

Research Article

Cite this article: Hamid, H., Nani, K. L., Wahyudi, D., Muchsin, S. B., & Ruhama, M. A. (2025). Mathematical Self-Explanation of First-Year Students at the State University of North Maluku through the Utilization of GeoGebra. *Educational Process: International Journal*, 17, e2025331.
<https://doi.org/10.22521/edupij.2025.17.331>

Received March 17, 2025

Accepted May 17, 2025

Published Online July 14, 2025

Keywords: Self-Explanation, GeoGebra, calculus, mathematics education

Author for correspondence:

Hasan Hamid

✉ hasan.hamid66@gmail.com

✉ Universitas Khairun, Indonesia

Mathematical Self-Explanation of First-Year Students at the State University of North Maluku through the Utilization of GeoGebra

Hasan Hamid^{ID}, Karman La Nani^{ID}, Dahlan Wahyudi^{ID}, Sitti Busyrah Muchsin^{ID}, Mustafa A.H. Ruhama^{ID}

Abstract

Background/purpose. This research is motivated by the difficulties often experienced by students in adapting to their first year of college, especially related to learning calculus. To address these difficulties, a solution needs to be found so that these problems can be resolved. One solution that is expected to yield optimal results in addressing these issues is implementing self-explanatory learning by utilizing the GeoGebra application in studying integral calculus.

Materials/methods. This study involved 25 first-year students enrolled in the Mathematics Education Program at the Faculty of Teacher Training and Education, Khairun University—the only public university in North Maluku Province, Indonesia. Given the institutional context, where only one relevant class was available, a quasi-experimental one-group pretest-posttest design was adopted. This design was selected due to practical limitations that precluded the inclusion of a control group. Despite this constraint, the chosen approach was considered suitable for assessing the impact of the intervention while adhering to ethical and logistical considerations in the research setting.

Results. The results of this study indicate that the integration of GeoGebra into the instructional process significantly contributes to the enhancement of students' self-explanation skills. This is evidenced by a normalized gain score ($N < g >$) of 0.71, categorized as a high improvement. These findings suggest that GeoGebra effectively facilitates the strengthening of students' mathematical understanding through immersive visual interactions. Furthermore, the effectiveness of GeoGebra is demonstrated by an effect size value of 3.72, which statistically exceeds the threshold for a "large" effect. This result indicates that the learning intervention using GeoGebra has a very strong impact on improving self-explanation abilities. Such improvement is attributed to the learning process that leverages GeoGebra's dynamic, exploratory, and manipulable visualization features, thereby promoting more active cognitive and metacognitive engagement among students.

Conclusion. Based on the data analysis results, the findings of this study indicate that learning by utilizing GeoGebra significantly helps students in developing and enhancing their self-explanation abilities. This technology provides students with opportunities to try new things, engage in independent exploration and visualization, and reduce their mental load in learning calculus during their first year at Khairun University



OPEN ACCESS

© The Author(s), 2025. This is an Open Access article, distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>), which permits unrestricted re-use, distribution, and reproduction, provided the original article is properly cited.

1. Introduction

Each student enrolled in any class and in any academic year at Khairun University, particularly in the Mathematics Education program of the Faculty of Teacher Training and Education, brings a diverse background of knowledge. This variation arises from their previous education, which was provided by teachers with various backgrounds and diverse teaching methods. This diversity, which is a distinctive characteristic of the learning community in this program, offers intriguing research opportunities. Learning mathematics at the university level requires a different approach to thinking than previously, which often poses a significant challenge for students. For example, Calculus is one of the courses offered at the university (Santosa et al., 2019). The transition from secondary school mathematics education to university, known as the secondary-tertiary transition (STT), explores the significant changes in thinking experienced by students (Tall et al., 1981). Research by Tall has motivated further studies focusing on students' difficulties in the STT. Additionally, the research by De Guzman et al. (1998) has identified three main categories of difficulties faced by students in the STT: epistemological-cognitive issues, sociological-cultural issues, and didactic issues.

The difficulties encountered by students when transitioning from school mathematics education to the approach of mathematics education in college, which integrates science and technology with mathematics and engineering in real-world contexts, known as STEM, remain a phenomenon that is not yet fully understood. This phenomenon can even challenge students who have previously been considered excellent in mathematics (Di Martino & Gregorio, 2018). This view is supported by Rach & Heinze's (2016) studies highlighting the differences in the trajectories of learning mathematics from school to university. (Tall, 2008) It has been suggested that the way of mathematical thinking at the university level is significantly different from students' previous experiences, involving a transformation from fundamental to advanced mathematical thinking that emphasizes logic and rigor. These differences are characteristic of mathematical learning materials, known as the intrinsic nature of the subject matter (Ayres & Paas, 2012; Sweller et al., 2011). This shift in thinking is particularly evident in calculus courses. Additional difficulties may arise from the design of teaching or the way material is presented, which relates to the teaching methodology, techniques, or approaches applied by lecturers (Fukawa-Connelley, 2012; Intaros et al., 2014; Lopez, 2007; Rukavina et al., 2012).

Addressing the epistemological challenges in calculus education, recent studies advocate the integration of interactive visual tools—particularly GeoGebra—as a means to enhance students' conceptual understanding. GeoGebra, as a dynamic mathematics software, facilitates the construction of geometric figures through lines, points, and conic sections, while offering functionalities for computing geometric properties (e.g., area, volume, angles, and lengths) and displaying corresponding equations during visualization (Gurmu Fikru et al., 2024). Complementing this, the self-explanation strategy has demonstrated effectiveness in reinforcing both conceptual and procedural knowledge. Nevertheless, its implementation remains challenging due to the demand for intensive instructional support. To address this, the development of self-explanation-based automated feedback systems has been proposed, aligning with SDG 4's objective of promoting quality and sustainable education. These systems incorporate personalized feedback, quality evaluations, and peer-generated exemplars (Nakamoto et al., 2023). The combined application of GeoGebra and self-explanation strategies fosters an active, reflective learning environment, supporting students in overcoming transitional difficulties from secondary to tertiary education, particularly in mastering calculus concepts.

2. Literature Review

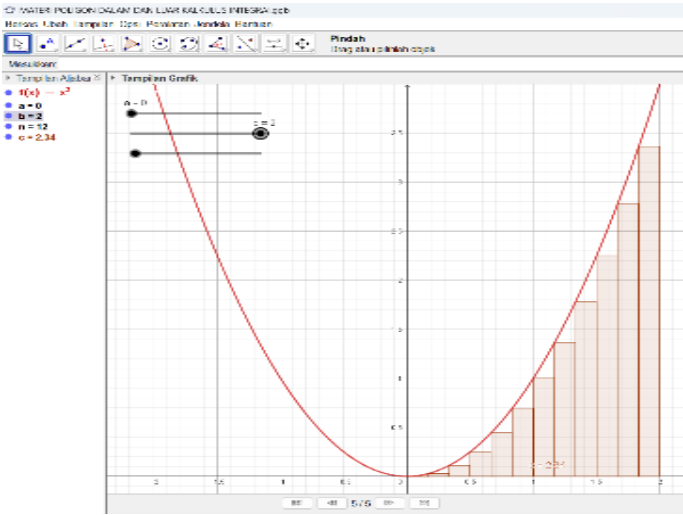
Self-explanation is the process by which an individual explains concepts to themselves by elaborating on information that is not explicitly contained in the existing material and drawing

