

## Research Article

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## Author for correspondence:

Supratman

✉ [supratman21034@mhs.unesa.ac.id](mailto:supratman21034@mhs.unesa.ac.id)

✉ Mathematics Education Department, Faculty of Mathematics and Natural Sciences, Universitas Negeri Surabaya, Surabaya, Indonesia

✉ [supratmanmathusnkolaka@gmail.com](mailto:supratmanmathusnkolaka@gmail.com)

✉ Mathematics Education Study Program, Faculty of Teacher Training and Education, Universitas Sembilanbelas November Kolaka, Kolaka, Indonesia



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# Probabilistic Thinking Process in Probability Problem-Solving Prospective Mathematics Teacher with Field-Independent based on Polya's Three Stages

Supratman<sup>ID</sup>, I Ketut Budayasa<sup>ID</sup>, Endah Budi Rahaju<sup>ID</sup>

## Abstract

**Background/purpose.** This study investigates the probabilistic thinking process in solving probability problems by prospective mathematics teachers with a field-independent cognitive style. The objective is to explore how individuals with this cognitive style approach problem-solving based on the three stages of Polya's framework: understanding the problem, devising a plan, and carrying out the plan.

**Materials/Methods.** A descriptive qualitative approach with a case study design was employed. The participant was a female student enrolled in the Mathematics Education Program at the Universitas Sembilanbelas November Kolaka, identified as having a field-independent cognitive style through the Group Embedded Figures Test (GEFT). The research instruments included a probability problem-solving task constructed based on indicators of probabilistic thinking and Polya's stages, along with semi-structured interview guidelines. Data were collected through task-based problem-solving and interviews, and were analyzed using data reduction, data display, and conclusion drawing.

**Results.** The participant demonstrated a systematic probabilistic thinking process across all three stages. During the problem-understanding phase, she successfully identified known and unknown information and accurately interpreted mathematical symbols. In the planning phase, she applied deductive reasoning to develop appropriate strategies. In the execution phase, she implemented the procedures precisely and arrived at logical conclusions.

**Conclusion.** The findings suggest that solving binomial distribution problems requires more than procedural knowledge; it also involves conceptual understanding, logical reasoning, and the capacity to manage uncertainty. Prospective mathematics teachers with a field-independent cognitive style exhibit strong potential in integrating structured probabilistic thinking with Polya's problem-solving stages.

## 1. Introduction

Probabilistic thinking is a form of thinking that involves understanding probability and uncertainty in various contexts, both formal and informal. According to Jones et al. (1999), probabilistic thinking includes the ability to construct a sample space, determine the probability of an event, compare probabilities, and understand conditional probability. This ability is crucial because it enables individuals to make more rational and logical judgments when faced with uncertain situations (AlAli et al., 2023). In everyday life, probabilistic thinking is highly relevant. Individuals are faced with situations that require estimates of probability, such as predicting rain based on weather forecasts, evaluating risks in financial decision-making, or estimating the likelihood of success of an action based on available data. Those with good probabilistic thinking skills tend to be more cautious, critical in decision-making, and able to assess uncertain information objectively.

Mathematics education plays an important role in developing probabilistic thinking skills from an early age. Learning that emphasizes concrete and contextual understanding of probability concepts can help students maintain strong probabilistic intuition. Furthermore, the use of games, simulations, or data-based case studies can strengthen understanding of sample space concepts, independent events, and conditional events, as well as prevent common misconceptions about probability. One study, Groth et al. (2021), stated that sample space misconception leads students to speak as if specific outcomes are guaranteed when they are not. Moreover, two studies, Elbehary (2022) and Martin et al. (2022), emphasize that probabilistic thinking is closely related to statistical literacy because both emphasize understanding uncertainty and data variability. Therefore, the development of probabilistic thinking is not only important in mathematics education but also in various other disciplines such as science, economics, information technology, and public policy. Moreover, in today's digital age, where data and information are disseminated in large quantities and at high speeds, the ability to think probabilistically has become increasingly crucial. Many important decisions in the fields of health, business, and technology are based on probabilistic models or predictions. Thus, education that fosters probabilistic thinking will directly contribute to strengthening numeracy literacy and developing critical thinking skills essential for the 21st-century generation (Wijaya et al., 2024) and (Abbasi et al., 2025)

Probabilistic thinking not only requires mathematical skills but also interpretive and affective skills, such as understanding the context of data, considering the sources of information, and having a critical disposition toward conclusions. Therefore, probabilistic thinking must be seen as an integral part of statistical literacy, which is the ability to understand and use data-based information in real life. In a modern society filled with information and data, this ability is important to avoid decision-making errors that can have social, economic, and personal impacts. Furthermore, probabilistic thinking does not arise automatically from everyday experience. Studies show that although humans are often faced with uncertain situations, they tend to use inaccurate heuristics in estimating probabilities (Savard, 2014). This emphasizes the need for explicit teaching of probabilistic thinking concepts and processes in secondary and higher education onwards.

In the context of mathematics learning, probabilistic thinking is categorized as part of higher-order thinking skills (HOTS) because it involves analysis, synthesis, and evaluation of uncertain information. Shaughnessy et al. (2021) state that understanding probability requires a shift from deterministic thinking to statistical thinking, which is not easy to achieve because it requires acceptance of uncertainty in results. This differs from mathematical operations such as arithmetic and algebra, which are deterministic in nature. Savard (2014) adds that probabilistic thinking consists of three main components: (1) understanding the random nature of an event; (2) the ability to recognize and interpret patterns or trends in data; and (3) the ability to argue probabilistically, that is, to explain and defend decisions based on probability estimates. These three aspects are crucial in preparing students, particularly future teachers, to face the complexities of the real world.

However, one study, Rodríguez-Alveal et al. (2022), indicates that many students, including prospective mathematics teachers, still hold misconceptions about probability concepts. Common difficulties include errors in constructing sample spaces, inaccuracies in understanding the nature of exclusive and inclusive events, and misunderstandings of conditional probability. This situation underscores the urgency of conducting in-depth research on probabilistic thinking processes, particularly in the context of problem-solving. One approach that can be used to explore the thinking process in problem-solving is Polya's approach (Polya, 1945), which consists of four stages: (1) understanding the problem, (2) devising a plan, (3) carrying out the plan, and (4) looking back. In this study, the focus is on the three stages, which reflect the core components of the probabilistic thinking process. The stage of understanding the problem requires the identification of random elements, such as sample space or possible events. The stage of designing a plan involves selecting the appropriate strategy, such as multiplication rules or tree diagrams. Meanwhile, the stage of implementing the plan requires precision in applying the strategy until a logical solution is obtained (Taamneh et al., 2024).

However, based on initial observations, it was found that prospective mathematics teacher showed low learning outcomes, especially in discrete probability topics such as the binomial distribution, even though it is a fundamental concept in discrete probability that represents repeated events with two possible outcomes (success and failure) occurring independently (García-García et al., 2022). This material requires an understanding of random variables, distribution parameters, and the connection between mathematical models and real-world contexts (Ingram, 2022; Batanero & Álvarez-Arroyo, 2024). Observations indicate that many students struggle to translate probability models into real-world situations, such as determining the exact probability of  $K$  successes out of  $n$  trials, identifying the values of  $p$  and  $q$ , and understanding the meaning of symbols and parameters in the binomial distribution formula. Most students also appear to have difficulty interpreting results contextually, which should be the core of probabilistic thinking. Muhaimin et al. (2024) stated that some of the causes of these difficulties are believed to stem from a lack of systematic problem-solving strategies. Students tend to directly apply formulas without understanding the concept of sample space and the nature of events. Additionally, cognitive style factors also play a role. Students who lack a Field-Independent (FI) style often struggle to organize complex information and identify key elements of a problem. The FI style, as explained by Witkin et al. (1977), refers to an individual's ability to process information analytically, isolate key elements from the context, and think independently and structurally.

FI-style individuals generally perform better in mathematical problem-solving that requires logical structure and deep reasoning. One study (Wang et al., 2003) stated that FI style is positively correlated with critical and conceptual thinking abilities, including understanding probability concepts. Furthermore, Ali et al. (2022) stated that "gender roles are a set of expectations that determine how women or men should think, act and feel". Furthermore, Santrock (2019) and Valsecchi et al. (2023) added that gender type "refers to the acquisition of traditional masculine or feminine roles, for example, aggression characterizes more traditional masculine roles, while nurturing characterizes more traditionally feminine roles". Therefore, gender in this study is a characteristic that distinguishes between men and women, formed by socio-cultural and psychological factors, with characteristics of men tending to be masculine and women tending to be feminine. Other than that, males have substantially more positive attitudes toward mathematics and higher levels of participation in non-mandatory levels of mathematics than females. However, the ability of female students in cognitive terms is much better (Hall, 2012). This study focused on how the probabilistic thinking process of a female student who had a field-independent cognitive style could solve probability problems based on Polya's Three Stages, namely the stage of understanding the problem, devising a Plan, and carrying out the plan. The variable of gender was added based on

a recent study by Xie et al. (2023) and Twohill et al. (2023), who stated that gender has a role in improving the learning process and students' confidence in understanding mathematics. This study is expected to contribute significantly to the development of more adaptive, contextual, and inclusive mathematics learning strategies, in order to train teachers who are capable of developing numeracy literacy evenly and equitably in the future.

