996). *Fundamentals of educational research*. Brown & Benchmark.
- Dane, A., Çetin, Ö. F., Bas, F., & Sagirli, M. Ö. (2016). A Conceptual and Procedural Research on the Hierarchical Structure of Mathematics Emerging in the Minds of University Students: An Example of Limit-Continuity-Integral-Derivative. *International Journal of Higher Education*, 5(2), 82 -91. <https://doi.org/10.5430/ijhe.v5n2p82>
- Davis, R., & Vinner, S. (1986). The Notion of Limit: Some Seemingly Unavoidable Misconception Stages. *The journal of mathematical behavior*, 5(3), 281-303.
- Dagley, M. A., Gill, M., Saitta, E., Moore, B., Chini, J., & Li, X. (2018). Using Active Learning Strategies in Calculus to Improve Student Learning and Influence Mathematics Department Cultural Change. *Proceedings of the Interdisciplinary STEM Teaching and Learning Conference (2017-2019)*, 2(8). <https://doi.org/10.20429/stem.2018.020108>
- Delozier, S. J., & Rhodes, M. G. (2016). Flipped classrooms: a review of key ideas and recommendations for practice. *Educ. Psychol. Rev.* 29, 141–151 <https://doi.org/10.1007/s10648-015-9356-9>.
- Denbel, D. (2015). Some Conceptual Difficulties of Students on Derivation. *Journal of Educational and Management Studies*, 5(4), 211-214. <http://www.science-line.com>.
- Dubinsky, E., & Schwingendorf, K. (1991). *Calculus, concepts and computers*. St Paul. West.
- El-khateeb, M. (2015). Perceptions and Performance of King Saud University Students' about Concept and Finding Limit of Functions Graphical and Symbolic. *Journal of Education and Learning*, 4(4), 25–37.
- Ellis, J., Kelton, M., & Rasmussen, C. (2014). Student perceptions of pedagogy and associated persistence in calculus. *ZDM Mathematics Education*, 46, 661-673. <https://doi.org/10.1007/s11858-014-0577-z>
- Engelke, N. (2004). Related rates problems: Identifying conceptual barriers. In D. McDougall (Ed.), *Proceedings of the 26th Annual Conference of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 455-462).
- Fernández-Plaza, J. A., Rico, L., & Ruiz-Hidalgo, J. F. (2013). Concept of finite limit of a function at a point: Meanings and specific terms. *International Journal of Mathematical Education in Science and Technology*, 44(5), 699-710. <https://doi.org/10.1080/0020739x.2013.805887>

- Ferrini-Mundy, J., & Graham, K. (1994). Research in calculus learning: Understanding of limits, derivatives, and integrals. *MAA notes*, 31-46.
- Fraenkel, J. R., & Wallen, N. E. (2008). *How to design and evaluate research in education*. McGraw-Hill Higher Education.
- García-García, J., & Dolores-Flores, C. (2021). Preuniversity students' mathematical connections when sketching the graph of derivative and antiderivative functions. *Mathematics Education Research Journal*, 33, 1-22. <https://doi.org/10.1007/s13394-019-00286-x>
- Garofalo, J., (1989). Beliefs and their influence on mathematical performance. *The Mathematics Teacher*, 82(7), 502-505.
- Gass, S. M., & Mackey, A. (2007). *Data elicitation for second and foreign language research*. Lawrence Erlbaum Associates. <https://doi.org/10.4324/9780203826102>
- Gay, L. R., & Airasian, P. (2000). *Educational research: Competencies for analysis and application* (6th ed.). Prentice Hall.
- Gay, L. R., & Airasian, P. W. (2003). *Educational research: Competencies for analysis and applications* (7th ed.). Mer-rill/Prentice Hall.
- Gordon, S. P. (2004). Mathematics for the new millennium. *The International Journal of Computer Algebra in Mathematics Education*, 11(2), 37-44.