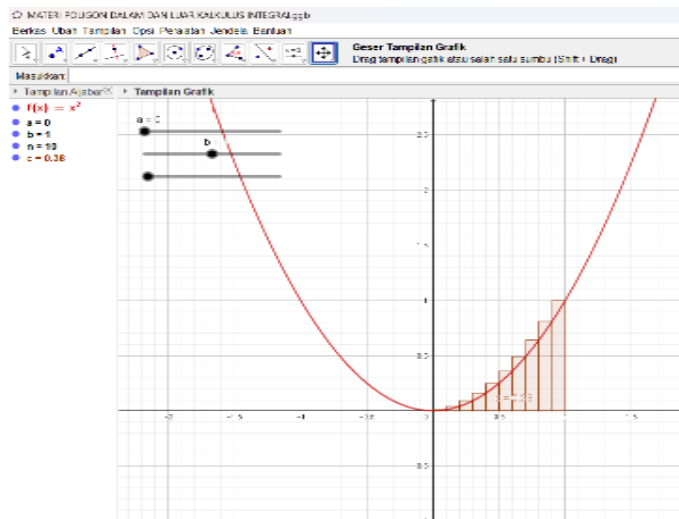
conclusions to organize that information for a better understanding. It is a productive activity that occurs during the learning process (Chi, 2000; Chi, 2009; Fiorella et al., 2016). Thus, self-explanation is a learning technique where students are expected to re-explain the material and learning processes to themselves while studying. This technique has been proven to enhance conceptual understanding and problem-solving skills. By integrating self-explanation and GeoGebra, it is expected that students will not only gain a deeper understanding of mathematical concepts but also be able to apply them in various situations.

The adoption of technology in mathematics education is rapidly increasing, providing new methods for teaching complex concepts to students. At Khairun University, which continually strives to stay up-to-date with the latest trends, GeoGebra has been implemented as a supportive tool in teaching mathematics to first-year students. The combination of using GeoGebra and the self-explanation method is expected to enhance students' understanding of mathematical concepts, as well as strengthen their analytical and problem-solving abilities. GeoGebra, as an educational tool, is mathematical software that integrates elements of geometry, algebra, and calculus. Its special feature is the ability to provide dynamic visualizations of complex mathematical objects, which is highly beneficial for students to understand mathematical structures and relationships better. At Khairun University, GeoGebra has been applied in various fundamental mathematics courses, including algebra and calculus, where students can directly see how changes in variables affect graphs or geometric objects.

Therefore, it is crucial to enhance the quality of teaching, including by implementing the GeoGebra application in integral materials within the Integral Calculus course. This aims to assist prospective mathematics teachers in understanding and independently explaining mathematical concepts. As a step to support the improvement of students' self-explanation abilities in mathematics, a brief explanation of the use of GeoGebra in learning integral calculus concepts will be provided below.

Table 1. An Example of Material That Will Be Taught Using GeoGebra

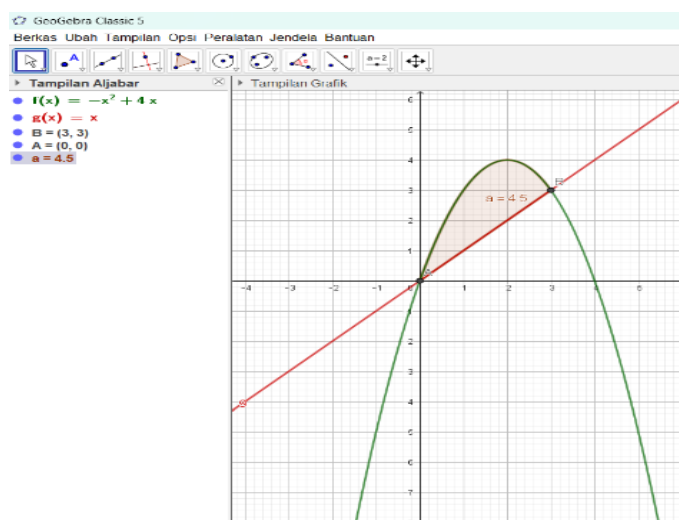
Some Concepts	Sample Course Material
Area of the area under the curve	



Explanation Analyzing the area under the curve using inscribed and circumscribed polygons, which is then developed into a more specific definition. Here are the steps to use GeoGebra to draw the desired graph:

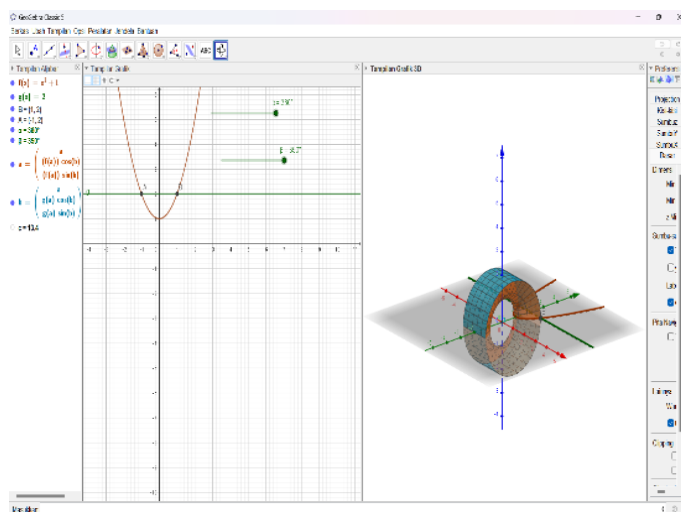
- (i) Open GeoGebra;
- (ii) Draw the graph of the function $f(x) = x^2$; for $0 \leq x \leq 2$ by typing $f(x) = x^2$ in the input;
- (iii) Next, create the boundaries on the x-axis by typing in the input $a = 0$ dan input $b = 2$;
- (iv) To create the inscribed and circumscribed polygons of the curve, click the slider menu to open the dialog box. Select "number" and type n. Next, click "interval" and enter 1 in the minimum box, 100 in the maximum box, and 1 in the increment box. After entering this data, click OK;
- (v) If you want to create the inscribed polygon, type **lowerSum(f, a, b, n)** in the **input box**, and if you want to create the circumscribed polygon, type **upperSum(f, a, b, n)** in the **input box**.

Area Between Two Curves



Explanation	Determining the area between two curves using GeoGebra as follows: (i) Open Geogebra; (ii) Type the function $f(x) = -x^2 + 4x$, in the input and press enter (iii) Type the following function in the input $g(x) = x$ (iv) Next, determine the intersection points of the two curves by typing in the input: Intersect (f, g) or alternatively, click the "Intersect Two Objects" menu and select functions f and g (v) • Once the intersection points of the two curves are found, we can determine the area between the two curves by typing in the input: IntegralBetween$(f, g, 0, 3)$, the result will then appear. The students can further develop the steps for determining the area between two curves.
-------------	---

The Volume of a Rotated Solid



Explanation	Analyzing the area under the curve using inscribed and circumscribed polygons, which is then elaborated into a more specific definition. Utilize GeoGebra to draw the desired graph with the following steps: (i) Open Geogebra; (ii) Draw the graph of the function $f(x) = x^2 + 1$; by typing $f(x) = x^2 + 1$, in the input and pressing enter; (iii) Next, type the following function in the input: $g(x) = 2$ (iv) Then, determine the intersection points of the two curves by typing in the input: Intersect (f, g), and pressing enter; (v) Create sliders to determine angles <i>alpha</i> and <i>beta</i>, by clicking the slider menu, selecting angle in the dialog box the angle α. For the interval, set the minimum to 0° and the maximum to 360°, with an increment of 5°. Repeat the same steps to create angle β; (vi) Since the goal is to display a three-dimensional solid based on the rotation about the x-axis with the obtained boundaries on the x-axis, the next step is to set up the screen for three-dimensional viewing by clicking the view menu and selecting 3D graphics;
-------------	--

(vii) Next, create the functions $f(x) = x^2 + 1$ and $g(x) = 2$ in the 3D graphics view by typing in the input: **Surface($a, f(a)\cos(b), f(a)\sin(b), a, x(A), b, 0, \alpha$)**, and pressing enter;

(viii) For $g(x) = 2$, type in the input: **Surface($a, g(a)\cos(b), g(a)\sin(b), a, x(A), b, 0, \beta$)**

The students can further develop these steps.