## 2. Literature Review

Thinking is a cognitive process used to understand the world and plays an important role in guiding a person to find new solutions that can have a positive impact on their life. Limbach Barbara (2022) stated that thinking is a cognitive process used to understand the world, where questioning everyday assumptions will lead students to new solutions that can have a positive impact on their quality of life. In line with this, A study by Ruggiero V.R. (2012) mentions that thinking is a term used for various activities, ranging from daydreaming to reflection and analysis. This statement shows that if someone dreams of something and tries to realize it to the stage of evaluation and analysis, then that person has engaged in thinking. Additionally, Keshav et al. (2022) reveal that thinking is a natural human activity because, through it, a person can generate knowledge for themselves that can then be shared with others. In the view of Jacinto and Carreira (2023), thinking is a cognitive activity that occurs within a person's mind. Although not directly observable, the thinking process can be inferred through observable behavior.

According to Solso et al. (1995), thinking is the process of forming new mental representations that occur through the transformation of information involving the complex interaction of various mental attributes such as judgment, abstraction, reasoning, imagination, and problem-solving. In relation to uncertainty and decision making, probabilistic thinking plays an important role. According to two studies, Maxine & Constance (1996) and Zhang et al. (2016), probabilistic thinking is a new approach to processing information that emerged with the paradigm shift from deterministic to probabilistic. Hokor et al. (2023) describe probabilistic thinking as a mode of reasoning aimed at quantifying uncertainty and used as a tool in decision-making. Hokor et al. (2021) state that probabilistic thinking has several important characteristics, namely not seeking direct relationships between events, explaining and describing, not being based on a single set of data, and working using models. Probabilistic thinking does not immediately attempt to connect events in individual cases but instead seeks patterns from possible repetitions, thus not merely guessing but using specific calculations (Batanero et al., 2021). Additionally, this thinking aims to explain and describe variations in probability models, not only focusing on a single data point but also considering the experimental mechanisms that generate various possibilities, and working with models that envision multiple possible outcomes rather than a fully known reality.

Lau and Ranyard (1999) explained that probabilistic thinking is a tendency to view the world in terms of uncertainty, assigning uncertainty values both verbally and numerically. Furthermore, Jones et al. (1999) stated that probabilistic thinking is used to describe how a person thinks in response to probabilistic situations. This is reinforced by Mooney et al. (2014) and Groth et al. (2021), who both state that probabilistic thinking reflects cognitive responses to various probabilistic problems. Jolliffe (2005) mentions that probabilistic thinking activities involve understanding how models are used to simulate random phenomena, how data is collected to estimate probabilities, and how symmetry and other properties of a situation can help in determining probability values. In the process of learning mathematics in higher education, mathematical problems are challenges that require flexible and creative thinking (Scheunemann et al., 2022), as they typically cannot be solved using routine procedures previously mastered. Instructors often present problems in the form of questions or assignments. However, a question only becomes a problem for a student if it is challenging and cannot be solved using procedures they are familiar with. In other words, a question can be a challenge for one student but not for another. Jones et al. (1999) explain that probabilistic situations

or problems involving uncertainty are random activities or experiments that can produce various possible outcomes. However, the final result cannot be determined with certainty. Nevertheless, the set of all possibilities can be determined through long-term distribution. One form of error in understanding probability is equilibrium bias, which is the tendency to believe that every random process will produce a fair distribution, with equal chances for every possible outcome. This was highlighted by Moreno & Cardeñoso (2014) in their study of 908 prospective mathematics and biology teachers in Mendoza Province, Argentina. In the study, the prospective teachers were asked to provide reasons for their answers to twelve questions about probability covering contexts such as games, daily life, and physical-natural phenomena. Probability problems that cannot be solved using routine procedures can be addressed using Polya's problem-solving model.

Polya (1945) proposed four steps in the problem-solving process: understanding the problem, devising a plan, carrying out the plan, and looking back. The first step, understanding the problem, is a very important foundation. According to Polya, students need to identify what is unknown, what data is available, and what conditions are given. This stage helps students analyze the problem and determine the appropriate direction for solving it. In solving probability problems, especially those that cannot be addressed using routine procedures, Polya's structured steps offer a practical framework. However, this study focuses only on the three steps, as they involve the main cognitive activities that are directly engaged in the problem-solving process. However, this study focuses only on the three steps because these stages involve the main cognitive activities directly engaged in the problem-solving process itself. The fourth step, Looking Back, although important for reflection, evaluation, and generalization, mainly concerns reviewing the solution after it has been completed. This step does not significantly contribute to the initial process of forming the solution, which is the primary focus of this study. In other words, this study aims to observe and analyze how students think and act during the process of understanding the problem, designing a solution, and carrying out that plan, that is, during the active phase of problem-solving (Baiduri et al., 2020). It can be used as a basis of reference in carrying out mathematics learning at any level (Kholid & Aisyah, 2025). Therefore, the three steps are considered sufficient to describe the students' probabilistic thinking process, mainly when analyzed based on cognitive style and gender differences.

Individual cognitive styles greatly influence the ability to understand and solve problems. Research findings by Lo and Tsai (2022) indicate that individuals with a field-independent (FI) cognitive style demonstrate better problem-solving abilities than those with a field-dependent (FD) style, as they possess superior reasoning skills. This is supported by the findings of another recent study Sutama et al. (2021), stating that students with an FI style achieve higher results in solving complex problems. The FI style is characterized by the ability to analyze and connect concepts with situations, focus on the main aspects of the problem, and not be easily influenced by distracting elements (Vecchione et al., 2023). Conversely, individuals with an FD style tend to think globally and passively, and have difficulty finding connections between situations, concepts, representations, formulations, and solution strategies. These limitations hinder the development of probabilistic thinking in FD-style individuals. Therefore, cognitive style has a close relationship with a person's ability to think probabilistically.

In addition to cognitive style, gender also influences thinking and problem-solving processes. Each individual has different abilities in solving probability problems, which biological differences between men and women can influence. In general, women's brains are smaller than men's brains. One key distinguishing feature is the inferior parietal lobe, which is involved in spatial processing and three-dimensional visualization, where men have a larger size compared to women (Ashkenazi et al., 2024). Menzler et al (2011) state that these structural brain differences can lead to variations in behavior, development, and cognitive processes between men and women. These differences then

have implications for student learning outcomes, as learning outcomes are not only influenced by ability factors but also by behavioral development and cognitive processing.

Probabilistic thinking is a complex cognitive activity, as various factors, both internal and external to the individual, influence it. Internal factors, such as a field-independent cognitive style, play a role in determining how one processes information and makes decisions independently. On the other hand, external factors such as gender differences also contribute to variations in strategies and tendencies when solving problems, including those related to probability. The interaction between these two factors can affect an individual's effectiveness in dealing with uncertainty-based problems. In the context of mathematics education, particularly in the topic of probability, these factors should be considered when designing instructional strategies that aim to develop students' analytical, logical, and reflective thinking skills. Therefore, an approach is needed that not only emphasizes conceptual mastery but also incorporates structured stages of thinking. Figure 1 illustrates that the probabilistic thinking process of prospective mathematics teachers occurs through three main stages as proposed by Polya: understanding the problem, devising a plan, and carrying out the plan. Each of these stages is closely linked to indicators of probabilistic thinking, such as identifying random experiments, determining the sample space, recognizing events, and calculating the probability of an event.