- Habre, S., & Abboud, M. (2006). Students' conceptual understanding of a function and its derivative in an experimental calculus course. *The Journal of Mathematical Behavior*, 25(1), 57-72. <https://doi.org/10.1016/j.jmathb.2005.11.004>
- Haciomeroglu, E. S., & Andreasen, J. B. (2013). Exploring calculus with dynamic mathematics software. *Mathematics and Computer Education*, 47(1), 6.
- Haciomeroglu, E. S., Aspinwall, L., & Presmeg, N. C. (2010). Contrasting cases of calculus students' understanding of derivative graphs. *Mathematical Thinking and Learning*, 12(2), 152-176. <https://doi.org/10.1080/10986060903480300>
- Hart, A., Daucourt, M., & Ganley, C. (2017). Individual differences related to college students' course performance in calculus II. *Journal of Learning Analytics*, 4(2), 129– 153. <http://dx.doi.org/10.18608/jla.2017.42.11>
- Hauk, S., & Hsu, P. S. (2022). Undergraduate and instructor perspectives on learning in first-year mathematics courses in the United States: A case study in calculus. *La Matematica*, 1. 583–617. <https://doi.org/10.1007/s44007-022-00022-1>
- Hegedus, S. J., Dalton, S., & Tapper, J. R. (2015). The impact of technology-enhanced curriculum on learning advanced algebra in US high school classrooms. *Educational Technology Research and Development*, 63, 203–228. <https://doi.org/10.1007/s11423-015-9371-z>
- Heim, B., Rupp, F., Viet, N., Stockhausen, P. V., Gallenkämper, J., & Kreuzer, J. (2015). Driving studentcentered calculus: results of a comprehensive case study for Kaizen learning in the Sultanate of Oman. *International Journal of Mathematical Education in Science and Technology*, 46(3), 354–369. <https://doi.org/10.80/0020739X.2014.979897>.
- Hiyam, B., Zoubi, A., & Khataybeh, A. (2019). Utilizing MATHEMATICA software to improve students' problem solving skills of derivative and its applications. *International Journal of Education and Research*, 7(11), 57-70.

- Hohenwarter, M., Hohenwarter, J., Kreis, Y., & Lavicza, Z. (2008). Teaching and learning calculus with free dynamic mathematics software GeoGebra. *Proceedings of conference: 11th International Congress on Mathematical Education (ICME 11)* At: Monterrey, Mexico
- Hughes-Hallett, D. (1991). Visualization and calculus reform. In W. Zimmermann & S. Cunningham (Eds.), *Visualization in teaching and learning mathematics* (pp. 121–126). Washington, DC: Mathematical Association of America
- Ibrahim, B., & Rebello, N. S. (2012). Representational task formats and problem solving strategies in kinematics and work. *Physical Review Special Topics Physics Education Research*, 8(1), 010126. <https://doi.org/10.1103/PhysRevSTPER.8.010126>
- Illanes, M. K. G., Breda, A., Manríquez, D. D. C., & Martínez, H. A. A. (2022). Analysis of a teaching learning process of the derivative with the use of ICT oriented to engineering students in Chile. *Eurasia Journal of Mathematics, Science and Technology Education*, 18(7), em2130. <https://doi.org/10.29333/ejmste/12162>
- Jones, S. R. (2017). An exploratory study on student understandings of derivatives in real-world, nonkinematics contexts. *The Journal of Mathematical Behavior*, 45, 95-110. <https://doi.org/10.1016/j.jmathb.2016.11.002>
- Jungic, V., Kaur, H., Mulholland, J., & Xin, C. (2015). On flipping the classroom in large first year calculus courses. *Int. J. Math. Educ. Sci. Technol.*, 46, 508–520. <https://doi.org/10.1080/0020739x.2014.990529>
- Juter, K. (2005). Limits of functions: Traces of students' concept images. *Nordic Studies in Mathematics Education*, 10(3-4), 65-82.
- Kamau, L. M. (2014). *Technology adoption in secondary mathematics teaching in Kenya: An explanatory mixed methods study*. [Doctoral dissertation, Syracuse University].