As observed in Table 1, students are provided with an understanding of the mathematical procedures for solving the given problems, connecting previous concepts to solve mathematical problems, generalizing specific examples into general rules, identifying and correcting any errors found, reflecting on procedures and concepts based on their understanding, using their own language to formulate the procedures and concepts they have created, and explaining why they chose the steps used in solving the given problems. This aligns with the concept of self-explanation, which requires learners to think more deeply about the information provided in the tasks. This thinking can focus on the logic used by experts in the given examples or texts, as well as in the problem-solving process (Bisra et al., 2018; Rattle-Johnson et al., 2017; Renkl, 2017). Research has shown that using varied learning examples and self-explanation is effective in enhancing students' mathematics learning. Self-explanation supports learning in two ways: 1) it aids understanding by reducing the difficulty of integrating knowledge (Chi, 2009), and 2) it assists in recognition and application by focusing attention on structural aspects rather than surface features of the material being learned. This supports both understanding and application by directing focus toward the structural characteristics rather than just the basic aspects of the content being studied (Rettle-Johnson, 2006; McEldoon et al., 2013).

The integration of GeoGebra and self-explanation strategies in mathematics instruction, particularly in calculus, has demonstrated effectiveness in enhancing the understanding of abstract concepts. GeoGebra enables students to interact directly with graphical representations such as function graphs, integrals, and derivatives, and to visually explore relationships among mathematical concepts. However, a persistent challenge lies in students' difficulty in linking these visual representations to the more formal and symbolic expressions of mathematical ideas, as reported by several studies (Bedada, 2022; Veasna, 2024). In this context, self-explanation plays a crucial role by encouraging students to reflect on and articulate the content in their own words, thus making abstract information more concrete and comprehensible (Miller-Cotto, 2022). When these two approaches are combined, students' conceptual understanding can be fostered through the integration of interactive visualizations and reflective verbal explanations. Nevertheless, recent studies indicate that the effectiveness of this integration remains mixed; while some students exhibit improved understanding, others struggle to verbally articulate their comprehension after engaging with visual materials (Almuwaiziri, 2023). This suggests that visualization alone does not automatically bridge the gap in students' verbal understanding of mathematical concepts. On the other hand, the implementation of GeoGebra in interactive and collaborative learning environments has proven beneficial for mastering a range of advanced mathematical topics, including calculus, statistics, and linear algebra (Ziatdinov et al., 2022). Furthermore, the use of GeoGebra in pre-class activities offers students the opportunity to independently explore course materials, thereby supporting deeper reflection and conceptual understanding (Ishartono, 2023). Therefore, for the integration of GeoGebra and self-explanation to yield optimal results, instructional design must be carefully balanced, not only leveraging technology effectively but also providing space for active reflection to strengthen both the understanding and application of mathematical concepts.

3. Methodology

The design of this study is a one-group pre-test post-test design, which is conducted on a single group with the sample taken using a purposive sampling technique. The research design scheme is illustrated in the following table:

Table 2. One-Group Pre-Test Post-Test Design Scheme

Pre-test	Treatment	Post-test
T_1	X	T_2

T_1 : The pre-test is conducted before the treatment

X: The treatment given includes GeoGebra and Mathematical Self-Explanation

T_2 : The post-test is conducted after the treatment.

This study involved 25 first-year students from the Mathematics Education Study Program, Faculty of Teacher Training and Education, Khairun University, who were enrolled in the Calculus course. Participants were selected through purposive sampling based on specific criteria: (1) active enrollment in the course; (2) full attendance in all GeoGebra-based instructional sessions; and (3) willingness to engage in the data collection process, including problem-solving tasks, written self-explanations, and, if needed, participation in interviews or discussions. The purposive approach ensured that the selected participants were directly involved in the intervention, allowing for the collection of meaningful and contextually relevant data regarding students' achievement in mathematical self-explanation indicators.

In order to verify the accuracy and consistency of the instrument in measuring students' mathematical self-explanation skills, the study employed both content validation and reliability analysis. Content validation was conducted through expert judgment involving three professionals: a lecturer with expertise in Calculus, a specialist in technology-integrated instruction, and an expert in educational assessment. These experts reviewed each item in terms of its alignment with self-explanation indicators, linguistic clarity, and relevance to the Calculus learning context. The assessment results showed that the average content validity score reached 91%, indicating a very high degree of validity. Reliability analysis was performed through a pilot test involving 20 students with similar characteristics to the main sample, but who were not part of the primary data collection. The internal consistency of the instrument, as calculated using Cronbach's Alpha, was found to be $\alpha = 0.86$. This result reflects high reliability according to standard interpretation guidelines ($\alpha > 0.80$), suggesting that the instrument is suitable for reliably assessing students' mathematical self-explanation ability.

4. Results

The data collected from the pre-test and post-test results from 25 students will be subjected to normality and homogeneity tests. The purpose of these tests is to determine whether the data from this research has normal and homogeneous variances, as well as to perform a t-test. Based on the analysis conducted, the results are presented in the following table:

Table 3. Results of Pretest and Posttest Normality Tests

Tests of Normality			
	Shapiro-Wilk		
	Statistic	df	Sig.
Pretest Self-Explanation Mathematics	.932	25	.096
Posttest Self-Explanation Mathematics	.932	25	.094

Based on Table 3 above, the Sig value for the Pretest Self-Explanation Mathematics is 0.096 and the Sig value for the Posttest Self-Explanation Mathematics is 0,094. This indicates that both the pretest and posttest Sig values are greater than $\alpha=0,05$, Therefore, the data for the Pretest Self-Explanation Mathematics and the Posttest Self-Explanation Mathematics are typically distributed.

Table 4. Homogeneity Test Results

Test of Homogeneity of Variances			
Score			
Levene Statistic	df1	df2	Sig.
.040	1	48	.842

Table 4 shows that the homogeneity test using Levene's test resulted in a Sig value of 0.842. This indicates that both the pretest and posttest Sig values are greater than $\alpha=0,05$, therefore, the data for the Pretest Self-Explanation Mathematics and the Posttest Self-Explanation Mathematics are homogeneous.