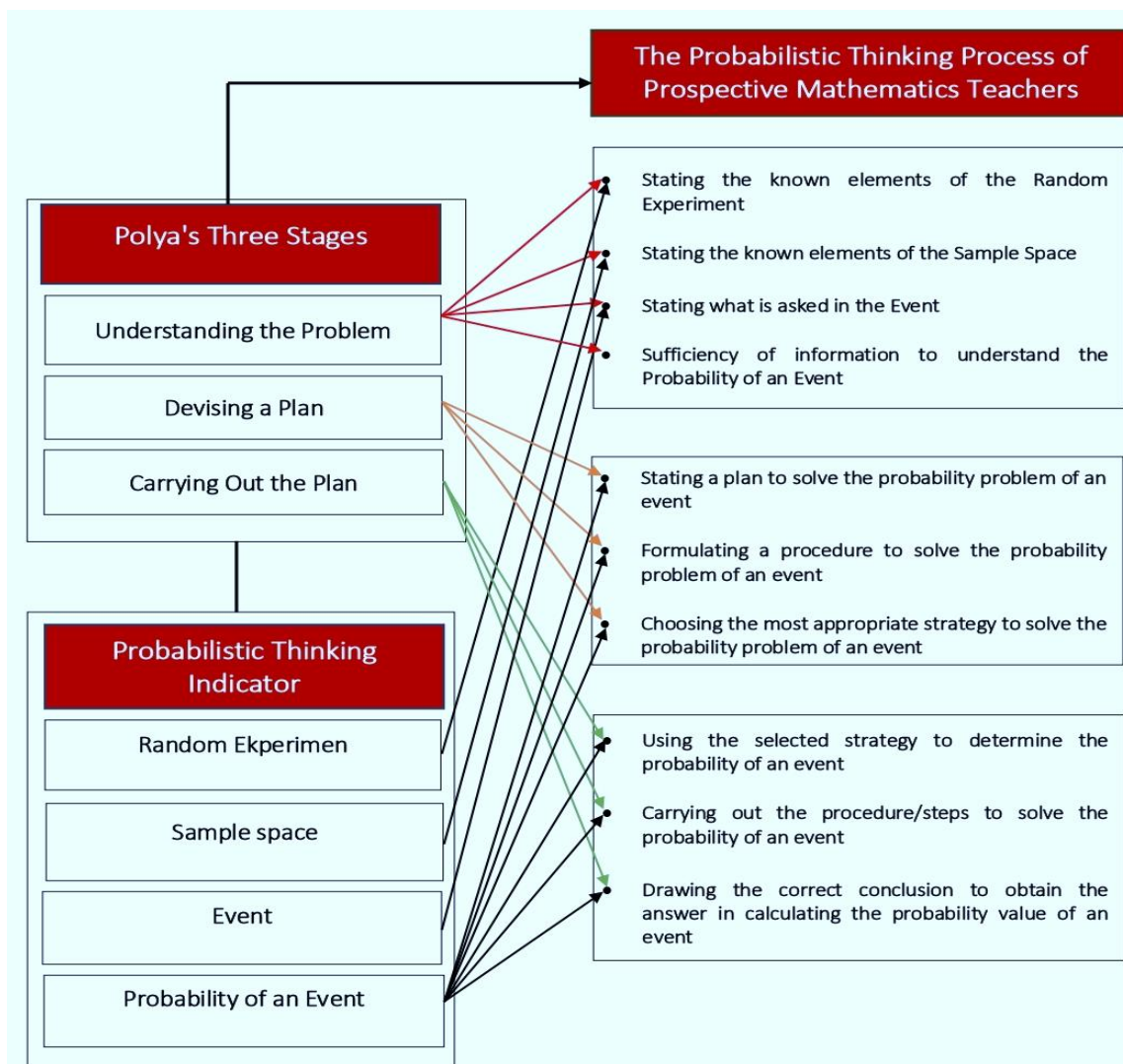


**Figure 1.** Probabilistic Thinking Model Based on Polya's Three Stages

### 3. Methodology

#### 3.1. Research Design

The objective of this study was to explore the probabilistic thinking of prospective mathematics teachers who exhibit a Field Independent (FI) cognitive style. Specifically, the study investigated their thinking processes through the lens of Polya's three problem-solving stages: understanding the problem, devising a plan, and carrying out the plan. The approach used in this study was descriptive qualitative, as described by Fraenkel et al. (2011), because it aimed to provide an in-depth understanding of the participants' thought processes and behaviors. This study employed a qualitative research design, utilizing a case study model to obtain a nuanced and in-depth understanding of the phenomenon. The theoretical framework underpinning this study was based on Field Independent (FI) cognitive style theory and Polya's problem-solving theory, which served as a framework for analyzing the probabilistic thinking processes of prospective mathematics teachers. By utilizing these theories, this study is expected to provide deeper insights into how cognitive style and problem-solving strategies influence probabilistic thinking.

#### 3.2. Participants

The subject of this research was a female prospective mathematics teacher who was a 6th semester student of the Mathematics Education program at Universitas Sembilanbelas November Kolaka at the time the study was conducted, selected based on her high mathematical ability and strong Field Independent (FI) cognitive style, as measured by the Group Embedded Figures Test (GEFT) with a score of 15 and the UTBK SBMPTN mathematics ability test with a score of 77. The selection of this subject also considered high mathematical ability and the ability to scrutinize information independently and solve problems effectively. The UTBK SBMPTN Mathematics Test and GEFT were utilized in this study due to their reliability and validity. The UTBK SBMPTN Mathematics Test is a standardized assessment that measures mathematical abilities, providing an accurate picture of the research subject's mathematical capabilities. The mathematics ability test referred to in this study was a general mathematics test, consisting of questions in statistics, geometry, and algebra. Meanwhile, the GEFT is a valid tool for measuring cognitive styles, specifically Field Independent (FI) or Field Dependent (FD). Both tests have been widely used in educational research, making their results dependable and trustworthy. The reason for choosing the female research subject was due to the limited number of male subjects, so the female subject was selected to meet the research objectives and provide a more accurate picture of the mathematical thinking and problem-solving processes in individuals with a Field Independent (FI) cognitive style. By focusing on one subject, this research was able to conduct an in-depth analysis of the subject's mathematical abilities and cognitive style. The study selected a student in her 6th semester because she had already studied the binomial distribution material, which was relevant to the research focus, allowing for a more informed assessment of her mathematical thinking and problem-solving skills. The recruitment was also done based on interviews with the lecturer supervising the Mathematical Statistics I course. This was because one of the contents of the material in it, namely probability, was included in the Mathematical Statistics I course. Further details can be seen in Table 1:

**Table 1.** Research subjects with characteristics and codes:

| Subject Initials | Math Ability | Gaya Kognitif |          | Subject Code |
|------------------|--------------|---------------|----------|--------------|
|                  | Score        | Score         | Category |              |
| FJA              | 77           | 15            | FI       | MPI          |

### 3.3. Research Tools

The main instrument of this research was the researcher himself, who played a role in planning and collecting data, analyzing and summarizing data, and reporting research results. The supporting instrument in this research is the Probability Problem-Solving Task (TPMP). The TPMP was designed to reveal students' probabilistic thinking activities through non-routine problems that were challenging and stimulated creative thinking. To ensure the quality and reliability of the TPMP, several important steps were taken. First, the TPMP was tested for content, construct, and face validity through rational analysis by experts. Content validity ensured that the material used in the TPMP was relevant to students' level of understanding and aligned with the applicable curriculum. Construct validity ensured that the TPMP is designed to measure probabilistic thinking ability accurately, while face validity ensured that the TPMP was presented clearly and unambiguously. Second, the TPMP was validated by three experienced mathematics education lecturers. This validation aimed to ensure that the TPMP was not only theoretically valid but also effective in measuring students' probabilistic thinking ability in practice. Thus, the TPMP used in this study had undergone a rigorous testing and validation process to ensure that the results were reliable and meaningful. This resulted in a valid Probability Problem-solving Task, so the researcher focused only on part (a) of the task, which was presented in Figure 2 below.

A circular arrow target board has a small circular center called a bullseye, the center of the target in area 1 consists of two parts, the inner part is red (Double Bull) and the outer part is green (Bull), whereas the 20 circular areas given numbers 1 to 20 are located around the board. Each circular area is divided into three parts, namely, in area 2 it covers the target boundary line that is assumed to be part of the circle and gets a double value 3, in area 3 it includes the target border line which is presumed to enter the inside of the circles and gets the double value 2, whereas in area 4 it is the most outer area of the goal board, so that if a person throws the arrow and touches a particular number, he gets the same value as the number, twice that number, or three times those numbers, depending on which portion of the area affected in sequence on the outside, the middle part, and the portion inside which the archer is assuming that the two plates are not reached so that the target can be reached next to the target. If a person's chance of touching the center of the target is 0.01, the chance of a double value of 3 is 0.25, the probability of a duplicate value of 2 is 0.15, and the probabon of an outermost area of the goal board is 0.09, as illustrated in the following figure.