- Körtesi, P., Simonka, Z., Szabo, Z. K., Guncaga, J., & Neag, R. (2022). Challenging examples of the wise use of computer tools for the sustainability of knowledge and developing active and innovative methods in STEAM and mathematics education. *Sustainability*, 14(20), 12991. <https://doi.org/10.3390/su142012991>
- Kuh, G. D., Kinzie, J., Schuh, J. H., & Whitt, E. J. (2011). *Student success in college: Creating conditions that matter*. John Wiley & Sons.
- Lang, X. (1999). CAI and the reform of mathematics education in China. *International Journal of Mathematical Education in Science and Technology*, 30(3), 399–404. <https://doi.org/10.1080/002073999287914>
- Leng, N. W. (2011). Using an advanced graphing calculator in the teaching and learning of calculus. *International Journal of Mathematical Education in Science and Technology*, 42(7), 925-938. <https://doi.org/10.1080/0020739X.2011.616914>
- Maciejewski, W. (2015). Flipping the calculus classroom: an evaluative study. *Teach. Math. Appl*, 1–15. <https://doi.org/10.1093/teamat/hrv019>
- Maharaj, A., & Wagh, V. (2016). Formulating tasks to develop HOTS for first-year calculus based on Brookhart abilities. *South African Journal of Science*, 112(11/12), 1- 6. <http://orcid.org/0000-0003-4808-551X>
- Marshall, N., Buteau, C., Jarvis, D., & Lavicza, Z. (2012). Do mathematicians integrate computer algebra systems in university teaching? Comparing a literature review to an international survey study. *Computers & Education*, 58, 423-434. <https://doi.org/10.1016/j.compedu.2011.08.020>

- Marsitin, R. (2019). Analysis of Differential Calculus in Economics. *Journal of Physics Conference Series*. 1381(1), 012003. <https://doi.org/10.1088/1742-6596/1381/1/012003>
- Mastorides, E., & Zachariades, T. (2004). *Secondary mathematics teachers' knowledge concerning the concept of limit and continuity*. Proceedings of the 28th conference of the international group for the Psychology of mathematics education, 4, 481 – 488.
- McLoughlin, M. (2009). *Incorporating inquiry-based learning in the calculus sequence: A most challenging endeavor* [Paper presentation]. The Annual Joint Meetings of the American Mathematical Society & the Mathematical Association of America.
- Mkhatshwa, T. (2018). Business calculus students' interpretations of marginal change in economic contexts. In Hodges, T. E., Roy, G. J., & Tyminski, A. M. (Eds.), *Proceedings of the 40th Annual Conference of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 564-571). Greenville, South Carolina.
- Mkhatshwa, T. (2020). Calculus students' quantitative reasoning in the context of solving related rates of change problems. *Mathematical Thinking and Learning*, 22(2), 139-161. <https://doi.org/10.1080/10986065.2019.1658055>
- Mkhatshwa, T. (2021). An investigation of students' content understanding, perception changes, and experiences in a flipped precalculus course. *Association for University Regional Campuses of Ohio Journal*, 27, 41-73.
- Mkhatshwa, T. (2023a). A quantitative and covariational reasoning investigation of students' interpretations of partial derivatives in different contexts. *International Journal of Mathematical Education in Science and Technology*, 54(4), 511-533. <https://doi.org/10.1080/0020739X.2021.1958941>
- Mkhatshwa, T. (2023b). Calculus instructors' perspectives on effective instructional approaches in the teaching of related rates problems. *Eurasia Journal of Mathematics, Science and Technology Education*, 19(11), 1-15. <https://doi.org/10.29333/ejmste/13658>
- Monk, S., & Nemirovsky, R. (1994). The case of Dan: Student construction of a functional situation through visual attributes. *CBMS Issues in Mathematics Education*, 4, 139-168. <https://doi.org/10.1090/cbmath/004/07>
- Moru, E. K. (2009). Epistemological obstacles in coming to understand the limit of a function at undergraduate level: A case from the National University of Lesotho. *International Journal of Science and Math Education*, 7, 431–454. <https://doi.org/10.1007/s10763-008-9143-x>
- Murphy, J., Chang, J. M., & Suaray, K. (2015) Student performance and attitudes in a collaborative and flipped linear algebra course. *Intl J. Math. Edu. Sci. Technol.* <https://doi.org/10.1080/0020739X.2015.1102979>
- Muzangwa, J., & Chifamba, P. (2012). Analysis of errors and misconceptions in the learning of calculus by undergraduate students. *Acta Didactica Napocensia*, 5(2), 1–10.