Table 5. ANOVA Test Result

ANOVA					
Score					
	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	11827.220	1	11827.220	168.880	.000
Within Groups	3361.600	48	70.033		
Total	15188.820	49			

Based on Tables 4 and 5, there is consistency between the homogeneity test and the ANOVA test results, indicating that the variance from the ANOVA test is homogeneous since Sig. is $0.000 \leq 0.05$. Based on the data in Tables 3, 4, and 5, a paired t-test can be conducted on the pretest and posttest data to assess the effectiveness of using GeoGebra in improving students' mathematical self-explanation. The results of the paired t-test analysis are as follows:

Table 6. Paired T-Test Results

Paired Samples Test					
	Paired Differences		t	df	Sig. (2-tailed)
	95% Confidence Interval of the Difference				
	Lower	Upper			
(Pair 1) Pretest Self-Explanation Mathematics - Posttest Self-Explanation Mathematics	-33.696	-27.824	-21.620	24	.000

The analysis results in Table 6 show that Sig (2-tailed) is $0,00 < \alpha = 0,05$, which means that there is a significant difference indicating that the use of GeoGebra can improve students' mathematical self-explanation abilities. In this study, a normalized gain ($N\langle g \rangle$) test was conducted to measure the category of improvement the students achieved after implementing GeoGebra to enhance their mathematical self-explanation, and the effect size obtained. The analysis results indicate that $N\langle g \rangle = 0,71$ and the effect size is 3.72. This shows that the category of improvement and the effectiveness of using GeoGebra are in the high category.

5. Discussion

Based on Tables 3, 5, and 6, the analysis results show that the pretest and posttest data for students' mathematical self-explanation with the aid of GeoGebra are normally distributed and homogeneous. The paired t-test results indicate a significant difference, showing that the use of GeoGebra can improve students' mathematical self-explanation abilities. Additionally, there is a significant improvement in students' mathematical self-explanation after being provided with an understanding of GeoGebra in studying integral calculus, categorized as high. The impact of using GeoGebra as a tool provides a very good contribution. This is supported by research (Chan et al., 2014) that found an overall effect size of 1.02 when they analyzed 587 students in a study related to the influence of GeoGebra software on improving mathematics learning outcomes. This is also consistent with the results of a study (Juandi et al., 2021) involving 57 students, which showed that the use of GeoGebra software in mathematics learning can significantly enhance students' mathematical abilities and has a very high effect.

The following are the results of student work designed to guide them in using GeoGebra to learn the concepts of integral calculus: (1) Students were asked to draw the graph of a function related to the method of calculating the area using inscribed and circumscribed polygons; (2) Based on the resulting graph, students were asked to analyze the outcomes they obtained; and (3) Students were asked to draw conclusions from their work. The students still encountered difficulties in determining the value of the integral using the definition of a definite integral by utilizing the given intervals $\int_0^4 f(x) dx = \int_0^1 f(x) dx + \int_1^2 f(x) dx + \int_2^4 f(x) dx$, by utilizing the definition of a definite integral $\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x_i$ there is, it can be said that f integrated over the interval $[a, b]$.

Find the area under the curve of $\int_0^4 f(x) dx$ using inscribed and circumscribed polygons, and draw the graph of the function $f(x) = \begin{cases} 1 & \text{if } 0 \leq x < 1 \\ x & \text{if } 1 \leq x < 2 \\ 4 - x & \text{if } 2 \leq x \leq 4 \end{cases}$

First, the students are asked to plot the function. Next, they are asked to show the area under the curve using inscribed and circumscribed polygons with varying numbers of polygons. This is intended to provide an understanding of the concept of approximating the area by partitioning the given interval into n subintervals.

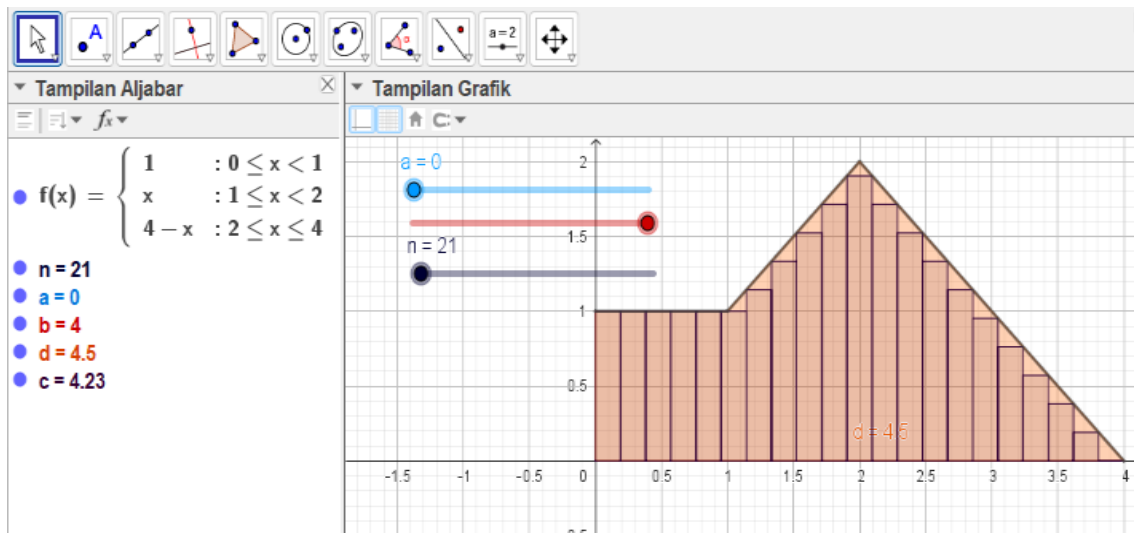


Figure 1. Graph of the Function $f(x)$ with an Inscribed Polygon Partition of $n = 21$

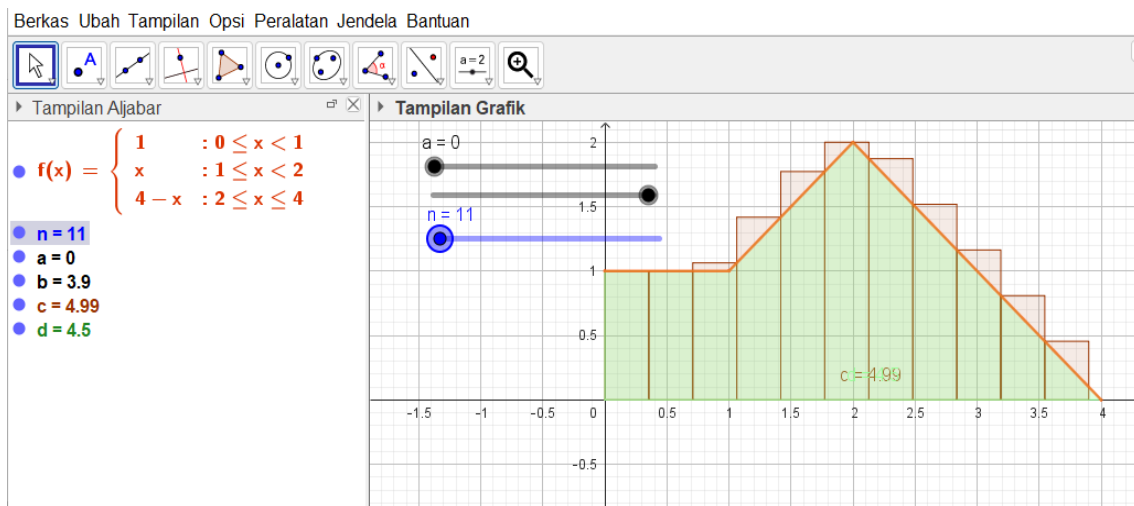


Figure 2. Graph of the Function $f(x)$ with a Circumscribed Polygon Partition $n = 11$.