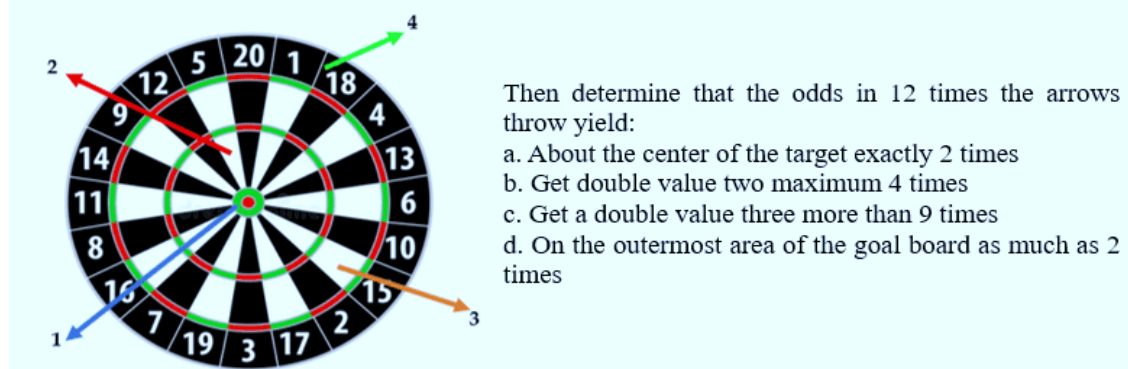


Figure 2. Probability problem-solving task

### 3.4. Data Collection Procedure

The researcher collected data from prospective mathematics teachers enrolled in the undergraduate Mathematics Education program who were given probability problem-solving tasks. The prospective teachers were asked to complete the tasks while verbalizing their thoughts aloud using the Think Aloud method. During this process, the researcher recorded their behaviors, expressions, and any unique actions displayed. After completing each task, the researcher conducted interviews to explore the probabilistic thinking of the prospective teacher, with all interviews recorded using a Sony ICD-PX240 voice recorder. The interviewer avoided giving hints, leading

questions, or any other interventions that might influence the participants' thinking processes. Additionally, the researcher transcribed the subjects' written work during problem-solving and noted their answers. Finally, the researcher documented both the subjects' responses and the interview results.

### 3.5. Data Analysis

After all the data had been collected, the next step was to manage and analyze the data systematically. The analysis process involved grouping data based on meanings related to probabilistic thinking in solving probability problem-solving tasks, presenting the data in the form of narratives or diagrams, and drawing conclusions that both the researcher and others easily understood. This study employed a qualitative approach, as outlined in the model proposed by Miles, Huberman, and Saldana (2014), which consists of five stages: data classification, data reduction, data display, data interpretation, and concluding. In the classification or categorization stage, data with similar meanings were selected and grouped, particularly data illustrating the probabilistic thinking process of prospective female mathematics teachers in solving probability problem-solving tasks. Next, during data reduction, the researcher summarized, selected, and focused on important data while identifying emerging themes and patterns from the thinking process. The reduced data were then presented in an easily understandable way, such as through brief narratives, diagrams, or relationships between categories, facilitating comprehension of the obtained data. In the interpretation stage, the researcher interpreted the meaning of the presented data and related it to the research objectives, which were to describe the probabilistic thinking of a prospective female mathematics teacher based on the Field Independence (FI) cognitive style. Finally, in the conclusion stage, the researchers provided a more precise and more in-depth description of the profile of probabilistic thinking of prospective female mathematics teachers in solving probability problems. The conclusions also included causal relationships, possible hypotheses, as well as similarities and differences based on the FI cognitive style, serving as answers to the research questions.

## 4. Results and Discussion

### 4.1. Result

The problem-solving abilities of prospective mathematics teachers in solving probability problems are presented in Table 2 below.

**Table 2.** Results of Probability Problem-Solving Tasks

| Subject Code | Gender             | Cognitive Style                  | TPMP Score | Probabilistic Problem-Solving Test Result Criteria | Description |
|--------------|--------------------|----------------------------------|------------|----------------------------------------------------|-------------|
| MPI          | Female<br>Feminine | <i>Field-Independent</i><br>(FI) | 87         | High                                               | A           |

The results of the study indicate that the probabilistic thinking process of the selected prospective mathematics teacher was good in solving probability problems. This is evidenced by the TPMP task score obtained, which was 87, classified as high. Based on her answers, three initial stages of problem-solving proposed by Polya were identified: understanding the problem, devising a plan, and carrying out the plan. The MPI subject demonstrated strong probabilistic thinking in the stage of understanding the problem, particularly in the context of random experiments, sample spaces, and events. The subject also demonstrated deep reflection on the meaning of the problem statements. Furthermore, the subject successfully devised a systematic problem-solving strategy to determine

the probability of an event by applying the correct formulas. In the execution stage, the subject effectively implemented the procedures and strategies, ultimately drawing accurate conclusions to determine the probability value of the given event.

The problem-solving responses of prospective mathematics teachers based on the three stages of Polya's framework are presented as follows:

**a) Research of Answers to Probability Problem-Solving task questions based on Understanding the Problem**

Based on the results of the probability problem-solving task, the MPI results on Polya's initial indicator, namely understanding the problem, can be seen in Figure 3.

a).

Diketahui :

- Banyaknya kesadisan ( $n$ ) = 12
- Variabel acak ( $x$ ) = 2
- $P_{\text{pusat sasaran}} = 0,01$
- $q_{\text{pusat sasaran}} = 1 - p$   
 $= 1 - 0,01$   
 $= 0,99$

Ditanyakan : Berapa Peluang mengenai Pusat sasaran tepat 2 kali ?

**Figure 3.** Results of student answers at the stage of understanding the problem

Next, the researchers conducted task-based interviews with the subjects to explore the problem-understanding stage, which included the subjects' ability to express known information, formulate questions about things they did not understand, and interpret the adequacy of the information in answering problems related to opportunities. The results of these interviews were then transcribed as shown in Table 3 below.

**Table 3.** Transcription of TPMP task-based interview results based on Polya's problem-solving stages

| Data Exposure |                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
|---------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| PTPMP1101     | Can you briefly tell me what you understand from the TPMP task that you have read?                                                                                                                                                                                                                                                                                                                                                                               |
| RS1TPMP1101   | Yes, Sir. What I understand from this problem is that a circular dart target board has a small circular center called the target center. The target center in area 1 consists of two parts: the red inner part and the green outer part. Additionally, 20 juried-shaped areas, numbered 1 to 20, are located around the board.                                                                                                                                   |
| PTPMP1102     | Okay, for you, what information can be known from the TPMP task?                                                                                                                                                                                                                                                                                                                                                                                                 |
| RS1TPMP1102   | Hmmmm... (while rereading TPMP in his own words), it can be identified that this is a binomial distribution task. The task states that the number of events ( $n$ ) is 12 dart throws. The probability of hitting the center of the target is 0.01, the probability of hitting double value 3 is 0.25, double value 2 is 0.15, and the outermost area of the target board is 0.09 (Subject 1 also recorded these known facts, as illustrated in Figure 3 above). |

| Data Exposure |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
|---------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| PTPMP1103     | Do you understand the meaning of the symbol $n$ that Feby wrote earlier?"                                                                                                                                                                                                                                                                                                                                                                                                       |
| RS1TPMP1103   | Yes, Sir, the symbol $n$ shows the number of events or trials, Sir.                                                                                                                                                                                                                                                                                                                                                                                                             |
| PTPMP1104     | Okay, can you explain the meaning of the number of events or experiments?                                                                                                                                                                                                                                                                                                                                                                                                       |
| RS1TPMP1104   | Well, Sir, I think the number of events is the number of events or results that may occur in an experiment. In contrast, a random experiment is an experiment whose results cannot be predicted with certainty.                                                                                                                                                                                                                                                                 |
| PTPMP1105     | Well, in the TPMP task part a, according to Feby, is it the number of events or the number of randomized trials?                                                                                                                                                                                                                                                                                                                                                                |
| RS1TPMP1105   | Okay, Sir, hmmm... In my opinion, part A of the TPMP assignment is about the number of incidents.                                                                                                                                                                                                                                                                                                                                                                               |
| PTPMP1106     | Next, why must the symbol $n$ be used? Why not another symbol?                                                                                                                                                                                                                                                                                                                                                                                                                  |
| RS1TPMP1106   | Yes, Sir. I used the symbol $n$ because it is a common, simple, and widely understood symbol in mathematics and statistics. It is conventionally used in textbooks and literature to represent the number of elements, events, trials, or sample size. Using $n$ helps maintain consistency, clarity, and effective communication. If I were to use unfamiliar symbols, it could lead to confusion or misinterpretation. Therefore, $n$ is the appropriate and standard choice. |
| PTPMP1107     | Okay, from the known things, you also wrote some symbols, such as $X_a$ , $X_b$ , $X_c$ and $X_d$ , now what is $X_a$ ?                                                                                                                                                                                                                                                                                                                                                         |
| RS1TPMP1107   | Yes, Sir. $X_a$ is a random variable                                                                                                                                                                                                                                                                                                                                                                                                                                            |
| PTPMP1108     | What is a random variable?                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
| RS1TPMP1108   | Well, Sir, (while thinking for a moment) I think a random variable is something whose value can change. The value can change depending on the results of different experiments.                                                                                                                                                                                                                                                                                                 |
| PTPMP1109     | In general, what is the random variable in TPMP task ?                                                                                                                                                                                                                                                                                                                                                                                                                          |
| RS1TPMP1109   | Yes, for me, the random variable in TPMP task is the number of successes of each question.                                                                                                                                                                                                                                                                                                                                                                                      |
| PTPMP1110     | Okay, in question part a, why did you write $X_a$ ?                                                                                                                                                                                                                                                                                                                                                                                                                             |
| RS1TPMP1110   | Yes Sir, I wrote $X_a$ because it shows the random variable in task part a.                                                                                                                                                                                                                                                                                                                                                                                                     |
| PTPMP1111     | Where did you get the number 2 that you wrote in the random variable?                                                                                                                                                                                                                                                                                                                                                                                                           |
| RS1TPMP1111   | Yes, Sir, the number 2 that I wrote was obtained from the TPMP task. The excerpt is as follows:                                                                                                                                                                                                                                                                                                                                                                                 |
|               | <b>a. Mengenai pusat sasaran tepat 2 kali</b>                                                                                                                                                                                                                                                                                                                                                                                                                                   |
| PTPMP1112     | Next, does TPMP task contain the sample space?                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| RS1TPMP1112   | Well, Sir. Although the TPMP does not present explicit question sentences, I interpret the instruction 'determine the probability in 12 dart throws' as implying several specific problems. These include: (1) the probability of hitting the center of the target exactly 2 times; (2) the probability of hitting                                                                                                                                                              |