- Nabb, K. A. (2010). CAS as a restructuring tool in mathematics education. In *Proceedings of the 22nd International Conference on Technology in Collegiate Mathematics* (pp. 247-259).
- National Council of Teachers of Mathematics (NCTM) (1980). *An Agenda for action: Recommendations for school mathematics of the 1980s*. Reston, VA: Author.
- National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, VA: Author.
- Oppenheim, A. (2001). *Questionnaire design, interviewing and attitude measurement*. Continuum.

- Peters, T., Johnston, E., Bolles, H., Ogilvie, C., Knaub, A., & Holme, T. (2020). Benefits to students of teambased learning in large enrollment calculus. *Problems, Resources, and Issues in Mathematics Undergraduate Studies*, 30(2), 211-229. <https://doi.org/10.1080/10511970.2018.1542417>
- Piccolo, C., & Code, W. J. (2013). Assessment of students' understanding of related rates problems. In S. Brown, G. Karakok, K. H. Roh, & M. Oehrtman (Eds.), *Proceedings of the 16th Meeting of the MAA Special Interest Group on Research in Undergraduate Mathematics Education* (pp. 607-609).
- Praslon, F. (1999). *Discontinuities regarding the secondary/university transition: The notion of derivative as a specific case*. Paper presented at the PME conference.
- Ramaglia, H. (2015). *The Flipped Mathematics Classroom: A Mixed Methods Study Examining Achievement, Active Learning, And Perception*. Dissertation For The Degree Doctor Of Philosophy Of Curriculum And Instruction, College Of Education, Kansas State University, Manhattan, Kansas.
- Rasmussen, C., Marrongelle, K., & Borba, M. C. (2014). Research on calculus: what do we know and where do we need to go? *ZDM – The International Journal on Mathematics Education*, 46, 507–515. <https://doi.org/10.1007/s11858-014-0615-x>
- Renfro, A. (2014). *Assessing The Effects Of A Flipped Classroom Approach On Student Achievement, Mathematical Thinking, Attitudes, And Teacher Perceptions In An Undergraduate Calculus Class Using A Participatory Action Research Approach*. Dissertation For The Degree Doctor Of Philosophy with a major in Instructional Management and Leadership, Robert Morris University.
- Roh, K., & Lee, Y. (2011). The Mayan activity: a way of teaching multiple quantifications in logical contexts. *Problems, Resources, and Issues in Mathematics Undergraduate Studies*, 21, 685–698. <https://doi.org/10.1080/10511970.2010.485602>
- Rosly, W. N. S. W. M., Abdullah, S. S. S., & Shukri, F. N. A. (2020). The uses of WolframAlpha in mathematics. *Articles of Teaching and Learning in Higher Education*, 1, 96-103.
- Sahin, A., Cavlazoglu, B., & Zeytuncu, Y. E. (2015). Flipping a College Calculus Course: A Case Study. *Educational Technology & Society*, 18(3), 142–152.
- Sammartino, N. (2023). *Teachers' perspectives of the effects of educational technology tools on high school Algebra I skills* [Doctoral dissertation, Saint Peter's University].