Figures 1 and 2, which are the visualization results from students using GeoGebra to enhance self-explanation in mathematics, show a significant understanding in solving problems related to integral calculus. The students were able to represent the problem using the concepts of inscribed and circumscribed polygons to determine the area under the curve. This aligns with the research conducted by Takači et al (2015), which found that students' process of constructing knowledge with the help of GeoGebra can produce new knowledge, thereby improving their understanding of calculus. Furthermore, Bedada (2022) reported that the use of GeoGebra significantly enhances students' conceptual and procedural understanding in differential calculus. Students who

participated in GeoGebra-based instruction achieved statistically higher posttest scores compared to those in the conventional teaching group. These findings highlight that the interactive and visual approach provided by GeoGebra supports students in developing a deeper understanding of mathematical concepts through exploratory learning processes.

Below is the work of one student in solving the problem:

Find the area under the curve of the following function $f(x) = \begin{cases} 1 & \text{if } 0 \leq x < 1 \\ x & \text{if } 1 \leq x < 2 \\ 4 - x & \text{if } 2 \leq x \leq 4 \end{cases}$ using the

definition of a definite integral:

Solution:

$$\begin{aligned} \int_0^4 f(x) dx, \text{ dengan } f(x) &= \begin{cases} 1 & \text{if } 0 \leq x < 1 \\ x & \text{if } 1 \leq x < 2 \\ 4 - x & \text{if } 2 \leq x \leq 4 \end{cases} \\ \text{Misalkan: } \Delta x &= \frac{4-0}{n} = \frac{4}{n} \\ x_0 &= 0 \\ x_1 &= \Delta x + x_0 = \frac{4}{n} + 0 \\ x_2 &= 2 \cdot \frac{4}{n} = \frac{8}{n} \\ x_3 &= 3 \cdot \frac{4}{n} = \frac{12}{n} \\ \dots \\ x_i &= i \cdot \frac{4}{n} = \frac{4i}{n} \\ &= \left(\lim_{n \rightarrow \infty} \sum_{i=1}^n 1 + \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{4i}{n} + \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(4 - \frac{4i}{n} \right) \right) \cdot \frac{4}{n} \quad \text{Karena } \sum_{i=1}^n i = \frac{n(n+1)}{2} \\ &\text{selanjutnya disubstitusikan kedalam persamaan diperoleh} \\ &= \left(\lim_{n \rightarrow \infty} \sum_{i=1}^n 1 + \lim_{n \rightarrow \infty} \frac{4}{n} \sum_{i=1}^n i + \lim_{n \rightarrow \infty} \sum_{i=1}^n 4 - \lim_{n \rightarrow \infty} \frac{4}{n} \sum_{i=1}^n i \right) \cdot \frac{4}{n} \\ &= \left(\lim_{n \rightarrow \infty} \sum_{i=1}^n 1 + \lim_{n \rightarrow \infty} \sum_{i=1}^n 4 \right) \cdot \frac{4}{n} = \left(\lim_{n \rightarrow \infty} n + \lim_{n \rightarrow \infty} 4n \right) \cdot \frac{4}{n} = \left(\lim_{n \rightarrow \infty} 4 + \lim_{n \rightarrow \infty} 16 \right) = 20 \end{aligned}$$

Figure 3. The Work of One Student (R₅)

Examining the student's work on the given problem reveals that there are still difficulties in understanding the interval boundaries of the given function. The student should not immediately determine $\Delta x = \frac{4}{n}$ from the overall function boundaries but should divide the interval of each function into segments based on the number of equations. The difficulty faced by the student is generally in the aspect of understanding mathematical concepts as evident in this problem. The student should be able to differentiate for the function $f(x) = 1$ with $0 \leq x < 1$, where $\Delta x = \frac{1}{n}$, thus $x_i = \frac{i}{n}$ can be determined, For the function $f(x) = x$ with $1 \leq x < 2$, $\Delta x = \frac{1}{n}$, thus $x_i = \frac{i}{n}$ can be determined, for the function $f(x) = 4 - x$ with $2 \leq x \leq 4$, $\Delta x = \frac{2}{n}$, thus $x_i = \frac{2i}{n}$. It can be determined. The next difficulty is that the student still struggles to use the properties of summation in an integral and to correctly apply formulas and steps in solving the problem. This is often referred to as procedural difficulty, as seen in Figure 3, where the student does not break down the integral solution. The correct approach the student should take is: $\int_0^4 f(x) dx = \int_0^1 f(x) dx + \int_1^2 f(x) dx + \int_2^4 f(x) dx$, with $\int_0^1 f(x) dx = \int_0^1 1 dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x_i$, with $\Delta x = \frac{1}{n}$ and $x_i = \frac{i}{n}$; $\int_1^2 f(x) dx = \int_1^2 x dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x_i$, with $\Delta x = \frac{1}{n}$ dan $x_i = \frac{i}{n}$, next for $\int_2^4 f(x) dx = \int_2^4 (4 - x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x_i$, with $\Delta x = \frac{2}{n}$ and $x_i = \frac{2i}{n}$. As observed in Figure 3, in solving the problem, the student skillfully utilized the formula, although the result differed from the

simulation shown on their GeoGebra worksheet. This indicates that conceptual difficulty was almost nonexistent in solving this problem.

Next, the following problem is: Using integrals, determine the area of the triangle with vertices at $(-1, 4)$, $(2, -2)$, and $(5, 1)$. For this problem, students are asked to visualize it on the GeoGebra worksheet, understand how to use GeoGebra to find the intersection points between two lines, and also determine the area of the triangle using definite integrals.