| Data Exposure |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
|---------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|               | the double-valued area 2 at most 4 times; (3) the probability of hitting the double-valued area 3 more than 9 times; and (4) the probability of hitting the outermost area of the target board exactly 3 times.                                                                                                                                                                                                                                                                                                                                               |
| PTPMP1113     | Okay, for this TPMP task, can you describe the sample space?                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| RS1TPMP1113   | Yes, Sir. In the TPMP task, the sample space for question (a) includes all possible outcomes from twelve dart throws at area 1; for (b), twelve throws at area 2; for (c), at area 3; and for (d), at area 4. Area 1 refers to the center of the target, area 2 is the inner ring with a double value of 3, area 3 is within the circle with a double value of 2, and area 4 represents the outermost part of the target board.                                                                                                                               |
| PTPMP1114     | Okay, then according to Feby, what is asked in TPMP,                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |
| RS1TPMP1114   | Well, Sir, after rereading the TPMP task, I noted that it consists of four questions. Part (a) asks about the probability of hitting the target center exactly 2 times. Part (b) concerns the probability of obtaining a double score of two at most 4 times. Part (c) addresses the probability of scoring a double three more than 9 times, and part (d) asks for the probability of hitting the outermost area of the target board exactly 3 times (Subject 1 also noted the things that were asked as in the following excerpt                            |
| PTPMP1115     | Okay, what do you think the event is?                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |
| RS1TPMP1115   | Yes, Sir. In my opinion, an event is the set of results that come from the sample space, because the sample space is the entire set of all possible results that can occur in an experiment. Thus, each event is always a <i>subset</i> of the sample space and is symbolized by $A \subseteq S$                                                                                                                                                                                                                                                              |
| PTPMP1116     | Then, what is the event in TPMP?                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
| RS1TPMP1116   | Well Sir, I think the event in TPMP part (a) is a <i>subset</i> of the sample space such that out of 12 darts thrown exactly 2 times hits area 1; the event in TPMP part (b) is a <i>subset of</i> the sample space such that out of 12 darts thrown at most 4 times hits area 2; the occurrence in TPMP part (c) is a <i>subset of</i> the sample space such that out of 12 darts thrown more than 9 times hits area 3; the occurrence in TPMP part (d) is a <i>subset of</i> the sample space such that out of 12 darts thrown exactly 3 times hits area 4. |
| PTPMP1117     | Why do you think that this is what is asked in TPMP?                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |
| RS1TPMP1117   | Well, Sir, although the TPMP is not presented in the form of explicit questions, I interpret the instruction 'determine the probability in 12 dart throws' as implicitly containing several specific inquiries, such as: (1) the probability of hitting the center exactly 2 times; (2) hitting the double-valued area 2 at most 4 times; (3) hitting the double-valued area 3 more than 9 times; and (4) hitting the outermost area of the target exactly 3 times.                                                                                           |
| PTPMP1118     | To what extent do you think the information provided in TPMP 2 helped you to understand and solve the questions? Can you explain why?                                                                                                                                                                                                                                                                                                                                                                                                                         |
| RS1TPMP1118   | Yes, Sir. In my opinion, the information presented in TPMP helps to understand and solve the questions. This is because the material provided                                                                                                                                                                                                                                                                                                                                                                                                                 |

| Data Exposure |                                                                                                                                                                                                                                                                                               |
|---------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|               | not only identifies sample spaces and events, but the task is also problem-solving in nature.                                                                                                                                                                                                 |
| PTPMP1119     | Is the information above enough?                                                                                                                                                                                                                                                              |
| RS1TPMP1119   | Yes, Sir. I think the information is enough.                                                                                                                                                                                                                                                  |
| PTPMP1120     | Why did you say that it was enough?                                                                                                                                                                                                                                                           |
| RS1TPMP1120   | Yes Sir, because all the variables needed to complete the task are already available. With the information available, I can determine the appropriate answer without having to look for additional information. Therefore, I am confident that I can complete this task well and effectively. |

**b) Research of Answers to Probability Problem-Solving task questions based on Devising a Plan**

Based on the results of the probability problem-solving task, the MPI results on Polya's initial indicator, namely Devising a Plan, can be seen in Figure 4.

Penyelesaian :

$$P(x, n) = C_x^n \cdot p^x \cdot q^{n-x}, \text{ dengan } C_x^n = \frac{n!}{(n-x)! \cdot x!}$$

$$= \frac{n!}{(n-x)! \cdot x!} p^x \cdot q^{n-x}$$

**Figure 4.** Results of student answers at the stage of Devising a Plan

After that, the researchers conducted a task-based interview with the subject to explore the stages of developing a solution plan, which included the subjects' ability to develop a solution plan for a problem arising from an event using symbols, develop a procedure for solving a problem arising from an event, and select the most appropriate strategy for solving a problem arising from an event. The results of these interviews were then transcribed as shown in Table 4 below.

**Table 4.** Transcription of TPMP task-based interview results based on Polya's problem-solving Devising a Plan stage

**Table 4.** Transcription of TPMP task-based interview results based on Polya's problem-solving Devising a Plan stage

| Data Exposure |                                                                                                                                                                                                                                                                                                                               |
|---------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| PTPMP1201     | Based on the information and your understanding of the problem, try to explain the plan that you will use to solve the problem!                                                                                                                                                                                               |
| RS1TPMP1201   | Yes, Sir. My plan is that first, I will re-identify what is known from the problem. Second, I will re-identify what is asked from the problem, and third, I will identify the symbols needed to choose the right formula in determining the probability of hitting the target center exactly 2 times in the TPMP task part a. |

| Data Exposure |                                                                                                                                                                                                                                                                                                                                                                                                 |
|---------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| PTPMP1202     | What is your objective in re-identifying the known items from TPMP task?                                                                                                                                                                                                                                                                                                                        |
| RS1TPMP1202   | Yes, Sir. My purpose in re-identifying the knowns was to better understand and organize the information in the task. By listing the given data, I could more easily determine what was important, input values into the correct formula, avoid errors, and solve the problem more systematically and efficiently.                                                                               |
| PTPMP1203     | Then, what is your purpose in re-identifying the things asked in the TPMP task?                                                                                                                                                                                                                                                                                                                 |
| RS1TPMP1203   | Yes, Sir. By recalling the things that are asked, you can understand what is expected from the task. Besides, the things that are asked are the core of the question.                                                                                                                                                                                                                           |
| PTPMP1204     | How do you distinguish a randomized experiment from a non-randomized experiment?                                                                                                                                                                                                                                                                                                                |
| RS1TPMP1204   | For a non-randomized experiment, the result can be determined beforehand, or we have full control over the result. Whereas a random experiment is uncertain, for example, like throwing a dice, choosing a card from a pile, or drawing a name. We know the possible outcomes, but we can't determine the exact outcome beforehand.                                                             |
| PTPMP1205     | Next, how do you determine the possible outcomes?                                                                                                                                                                                                                                                                                                                                               |
| RS1TPMP1205   | Well, Sir. I started by identifying the possible outcomes of one throw first. For example, if we only focus on whether the arrow hits the center or not, then the results of one throw can be two: "hit the center" (I write "P") or "did not hit the center" (I write "T").                                                                                                                    |
| PTPMP1206     | Next, how do you assign or determine the sample space for this problem?                                                                                                                                                                                                                                                                                                                         |
| RS1TPMP1206   | <p>Since each throw can hit the center of the target or not, each throw has two possibilities: "hit" or "not hit". Therefore, the sample space is all the combinations of 12 throws where each element can have these two possibilities.</p> <p>Mathematically, the sample space consists of <math>2^{12}</math> possible outcomes, since each toss has 2 outcomes and there are 12 tosses.</p> |
| PTPMP1207     | How do you relate this sample space to the event "hitting the center of the target exactly twice"?                                                                                                                                                                                                                                                                                              |
| RS1TPMP1207   | Okay, Sir. Of all these possibilities, the event "exactly 2 hits on the center" is a subset of the sample space, i.e., all outcomes in which there are exactly 2 throws that hit the center, and the rest do not. The number of outcomes for that event can be calculated by combination, i.e., $C_2^{12}$ means choosing 2 throws that hit the center out of 12 throws.                        |
| PTPMP1208     | Did you find any problems when setting the sample space in this problem?                                                                                                                                                                                                                                                                                                                        |
| RS1TPMP1208   | Yes, Sir. Sometimes, it is difficult to imagine all possible results because of the many combinations, especially if the number of throws is large like this. So, I focused on the basic principle that the sample space should include all                                                                                                                                                     |