- Sari, P., Hadiyan, A., & Antari, D. (2018). Exploring derivatives by means of GeoGebra. *International Journal on Emerging Mathematics Education*, 2(1), 65 - 78. <https://doi.org/10.12928/ijeme.v2i1.8670>
- Schoenfeld, A. H. (1995). A brief biography of calculus reform. *UME trends*, 6(6), 3-5
- Sebsibe, A. S., & Feza, N. N. (2020). Assessment of Students' Conceptual Knowledge in Limit of Functions. *International Electronic Journal of Mathematics Education*, 15(2), em0574. <https://doi.org/10.29333/iejme/6294>
- Sepriyanti, N., Fauzan, A., & Arnawa, I. M. (2017). Calculus Based On Contextual Learning Model To Cultivate Student ' s Activity , Interest And Mathematical Connection Ability. *International Journal of Scientific & Technology Research*, 6(10), 233–238.
- Serkan, C. G. (2013). *The good, the bad, and the ugly: The integration of computing technology into calculus classes* [Doctoral dissertation, The University of Georgia].

- Sevimli, E. (2016). Do calculus students demand technology integration into learning environment? case of instructional differences. *International Journal of Educational Technology in Higher Education*, 13, 37. <https://doi.org/10.1186/s41239-016-0038-6>
- Siyepu, S. W. (2013). An exploration of students' errors in derivatives in a university of technology. *The Journal of Mathematical Behavior*, 32(3), 577-592. <https://doi.org/10.1016/j.jmathb.2013.05.001>
- Skemp, P. R. (1976). Relational Understanding and instrumental Understanding. *Mathematics Teaching* 77, 20-26.
- Smith, D. A., & Moore, L. C. (1990). Duke university: Project calc. *Priming the calculus pump: Innovations and resources*. 51-74.
- Smith, D. A., & Moore, L. C. (1991). Project CALC: An integrated laboratory course. The laboratory approach to teaching calculus. *The Mathematical Association of America, Washington, DC*, 81-92.
- Speer, N. M., Smith III, J. P., & Horvath, A. (2010). Collegiate mathematics teaching: An unexamined practice. *The Journal of Mathematical Behavior*, 29(2), 99-114. <https://doi.org/10.1016/j.jmathb.2010.02.001>
- Strayer, J., Hart, J., & Bleiler-Baxter, S. (2016). Kick-Starting Discussions With the Flipped Classroom. *Mathematics Teacher*, 109(9), 662–668 <https://doi.org/10.5951/mathteacher.109.9.0662>
- Tall, D. (1992). Students' Difficulties in Calculus. *Proceedings of Working Group 3 on Students' Difficulties in Calculus*, ICME-7, Québec, Canada, 13-28.
- Tall, D. (1992). The transition to advanced mathematical thinking: Functions, limits, infinity and proof. *Handbook of research on mathematics teaching and learning*, 495-511.
- Tall, D. (2008). The Transition to Formal Thinking in Mathematics. *Mathematics Education Research Journal*, 20(2), 5-24. <https://doi.org/10.1007/BF03217474>
- Tall, D., & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics*, 12, 151-169. <https://doi.org/10.1007/BF00305619>
- Tall, D., Smith, D., & Piez, C. (2008). Technology and calculus. *Research on Technology and the Teaching and Learning of Mathematics*, 1, 207-258.
- Thompson, P. W. (1994). Images of rate and operational understanding of the fundamental theorem of calculus. *Educational Studies in Mathematics*, 26(2-3), 229-274. <https://doi.org/10.1007/BF01273664>
- Thompson, P. W., & Harel, G. (2021). Ideas foundational to calculus learning and their links to students' difficulties. *ZDM—Mathematics Education*, 53(3), 507–519. <https://doi.org/10.1007/s11858-021-01270-1>
- Tucker, A. C., & Leitzel, J. R. C. (1995). *Assessing calculus reform: A report to the community*. Washington, DC: Mathematical Association of America.
- Tucker, T. W. (Ed.). (1990). *Priming the calculus pump: Innovations and resources*. Washington, DC: Mathematics Association of America.