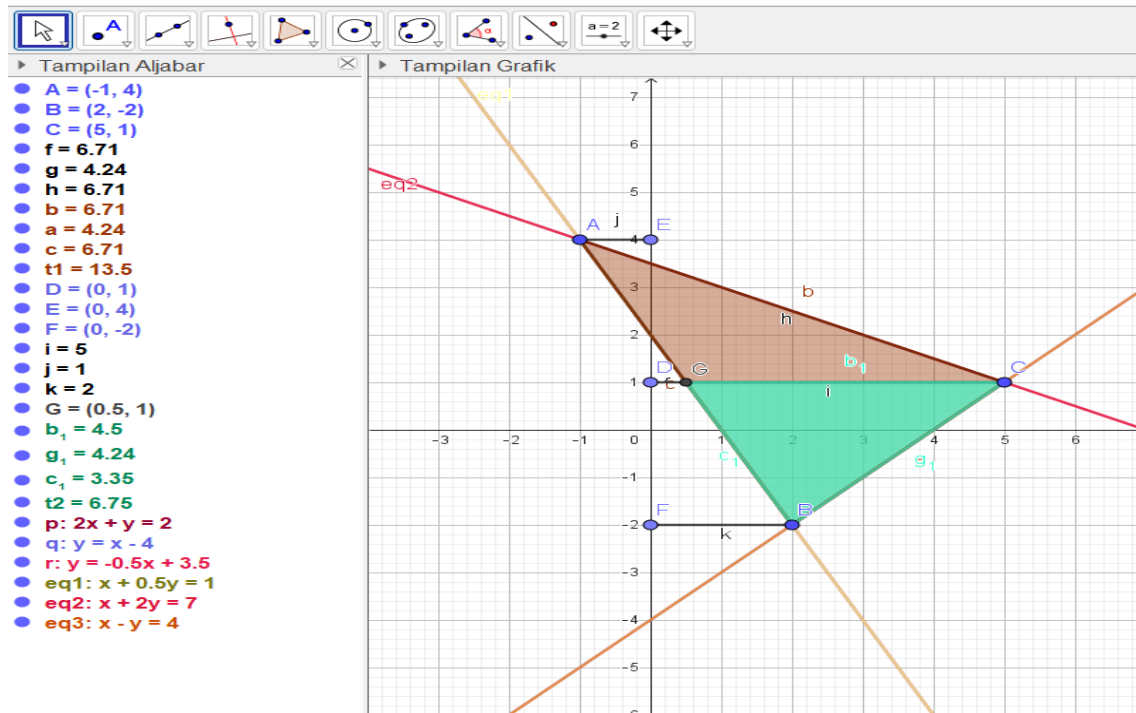


Figure 4. A Triangle with Known Vertices

As observed in Figure 4, using GeoGebra, it is clear that the students thoroughly understand how to determine the vertices of the triangle, the intersection points between two lines, and visualize the triangle by dividing it into two regions. This helps them in determining the area between two curves using definite integrals. This aligns with what Dahal and Pan Prasat (2021) stated: the GeoGebra application, created by Markus in 2001/2002 at the University of Salzburg, Austria, is an interactive software for geometry, algebra, statistics, and calculus. This application is designed as an ideal mathematical tool from elementary to university levels, to construct and illustrate mathematical concepts.

The findings from Figure 4 indicate that the use of GeoGebra greatly aids students in understanding key concepts such as determining the vertices of a triangle, finding intersection points between two lines, and visualizing the triangle by dividing it into two regions. This ability to represent the triangle geometrically helps students calculate the area between two curves using definite integrals, a critical skill in calculus. This supports the conclusions of (Dahal et al, 2022), who emphasized that GeoGebra, developed by Markus at the University of Salzburg in 2001/2002, is a highly interactive and versatile software designed for geometry, algebra, statistics, and calculus. The application is recognized as an ideal mathematical tool that supports learning from elementary through university levels, facilitating the construction and illustration of mathematical concepts. (Dahal et al, 2022) also highlighted GeoGebra's effectiveness in enhancing students' conceptual understanding through visual and interactive learning methods, which aligns with the observed improvements in student comprehension of mathematical topics in this study.

Below is the manual work of one student.

Solution:

Solusi:

Karena diketahui setiga dengan 3 titik yakni $(-1, 4)$, $(2, -2)$, dan $(5, 1)$, yang merupakan titik-titik sudut, maka dapat dimisalkan titik $A(-1, 4)$, $B(2, -2)$, dan $C(5, 1)$. Selanjutnya dapat ditentukan persamaan garis melalui dua titik sebagai berikut:

$$A(-1, 4), \text{ dan } B(2, -2); m_1 = \frac{-2-4}{2-(-1)} = -2;$$

$$l_1: y - y_1 = x - x_1 \Leftrightarrow y - 4 = -2(x - (-1))$$

$$l_1: y - 4 = -2x - 2 \Leftrightarrow y = -2x + 2;$$

$$B(2, -2), \text{ dan } C(5, 1); m_2 = \frac{1-(-2)}{5-2} = 1;$$

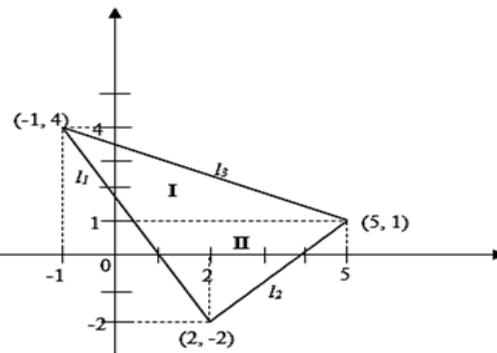
$$l_2: y - y_1 = x - x_1 \Leftrightarrow y - (-2) = x - 2$$

$$l_2: y - (-2) = x - 2 \Leftrightarrow y = x - 4;$$

$$A(-1, 4), \text{ dan } C(5, 1); m_3 = \frac{1-4}{5-(-1)} = \frac{-3}{6} = -\frac{1}{2};$$

$$l_3: y - y_1 = x - x_1 \Leftrightarrow y - 4 = -\frac{1}{2}(x - (-1))$$

$$l_3: y - 4 = -\frac{1}{2}(x - (-1)) \Leftrightarrow y = -\frac{1}{2}x + \frac{7}{2};$$



Luas Segitiga = Luas I + Luas II

$$\Leftrightarrow \int_1^4 \left[(-2y + 7) - \left(-\frac{1}{2}y + 1 \right) \right] dy + \int_{-2}^1 \left[(y + 4) - \left(-\frac{1}{2}y + 1 \right) \right] dy =$$

$$\int_1^4 \left[\left(-\frac{3}{2}y + 6 \right) \right] dy + \int_{-2}^1 \left[\left(\frac{3}{2}y + 3 \right) \right] dy = \left(-\frac{3}{4}y^2 + 6y \right) \Big|_1^4 - \left(\frac{3}{4}y^2 + 3y \right) \Big|_{-2}^1 =$$

$$\int_1^4 \left[\left(-\frac{3}{2}y + 6 \right) \right] dy + \int_{-2}^1 \left[\left(\frac{3}{2}y + 3 \right) \right] dy = \left(-\frac{3}{4}y^2 + 6y \right) \Big|_1^4 + \left(\frac{3}{4}y^2 + 3y \right) \Big|_{-2}^1 = 6\frac{3}{4} + 6\frac{3}{4} = 13\frac{1}{2}$$

Jadi luas segitiga adalah $13\frac{1}{2}$

Figure 5. The Work of One Student (R₁₇)

Based on Figure 4 above, it is evident that the students can reactivate the knowledge they have previously acquired. They can determine the equation of a line from two given points, organize the provided information, and integrate the representations from the obtained graph. This allows them to calculate the area of the triangle in question using definite integrals. These results show that students can determine the boundaries on the y-axis of the triangle and are skilled in solving the required integral. The reflective activity above is known as self-explanation, which aligns with the concept presented by (Durkin, 2011; Hodds et al., 2014; Kyungbin et al.; Rau et al., 2015) related to self-explanation, namely how students process information, understand concepts, recall material, and apply knowledge to solve problems, all of which are mental activities involved in acquiring and using knowledge. A study by Nongharnpituk et al (2022) demonstrated that the comprehensive use of GeoGebra in calculus learning—both in the classroom and at home—significantly fosters the development of essential student skills, including active engagement, independent exploration, visualization, and conceptual understanding. These findings support the perspective that technology does not diminish the roles of teachers and students, but rather serves as a powerful tool to encourage innovative learning practices and promote greater responsibility in the learning process.

6. Conclusion

Based on the data analysis results, the findings of this study indicate that learning by utilizing GeoGebra significantly helps students in developing and enhancing their self-explanation abilities. This technology provides students with opportunities to try new things, engage in independent

exploration and visualization, and reduce their mental load in learning calculus during their first year at Khairun University. For broader research, it is recommended to explore factors such as gender, learning styles, and motivation based on their initial abilities in improving self-explanation in mathematics.