| Data Exposure |                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
|---------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|               | possibilities, and used the combination formula to calculate certain subsets.                                                                                                                                                                                                                                                                                                                                                                               |
| PTPMP1209     | How did you define the event in the context of the problem?                                                                                                                                                                                                                                                                                                                                                                                                 |
| RS1TPMP1209   | I defined the event as "the arrow hits the center of the bullseye exactly 2 times". Mathematically, I express the event as $x=2$ , where $x$ is a random variable expressing the number of successes (arrows hitting the center of the target). Then, I express $x \sim B(n=12, p)$ where $n$ is the number of trials and $p$ is the probability of success in one trial.                                                                                   |
| PTPMP1210     | Now we will discuss how Feby developed the problem-solving procedure for the probability problem. The problem involves throwing a dart 12 times and hitting the center exactly 2 times. What type of experiment do you think we are dealing with?                                                                                                                                                                                                           |
| RS1TPMP1210   | Yes, Sir. I identify this as a repeated Bernoulli experiment. Because in each throw there are only two possibilities, namely <i>hitting the center</i> or <i>not hitting the center</i> , and it is done 12 times.                                                                                                                                                                                                                                          |
| PTPMP1211     | According to you, what are the characteristics of a Bernoulli experiment like this?                                                                                                                                                                                                                                                                                                                                                                         |
| RS1TPMP1211   | Well, Sir. The main characteristic is that there are two mutually exclusive results in each experiment. The result is only success or failure. And each trial is conducted independently, meaning that one throw does not affect the next. Here, we can consider "hitting the center" as success.                                                                                                                                                           |
| PTPMP1212     | Very good. Now that you recognize the type of experiment, how do you choose a strategy to solve this problem?                                                                                                                                                                                                                                                                                                                                               |
| RS1TPMP1212   | Well, Sir. I chose a strategy using the binomial distribution, because this is the appropriate probability model for the case of a repeated Bernoulli experiment.                                                                                                                                                                                                                                                                                           |
| PTPMP1213     | Okay, what is the formula for the binomial distribution?                                                                                                                                                                                                                                                                                                                                                                                                    |
| RS1TPMP1213   | Hmm, yes, Sir. The Binomial distribution formula is as follows (Subject 1 listed the binomial distribution formula in Figure 4 above). But to determine the value of $C_x^n$ , first I used the combination formula as follows (Subject 1 listed the binomial distribution formula in Figure 4 above).                                                                                                                                                      |
| PTPMP1214     | Okay, how do you plan to apply the binomial distribution formula in solving some of these problems?                                                                                                                                                                                                                                                                                                                                                         |
| RS1TPMP1214   | Well, Sir. To solve the problem, I plan to apply the binomial distribution formula in a structured manner. First, I will ensure the problem meets the binomial criteria: repeated independent trials, two possible outcomes, and a fixed number of trials. Then, I will set the parameters: $n=12$ , $p=0.01$ , and $x=2$ . After that, I will use the binomial formula to calculate the probability of obtaining exactly two successes from twelve trials. |
| PTPMP1215     | Where did you get these formulas from, Feby?                                                                                                                                                                                                                                                                                                                                                                                                                |

| Data Exposure |                                                                                                                                                                                                                                                                                                                                                                                                             |
|---------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| RS1TPMP1215   | Yes, Sir. I learned these formulas during my college studies through lectures, assignments, and discussions with lecturers and peers. In addition, I deepened my understanding by reading various references, such as textbooks, modules, and other learning media. This process has strengthened my mastery and confidence in applying the formulas to solve problems.                                     |
| PTPMP1216     | According to you, are there any other formulas to be used in solving the problem?                                                                                                                                                                                                                                                                                                                           |
| RS1TPMP1216   | Yes, Sir. I believe the formulas I have selected and studied are sufficient to solve the given problems accurately. They are relevant and adequate to answer all the questions without needing to add unfamiliar or unrelated formulas. This allows me to focus on the methods I understand well, ensuring an effective and efficient solution process.                                                     |
| PTPMP1217     | What is the reason why you used that problem-solving plan?                                                                                                                                                                                                                                                                                                                                                  |
| RS1TPMP1217   | Yes, Sir. I used that problem-solving plan because I believe the steps are appropriate and effective. Writing down known information helps me organize key data, identifying what is asked keeps me focused, and determining symbols and formulas allows for a systematic strategy. These steps helped me work more accurately, efficiently, and reach the correct solution.                                |
| PTPMP1218     | According to you, is there another plan that you are still thinking about?                                                                                                                                                                                                                                                                                                                                  |
| RS1TPMP1218   | No, Sir. In my opinion, the plan I created already includes all the essential steps to solve the problem accurately. It covers the process from understanding the information, identifying what is asked, to selecting appropriate symbols and formulas. I believe adding another plan would only complicate the process, so I chose to focus on the existing plan for more effective and efficient results |

***c) Research of Answers to Probability Problem-Solving task questions based on Carrying Out the Plan***

Based on the results of the probability problem-solving task, the MPI results on Polya's initial indicator, namely Carrying Out the Plan, can be seen in Figure 5 below. In addition, the subject's answers were coded using several steps, namely PMI(a)01, PMI(a)02, PMI(a)03, PMI(a)04, and PMI(a)05, which represent the stages in implementing a structured problem-solving plan. Each code reflects a specific aspect of the thinking process and problem-solving strategy used by the subject in addressing probability problems. The answers can be seen in the following figure.

a).

Diketahui : • Banyaknya kesadisan ( $n$ ) = 12

• Variabel acak ( $x$ ) = 2

•  $P$  pusat sasaran = 0,01

•  $q$  pusat sasaran =  $1 - p$   
 $= 1 - 0,01$   
 $= 0,99$

Ditanyakan : Berapa Peluang mengenai Pusat sasaran tepat 2 kali ?

Penyelesaian :

$$P(x, n) = C_x^n \cdot p^x \cdot q^{n-x}, \text{ dengan } C_x^n = \frac{n!}{(n-x)! x!}$$

$$= \frac{n!}{(n-x)! x!} p^x \cdot q^{n-x}$$

$$= \frac{12!}{(12-2)! 2!} (0,01)^2 \cdot (0,99)^{12-2} \quad \rightarrow \quad \text{PMI(a)01}$$

$$= \frac{12!}{10! 2!} (0,0001)^2 \cdot (0,9044) \quad \rightarrow \quad \text{PMI(a)02}$$

$$= \frac{12 \times 11 \times 10!}{10! 2!} (0,00009044) \quad \rightarrow \quad \text{PMI(a)03}$$

$$= 66 (0,00009044) \quad \rightarrow \quad \text{PMI(a)04}$$

$$= 0,00596904 \quad \rightarrow \quad \text{PMI(a)05}$$

Jadi Peluang mengenai Pusat sasaran tepat 2 kali adalah 0,00596904

**Figure 5.** Results of student answers at the stage of Carrying Out the Plan

Then, the researcher conducted task-based interviews with the subjects to explore the stages of implementing the solution plan, which included the subjects' ability to formulate procedures for solving problems arising from an event, implementing the chosen strategy in determining the probability of an event, and drawing the right conclusions to obtain answers in calculating the probability of an event. The results of these interviews were then transcribed as shown in Table 5 below.