- Ubuz, B. (2007). Interpreting a graph and constructing its derivative graph: Stability and change in students' conceptions. *International Journal of Mathematical Education in Science and Technology*, 38(5), 609-637. <https://doi.org/10.1080/00207390701359313>

- Uwurukundo, M. S., Maniraho, J. F., & Tusiime, M. (2022). Enhancing students' attitudes in learning 3-Dimension geometry using GeoGebra. *International Journal of Learning, Teaching and Educational Research*, 21(6), 286–303. <https://doi.org/10.26803/ijlter.21.6.17>
- Viirman, O., & Pettersson, I. (2022). A small-scale implementation of inquiry-based teaching in a single-variable calculus course for first-year engineering students. *Hiroshima Journal of Mathematics Education*, 15(2), 129-139.
- Vincent, B. (2016). *First Semester Calculus Students' Concept Definitions and Concept Images of the Tangent Line and How These Relate to Students' Understandings of the Derivative*. Dissertation Degree of Doctor of Philosophy in Mathematics at West Virginia University
- Wassie, Y. A., & Zergaw, G. A. (2019). Some of the potential affordances, challenges and limitations of using GeoGebra in mathematics education. *Eurasia Journal of Mathematics, Science and Technology Education*, 15(8), em1734. <https://doi.org/10.29333/ejmste/108436>
- White, P., & Mitchelmore, M. (1996). Conceptual knowledge in introductory calculus. *Journal for research in mathematics education*, 27, 79-95. <https://doi.org/10.2307/749199>
- Wilkinson, D., & Birmingham, P. (2003). *Using research instruments: A guide for researchers*. RoutledgeFalmer. London: Routledge. <https://doi.org/10.4324/9780203422991>
- Williams, S. R. (1991). Models of Limit Held by College Calculus Students. *Journal for Research in Mathematics Education*, 22(3), 237–251. <https://doi.org/10.2307/749075>
- Wu, L., & Li, Y. (2017). Project-based learning in calculus on the use of Maple software technology. *Journal of Mathematics and System Science*, 7, 142-47. <https://doi.org/10.17265/2159-5291/2017.05.002>
- Yerizon, S. F., Tasman, F., & Tasman, F. (2021). Development of a geogebra-assisted calculus worksheet to enhance students' understanding. *International Journal of Information and Education Technology*, 11(10), 456–463. <https://doi.org/10.18178/ijiet.2021.11.10.1550>
- Young, G. S. (1987). Present problems and future prospects. *Calculus for a new century*, 172- 175.
- Zachariades, T., Pamfilos, P., Christou, C., Maleev, R., & Jones, K. (2007). *Teaching introductory calculus: Approaching key ideas with dynamic software*. Paper presented at CETL-MSOR Conference on Excellence in the Teaching and Learning, Stats & OP, University of Birmingham, 10-11 September 2007.
- Zbarsky, E. S., Simundza, G., & Henriksen, M. (2021). Teaching calculus as a tool in the twenty-first century. In R. Buckmire, & J. M. Libertini (Eds.), *Improving applied mathematics education* (pp. 13-25). Springer. https://doi.org/10.1007/978-3-030-61717-2_2
- Zengin, Y. (2017). Investigating the Use of the Khan Academy and Mathematics Software with a Flipped Classroom Approach in Mathematics Teaching. *Educational Technology & Society*, 20(2), 89–100.
- Zengin, Y., Furkan, H., & Kutluca, T. (2012). The effect of dynamic mathematics software geogebra on student achievement in teaching of trigonometry. *Procedia-Social & Behavioral Sciences*, 31, 183–187. <https://doi.org/10.1016/j.sbspro.2011.12.038>
- Zulnaidi, H., Oktavika, E., & Hidayat, R. (2019). Effect of use of GeoGebra on achievement of high school mathematics students. *Education and Information Technologies*, 25(1), 51–72. <https://doi.org/10.1007/s10639-019-09899-y>

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