7. Suggestion

To broaden the scope and strengthen the generalizability of the findings, it is recommended that this study be replicated at other universities with diverse first-year student profiles. Similar research can also be extended to other mathematical domains, such as linear algebra or statistics, to examine GeoGebra's effectiveness across varied contexts. These findings may serve as a foundation for developing technology-integrated curricula, including training programs for lecturers on how to utilize GeoGebra to foster active learning and self-explanation. Future studies are encouraged to adopt a longitudinal design and incorporate qualitative methods, such as interviews or think-aloud protocols, to gain deeper insights into students' cognitive strategies when using GeoGebra for independent mathematical explanation.

Declarations

Author Contributions. H.H.: Literature review, conceptualization, design, analysis, writing, and securing funding, K.L.N.: Methodology, review-editing, and writing, D.W.: Critical revision of the manuscript, S.B.M.: Writing-original draft, original manuscript preparation, and M.R.: Analysis, writing, and securing funding. All authors have read and approved the published on the final version of the article.

Conflicts of Interest. The authors declare no conflict of interest.

Funding. This research was funded by Universitas Khairun

Ethical Approval. This research was conducted in accordance with scientific writing guidelines and ethics

Data Availability Statement. Data is available from the author upon request

Acknowledgements. We would like to extend our gratitude to the Rector of Khairun University, Dr. M. Ridha Ajam, M.Hum, who has initiated the assignment research program for the acceleration of the professor program at Khairun University

References

- Almuwaiziri, F., Trakulphadetkrai, N. V., & Williams, T. (2023). Visualisation to support children with attention- deficit/hyperactivity disorder learning to solve mathematical word problems: A randomised controlled trial. *British Journal of Special Education*, 50 (2), 314–324. <https://doi.org/10.1111/1467-8578.12466>.
- Ayres, P., & Paas, F. (2012). Cognitive load theory: new directions and challenges. *Applied Cognitive Psychology*, 26(6), 827–832. <http://doi.org/10.1002/acp.2882>.
- Bedada, B., & Machaba, F. M. (2022). The Effect of GeoGebra on Students' Abilities to Study Calculus. *Education Research International*. 2-14. <https://doi.org/10.1155/2022/4400024>.
- Bisra, K., Liu, Q., Nesbit, J. C., Salimi, F., & Winne, P. H. (2018). Inducing self-explanation: A meta-analysis. *Educational Psychology Review*, 30, 703-725. <https://doi.org/10.1007/s10648-018-9434-x>.
- Chan, K. K., & Leung, S. W. (2014). Dynamic geometry software improves mathematical achievement: systematic review and meta-analysis. *J. Educ. Comput. Res.* 51(3), 311–325. <http://dx.doi.org/10.2190/EC.51.3.c>.

- Chi, M. T. H. (2000). "Self-explaining expository texts: the dual processes of generating inferences and repairing mental models," in *Advances in Instructional Psychology*, ed. R. Glaser (Mahwah, NJ: Lawrence Erlbaum Associates), 161–238. <https://doi.org/10.1017/CBO9780511816819.018>.
- Chi, M. T. H. (2009). Active-constructive-interactive: a conceptual framework for differentiating learning activities. *Top. Cogn. Sci.* 1, 73–105. <https://doi.org/10.1111/j.1756-8765.2008.01005.x>
- Dahal, N., Pant, B. P., Shrestha, M. I., & Manandhar, K. N. (2022). Use of GeoGebra in Teaching and Learning Geometric Transformation in School Mathematics. *International Journal of Interactive Mobile Technologies (IJIM)*. 16 (08), 323-332. <https://doi.org/10.3991/ijim.v16i08.29575>.
- De Guzman, M., Hodgson, B. R., Robert, A., & Villani, V. (1998). Difficulties in the passage from secondary to tertiary education. In G. Fischer (Ed.), *Documenta mathematica: Proceedings of the international congress of mathematicians, Extra volume* (pp. 747–762). Geronimo. Retrieved September 30, 2022, from https://www.ime.usp.br/~vhgiusti/difcuidades_passagem.pdf.
- Di Martino, P., & Gregorio, F. (2018). The mathematical crisis in secondary-tertiary transition. *International Journal of Science and Mathematics Education*, 17(4), pp 825–843. <https://doi.org/10.1007/s10763-018-9894-y>.
- Durkin, K. (2011). The self-explanation effect when learning mathematics: A meta-analysis. In *Conference on Building and Educational Science: Investigating Mechanisms* (pp. 1–5). Washington, DC: Society for Research on Educational Effectiveness Spring 2011. <http://onlinelibrary.wiley.com/doi/10.1002/cbdv.200490137/abstract%5Cnhttp://www.sree.org/conferences/2011/program/download%20s/abstracts/150.pdf>.
- Fiorella, L., & Mayer, R. E. (2016). Eight ways to promote generative learning. *Educational Psychology Review*, 28(4), 717-741. <https://doi.org/10.1007/s10648-015-9348-9>.
- Fukawa-Connelly, T. (2012). Classroom socio mathematical norms for proof presentation in undergraduate in abstract algebra. *The Journal of Mathematical Behavior*, 31(3), 401–416. <http://doi.org/10.1016/j.jmathb.2012.04.002>.
- Gurmu, F., Tuge, C., & Hunde, B. A. (2024). Effects of GeoGebra-assisted instructional methods on students' conceptual understanding of geometry. *Cogent Education*, 11(1), 1-17. <https://doi.org/10.1080/2331186X.2024.2379745>.
- Hodds, M., Alcock, L., & Inglis, M. (2014). Self-explanation training improves proof comprehension. *Journal for Research in Mathematics Education*, 45(1), 62–101. <http://doi.org/10.5951/jresmetheduc.45.1.0062>.
- Intaros, P., Inprasitha, M., & Srisawadi, N. (2014). Students' problem solving strategies in problem solving-mathematics classroom. *Procedia - Social and Behavioral Sciences*, 116, 4119–4123. <http://doi.org/10.1016/j.sbspro.2014.01.901>.
- Ishartono, N., Nurcahyo, A., Waluyo, M., Razak, R., Sufahani, S., & Hanifah, M. (2022). GeoGebra-based flipped learning model: An alternative panacea to improve student's learning independency in online mathematics learning. *JRAMathEdu (Journal of Research and Advances in Mathematics Education)*, 7(3), 178-196. <https://doi.org/10.23917/jramathedu.v7i3.18141>.