**Table 5.** Transcription of TPMP task-based interview results based on Polya's problem-solving Carrying Out the Plan stage

| Data Exposure |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
|---------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| PTPMP01301    | After preparing the plan, you then began working on TPMP task part a (when the Subject was working. The researcher observed carefully and the Subject was given sufficient time). (After 15 minutes had passed) Has the TPMP part a task been completed or not?                                                                                                                                                                                                                                                                                                               |
| RS1TPMP01301  | Yes, Sir (the subject's answers to the probability questions in part a can be seen in Figure 5 above).                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| PTPMP01302    | According to you, what is the main consideration in choosing the binomial distribution formula after completing the TPMP task?                                                                                                                                                                                                                                                                                                                                                                                                                                                |
| RS1TPMP01302  | Yes, Sir. My main consideration in choosing the binomial distribution formula is the suitability of the problem characteristics with the binomial distribution requirements: First, I made sure that the problem had a fixed number of trials and that each trial resulted in only two possibilities, namely success or failure; Second, the probability of success in each trial must be fixed or constant, not changing throughout the series of trials; Third, the trials conducted must be independent, meaning that the results of one trial do not affect other trials. |
| PTPMP01303    | Answer. When working on the probability task in part a, you included the completion step <i>PM1(a)01</i> . My question is, can you explain where you got <i>PM1(a)01</i> from?                                                                                                                                                                                                                                                                                                                                                                                                |
| RS1TPMP01303  | Yes, Sir. I included the <i>PM1(a)01</i> solution step from the first substitution result is the combination formula Sir, where the combination formula is as follows, $C_x^n = \frac{n!}{(n-x)! \cdot x!}$ , then I applied the binomial distribution formula and substituted each value of $n$ and $x$ , so that $C_2^{12} = \frac{12!}{(12-2)! \cdot 2!} \cdot (0,01)^2 \cdot (0,99)^{12-2}$ , where the value of 0.01 is the result of substitution from $p^x$ , and the value of 0.99 is the result of substitution from $q^x$ .                                         |
| PTPMP01304    | Next, can you explain where you got the exponent value for 2 and the exponent (12 - 2)?                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
| RS1TPMP01304  | Yes, Sir. these values are obtained from the random variable $x=2$ which shows the number of successes in an experiment and $n-x$ , from the difference between the total number of trials and the number of successes $12-2 = 10$ , which shows the number of failures. In this case, $n = 12$ indicates the number of trials performed, while $x = 2$ indicates the desired number of successes.                                                                                                                                                                            |
| PTPMP01305    | Okay, now for the completion of step <i>PM1 (a)02</i> , where is it obtained from?                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
| RS1TPMP03205  | Yes, Sir. Hmmmm, for the results of step <i>PM1 (a)02</i> I obtained from the results of the multiplication of the value of $p^x = (0,001)^2$ , so I multiplied 0.01 by 0.01 and got the answer 0.0001 while the result of 0.9044 I obtained from the results of the multiplication of the value of $q^x (0,9044)^{12-2}$ , so I also multiplied between 0.99 and 0.99 10 times.                                                                                                                                                                                              |
| PTPMP01306    | Then, 10 times is obtained from where?                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |

| Data Exposure |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
|---------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| RS1TPMP01306  | Yes, Sir, 10 times is obtained from the difference of the exponent of $q$ (the chance of failure) then multiplied by the value of the principal number which is 0.99.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |
| PTPMP01307    | Okay. I see that the result of the combination substitution is still the same as in stage $PM1(a)01$ , why?                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
| RS1TPMP01307  | Yes, Sir. I haven't explained the value of the combination because I am still focused on solving $p^x = (0,001)^2$ and $q^x (0,9044)^{12-2}$ , this is because it makes it easier for me in the calculation process.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
| PTPMP01308    | Next, for the answer step $PM1(a)03$ , where did Feby get it from?                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
| RS1TPMP01308  | Yes Sir, for the result of step $PM1(a)03$ I got it from the result of substituting the combination of the previous binomial distribution formula, namely $C_2^{12} = \frac{12!}{(12-2)! \cdot 2!}$ , then I replaced the number 12 with the number 6, then crossed out 10! and the divisor was crossed out both 10! or 2! So that we get $\frac{12 \cdot 11 \cdot 10!}{(10)! \cdot 2!} \cdot (0,00009044)$                                                                                                                                                                                                                                                                                                                                    |
| PTPMP01309    | Okay, my next question is why did Feby cross out 12 and replace it with 6?                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
| RS1TPMP01309  | Yes, Sir. I crossed out 12 and replaced it with 6 because I simplified the fraction, specifically by dividing the numerator of 12 by the value of the factorial of 2, which is 2! Mathematically, 2! (two factorials) is equal to $2 \times 1 = 2$ . Since 12 is completely divisible by 2, I simplified it by: $\frac{12}{2!} = \frac{12}{2 \cdot 1} = 6$ This step aims to simplify further calculations Sir. For example, in the context of combinatorics or permutation assignments, I often need to simplify fractions between numerators and denominators involving factorials. By crossing out the number 12 and replacing it with 6, I have done direct division of the numerator, without changing the overall value of the fraction. |
| PTPMP01310    | Ok, then where did you get the result of solving $PM1(a)03$ ?                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| RS1TPMP01310  | Yes, Sir. The value of 0.00009044 I obtained from the multiplication result between $(0,001) \cdot (0,9044)$ which was calculated previously in the solution step $PM1(a)02$ . More precisely, the value is the result of the continuation of the calculation process using the output of $PM1(a)02$ Sir. In $PM1(a)02$ , a probability value was obtained (the result of a combinatoric calculation), and I then multiplied this value by a certain factor or value according to the task requirements in $PM1(a)03$ .                                                                                                                                                                                                                        |
| PTPMP01311    | Yes, then the result of solving $PM1(a)04$ is obtained from where?                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
| RS1TPMP01311  | Well, Sir, the result of solving $PM1(a)04$ I obtained from the result of solving the combination of $C_2^{12}$ which I multiplied by the result of the multiplication between $p^x = (0,001) \cdot q^x (0,9044)$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
| PTPMP01312    | Okay, then, where did you get the final answer in the solution step $PM1(a)05$ ?                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |

| Data Exposure |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
|---------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| RS1TPMP01312  | Yes, Sir. My final answer in Step <i>PM1(a)05</i> is 0.00596904, which I obtained by multiplying 66 by 0.00009044, or $66 \times 0.00009044 = 0.0059690466$ . The value 0.00009044 comes from Step <i>PM1(a)02</i> , where I multiplied 0.0001 by 0.9044. The number 66 was derived from the previous step, <i>PM1(a)04</i> , as a result of a combination calculation involving the numbers 6 and 11, which represent the number of possible outcomes. Therefore, the final result of 0.00596904 was reached through a series of connected and consistent steps from <i>PM1(a)02</i> to <i>PM1(a)05</i> .                                                                                                                                                                                                                       |
| PTPMP01313    | So, based on the problem-solving steps above, what conclusion did you reach?                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
| RS1TPMP01313  | Yes, Sir. Based on the completion of the task in part a, I concluded that: First, the probability of hitting the target center under the specified conditions is 0.00596904. This value is obtained through a stepwise calculation process, which involves multiplying the base probability and the outcome of a particular possible event according to the data in the task. In addition, I also understand that in this kind of joint probability calculation, it is important to pay attention to factorial simplification steps, as well as to ensure that any numbers used are actually derived from previous calculations, not from mere assumptions. Thus, the result reflects a very small probability of occurrence, indicating that hitting the center of the target is very rare in the context of the given problem. |
| PTPMP01314    | Okay. Can you explain what is meant by a very small probability of occurrence in the context of the calculation?                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
| RS1TPMP0114   | Yes, Sir. A very small probability of occurrence in the context of the calculation means that the chance of an event occurring is almost impossible or very rare. In numerical terms, this probability is usually below 0.2 or 20%. This means that if the probability of an event is very small, the probability of the event occurring is very low and can almost be ignored in practical calculations.                                                                                                                                                                                                                                                                                                                                                                                                                        |
| PTPMP01315    | So, what is the practical calculation?                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
| RS1TPMP01315  | In my opinion, practical calculation means the use of the probability results in real situations or everyday applications, where very small chances are often considered insignificant or can be ignored because the probability is very low.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
| PTPMP01316    | Okay, do you still want to say anything else?                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
| RS1TPMP01316  | Yes, Sir. I would like to state that the answers I have provided were based on my understanding of the material we studied together. I followed each step of the problem-solving process systematically starting from understanding the problem context, identifying the random experiment and its sample space, determining the related events, and calculating the probability using the appropriate method. Throughout the process, I ensured that every number used had a clear basis and that all calculations were carried out carefully. Therefore, I believe the answer I submitted complies with the correct procedures for solving TPMP Task                                                                                                                                                                           |

## 5. Discussion

### ***5.1. Probabilistic Thinking Process of Female Student as Prospective Mathematics Teacher in Probability Problem-Solving at the stage Problem Understanding***

In the problem-understanding stage, the subject demonstrated logical and systematic thinking skills in interpreting the information provided in TPMP. The subject was able to identify the structure of the target board and distinguish each target area, including the centre of the target and the double-value area, which formed the basis for interpreting the context of the opportunities. In line with Burgos et al. (2022), the subject explained that the board had different areas, such as the center of the target, areas with double and triple values, and the outer area, each of which had a certain probability. Research on probability teaching and learning was done to identify recent research trends, existing problems, and topics that need further study, especially related to mathematics knowledge and didactics of prospective mathematics teacher regarding probability concepts (Batanero & Álvarez-Arroyo, 2024). The disclosure of known information was done in a systematic manner, such as stating the number of trials (12 arrow throws), the probability of hitting each area, and symbols such as  $n$  and  $X_a$  used in binomial distribution calculations. The subject demonstrated that these symbols were not merely written as a formality but were understood in the context of probability, for example,  $n$  as the number of random trials and  $X_a$  as the random variable representing the number of successes in an event. Furthermore, the subject logically expresses the things being asked, even though the questions are not written in the form of questions. He is able to interpret the command 'determine the probability...' as an explicit question and break it down into four different events according to the context of the question.