- Juandi, D., Kusumah, S. Y., Tamur, M., & Perbowo, K. (2021). The Effectiveness of Dynamic Geometry Software Applications in Learning Mathematics: A Meta- Analysis Study. *International Journal of Interactive Mobile Technologies (IJIM)*, 15(02), 18-37. <https://doi.org/10.3991/ijim.v15i02.18853>.
- Kyungbin, K., Kumalasari, C. D., & Howland, J. L. (2011). Self-explanation prompts on problem-solving performance in an interactive learning environment. *Journal of Interactive Online Learning*, 10(2), 96– 112. <http://www.ncolr.org/jiol/issues/pdf/10.2.3.pdf>.
- Lopez, O. S. (2007). Classroom diversification: a strategic view of educational productivity. *Review of Educational Research*, 77(1), 28–80. <https://doi.org/10.3102/003465430298571>.
- McEldoon, K. L., Durkin, K. L., & Rattle-Johnson, B. (2013). Is self-explanation worth the time? A comparison to additional practice. *Br. J. Educ. Psychol.* 2013, 83, 615–632. DOI: 10.1111/j.2044-8279.2012.02083.x.
- Miller-Cotto, D., & Belenky, D. M. (2022). Sketching and verbal self-explanation: Do they help middle school children solve science problems? *Applied Cognitive Psychology*, 36(3), 552–563. <https://doi.org/10.1002/acp.3980>.
- Nakamoto, R., Flanagan, B., Dai, Y., Yamauchi, T., Takami, K., & Ogata, H. (2023). Enhancing Self-Explanation Learning through a Real-Time Feedback System: An Empirical Evaluation Study. *Sustainability*, 15, 15577. <https://doi.org/10.3390/su152115577>.
- Nongharnpituk, P., Yonwilad, W., & Khansila, P. (2022). The Effect of GeoGebra Software in Calculus for Mathematics Teacher Students. *Journal of Educational Issues*. 8(2), 755-770. <https://files.eric.ed.gov/fulltext/EJ1362030.pdf>.
- Rach, S., & Heinze, A. (2016). The Transition from School to University in Mathematics: Which Influence Do School-Related Variables Have? *International Journal of Science and Mathematics Education*, 15(7), 1-21. <https://doi.org/10.1007/s10763-016-9744-8>.
- Rittle-Johnson, B. (2006). Promoting transfer: Effects of self-explanation and direct instruction. *Child Dev.* 2006, 77, 1–15. DOI: 10.1111/j.1467-8624.2006.00852.x.
- Rittle-Johnson, B., Loehr, A. M., & Durkin, K. (2017). Promoting self-explanation to improve mathematics learning: A meta-analysis and instructional design principles. *ZDM* 2017, 49, 599–611. <http://dx.doi.org/10.1007/s11858-017-0834-z>.
- Rau, M. A., Aleven, V., & Rummel, N. (2015). Successful learning with multiple graphical representations and self-explanation prompts. *Journal of Educational Psychology*, 107(1), 30–46. <http://doi.org/10.1037/a0037211>.
- Renkl, A. (2017). Learning from worked-examples in mathematics: Students relate procedures to principles. *ZDM* 2017, 49, 571–584. <https://doi.org/10.1007/s11858-017-0859-3>.
- Rukavina, S., Zuvic-butovac, M., Ledic, J., Milotic, B., & Jurdana-sepic, R. (2012). Developing positive attitude towards science and mathematics through motivational classroom experiences. *Science Education International*, 23(1), 6– 19. <https://urn.nsk.hr/urn:nbn:hr:193:781542> .
- Santosa, C., Suryadi, D., Prabawanto, S., & Syamsuri, S. (2019). The role of worked-example in enhancing students' self-explanation and cognitive efficiency in calculus instruction. *Jurnal Riset Pendidikan Matematika*, 5(2), 168-180. <https://doi.org/10.21831/jrpm.v0i0.19602> .
- Sweller, J., Ayres, P., & Kalyuga, S. (2011). Cognitive load theory: explorations in the learning sciences, instructional systems and performance technologies. London, UK: Springer. <https://doi.org/10.1007/978-1-4419-8126-4> .

- Takači, D., Stankov, G., & Milanovic, I. (2015). Efficiency of learning environment using GeoGebra when calculus contents are learned in collaborative groups. *Computers & Education*, 82, 421-431. <https://doi.org/10.1016/j.compedu.2014.12.002>.
- Tall, D. (2008). The transition to formal thinking in mathematics. *Mathematics Education Research Journal*, 20, 5–24. <https://doi.org/10.1007/BF03217474>.
- Tall, D., & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics*, 12(2), 151–169. <https://doi.org/10.1007/BF00305619>.
- Veasna, S., & Heng, T. (2024). Improving Students' Visual Representation and Conceptual Understanding to Overcome Learning Difficulties in Geometry Using GeoGebra. *International Journal on Emerging Mathematics Education (IJEME)*. 8(2), 31-44. DOI: <http://dx.doi.org/10.12928/ijeme.v8i2.29834>.
- Ziatdinov, R., & Valles, J. R., Jr. (2022). Synthesis of Modeling, Visualization, and Programming in GeoGebra as an Effective Approach for Teaching and Learning STEM Topics. *Mathematics*, 10(398), 1-16. <https://doi.org/10.3390/math10030398>.

About the Contributor(s)

Hasan Hamid, Doctor of Mathematics Education, Associate Professor of Mathematics Education Program at Faculty of Teacher Training and Education, Khairun University. He serves as the Vice Rector for Academic Affairs. His research interests include pragmatics in mathematics education, proof in mathematics learning, as well as planning and assessment in mathematics instruction. He is actively involved in several national research projects on mathematics learning topics. Additionally, he frequently publishes articles in reputable national and international journals.

Email: hasan.hamid66@gmail.com

ORCID: <https://orcid.org/0009-0000-2711-265X>

Karman La Nani, Doctor of Mathematics Education, Assistant Professor of Mathematics Education Program at Faculty of Teacher Training and Education, Khairun University. He serves as the Coordinator of the Master's Program in Mathematics Education. His research interests include reasoning in statistics learning and competence in mathematics and statistics education. He is actively involved in several national research projects related to statistics learning. Additionally, he frequently publishes articles in reputable national and international journals.

Email: karmanlanani@gmail.com

ORCID: <https://orcid.org/0009-0003-5128-8665>

Dahlan Wahyudi, Master of Mathematics Education, Assistant Professor of Mathematics Education Program at Faculty of Teacher Training and Education, Khairun University. His research interests are related to the teaching of mathematics education, as well as the planning and assessment of mathematics learning. Additionally, he is actively publishing articles in reputable national and international journals.

Email: dahan.wahyudi@unkhair.ac.id

ORCID: <https://orcid.org/0000-0002-7437-2208>

Sitti Busyrah Muchsin, Master of Mathematics Education, Assistant Professor of Mathematics Education Program at Faculty of Teacher Training and Education, Khairun. His research interests are related to the teaching and curriculum of mathematics education in schools, as well as the planning

and assessment of mathematics learning. He is actively involved in several national research projects on mathematics learning topics. Additionally, he frequently publishes articles in reputable national and international journals.

Email: sittibusyrah@unkhair.ac.id

ORCID: <https://orcid.org/0009-0001-5632-0180>

Mustafa A.H Ruhama, Doctor of Mathematics Education, Associate Professor of Mathematics Education Program at Faculty of Teacher Training and Education, Khairun University. His research interests include learning styles in mathematics education, proof in mathematics learning, and competence in mathematics learning. He is actively involved in several national research projects related to mathematics learning. Additionally, he frequently publishes articles in reputable national and international journals.

Email: mustafaruhama@gmail.com

ORCID: <https://orcid.org/0009-0008-3425-0661>

Publisher's Note: *The opinions, statements, and data presented in all publications are solely those of the individual author(s) and contributors and do not reflect the views of Universitepark, EDUPIJ, and/or the editor(s). Universitepark, the Journal, and/or the editor(s) accept no responsibility for any harm or damage to persons or property arising from the use of ideas, methods, instructions, or products mentioned in the content.*