This shows that the subject has the ability to understand the hidden meaning of the question and relate it to the concepts of events and sample space. The explanation that events are subsets of the sample space, along with efforts to describe the sample space for each part of the question, strengthens their conceptual understanding of probability. This is in line with the study by Chernoff (2009) who stated that the sample space plays a fundamental role in understanding probability, as it helps identify all possible outcomes of an experiment. The way students partition the sample space influences their judgment of the likelihood of an event. The subject assessed that the information available in the question was sufficient to solve the problem. The subject did not only mention the information verbally but also linked it to the requirements for solving the problem, such as the variables needed to apply the binomial distribution formula. This reflects the sufficiency of information that supports a comprehensive understanding of the problem and readiness to move on to the planning stage.

Thus, the subject has fully completed the problem understanding stage by demonstrating the ability to identify important information, interpret probabilistic symbols and terms, and relate the problem context to the structure of probability. The clarity in distinguishing between what is known and what is asked, along with the recognition of the adequacy of information, indicates the subject's readiness to proceed to the next stage in mathematical problem-solving. This stage, within Polya's framework, has therefore been adequately addressed. The subject not only identified key elements of the problem but also began to demonstrate relevant probabilistic thinking, particularly in applying the binomial distribution approach to model the situation. These findings align with the study conducted by Supratman, Budayasa, and Rahaju (2025), which showed that subjects from the mathematics education background (MPI) exhibited deeper insights when following Polya's problem-solving stages. During the interviews conducted after a probability test, MPI participants reportedly experienced little difficulty in understanding each phase of Polya's framework. In addition, research by (Tohir et al., 2024) supports these observations by indicating that during "understanding the problem" stage, four types of mathematical thinking were generally well demonstrated. However,

the same study noted that processes such as “generalising” and “making assumptions” were mostly absent low-performing participants. In contrast, the subject in this case showed signs of effective problem analysis, though further observation would be needed to confirm the presence of those higher-level thinking strategies. Sari et al. (2024) stated that the ability to understand mathematical concepts influences probabilistic thinking skills and the types of learning obstacles experienced by students.

### ***5.2. Probabilistic Thinking Process of Female Student as Prospective Mathematics Teacher in Probability Problem-Solving at the stage of Devising a Plan***

When making a plan to solve the problem, the subject showed a logical and structured approach to dealing with the problem. They started by re-identifying the available information and clarifying the questions asked, as a basis for building the right solution strategy. (Taram et al., 2020) stated that the strategy used reflects a strong willingness to keep trying, despite obstacles such as limited time and the perception that the problems were more difficult than those in the previous year. Nevertheless, the subjects demonstrated the ability to develop good probabilistic thinking, along with strong motivation to solve problems systematically. This is evident from their efforts to identify and determine all the members of the sample space  $S$  completely. Sari, D.I. et al. (2017) which states that There were differences in responses between students with high and low mathematical abilities, Students with high mathematical ability were able to list all possible outcomes in one- and two-dimensional sample space tasks, while students with low mathematical ability failed to complete these tasks. This step is followed by selecting relevant symbols, such as  $n$  for the number of trials,  $p$  for the probability of success, and  $x$  for the number of expected successes. The subject is able to recognise the characteristics of the experiment as random, repetitive, and having only two possible outcomes (success or failure), which is then identified as a Bernoulli experiment. This understanding prompts them to choose the binomial distribution model as the appropriate approach for calculating the probability of a specific event. In the solution plan they develop, the subject also sets the conditions for using the binomial distribution and systematically arranges the sequence of formula usage, starting from parameter identification to the application of combinations in calculations. The subject's firm statement that no additional formulas are required demonstrates confidence in the sufficiency of the chosen strategy. Thus, the designed solution plan reflects analytical ability, efficient thinking, and metacognitive awareness in selecting and organising the steps of the solution appropriately.

Thus, at the stage of developing a solution plan, the subject has demonstrated the ability to design problem-solving steps in a sequential and logical manner, taking into account the nature of the problem and selecting the most relevant methods. Clarity in formulating strategies, accuracy in selecting the binomial distribution model, and confidence in using the formulas that have been mastered. This is in line with the results of a study Ozyildirim-Gumus & Ozyildirim (2020) which states that study conducted on 188 eighth-grade students showed a strong correlation between reflective thinking and probabilistic thinking, particularly female students. In the context of problem-solving, two aspects of probabilistic thinking stood out the ability to recognise random similarities and the ability to synthesise results and contributed significantly to the effectiveness of learning.

### ***5.3. Probabilistic Thinking Process of Female Student as Prospective Mathematics Teacher in Probability Problem-Solving at the stage of Carrying Out the Plan***

During the implementation stage of the solution plan, the subject demonstrated the ability to systematically execute the designed procedures to solve the probability problem. The process began by applying the combination formula and binomial distribution, and substituting relevant values such as the number of trials, probability of success, and expected number of successes. The subject explains each step of the calculation in a sequential manner, starting from determining the

combination values to the process of exponentiating the probability of success and failure. The precision in simplifying factorials and utilizing combinatorial principles demonstrates that the subject does not merely execute the formulas mechanically but also understands the underlying logical structure. The final result, a very small probability value, is calculated through the sequential multiplication of the previously determined probability components. Additionally, the subject provides mathematical reasoning behind each calculation decision, such as simplifying numbers and selecting efficient methods. The ability to explain the origin of numbers and draw conclusions from the results obtained indicates that the implementation of the plan is not only technically accurate but also conceptually reflective. Thus, the implementation of the designed strategy not only successfully produces the correct solution but also demonstrates mastery of probabilistic thinking processes in the context of the binomial distribution.

Thus, in the implementation stage of Carrying out the Plan, the subject not only demonstrated the ability to correctly execute technical steps but also showed a strong conceptual understanding of the probability calculation process. Accuracy in value substitution, precision in formula application, and the ability to explain the rationale behind each step indicate that the subject effectively and efficiently implemented the pre-designed strategy in accordance with the characteristics of the problem at hand. Moreover, Taamneh et al. (2024) stated that challenges begin to emerge in the advanced stages of the problem-solving process, particularly in the planning and implementation phases. At this stage, students still need guidance in designing appropriate strategies and implementing them effectively to solve the problems given. Additionally, two studies Geiger et al. (2022) and Kusuma et al. (2024) stated that this shows that strategic thinking and execution skills in the context of mathematical problem-solving still need to be developed more systematically through targeted learning. Therefore, in the step of implementing the solution plan, prospective mathematics teacher need to be equipped with probabilistic thinking skills to improve accuracy in applying strategies, the ability to analyse situations involving uncertainty, and accuracy in drawing conclusions based on available data and opportunities. This training should be an integral part of teacher education, particularly through curriculum design that emphasises the development of higher-order thinking skills (HOTS) and contextual problem-solving (Li et al., 2024); (Jailani et al., 2023) and (Lubis et al., 2023). As a result, graduates of mathematics education programmes will not only master concepts but also be able to guide students in understanding and effectively applying probability concepts in the classroom. Hokor (2020) and Ratri Rahayu & Kartono (2021) stated that teacher design activities that target probabilistic misconceptions, using a constructivist approach, asking critical questions, and reflecting on and evaluating the learning process. When teaching is linked to real-life contexts and teacher act as facilitators, the probabilistic reasoning abilities of students and prospective teacher can develop optimally. Greer (2001) states that probabilistic thinking encompasses three key aspects: (a) the role of intuition in mathematical and scientific thinking, highlighting that intuition serves as a fundamental basis for understanding uncertainty; (b) the development of probabilistic thinking, emphasizing that this ability evolves gradually from an early age and is shaped by learning experiences; and (c) the influence of instruction on its development, where effective teaching strategies can either reinforce or correct students' understanding of probability concepts. These three aspects form a crucial foundation for designing effective instruction aimed at fostering both conceptual and intuitive probabilistic reasoning. Moreno and Cardenoso (2014) stated that changes in the curriculum and instructional approaches are needed so that the concept of probability can be explored more deeply, by approaching probability theory from multiple perspectives in line with current educational and conceptual theoretical frameworks.

## 6. Conclusion

The findings of this study reveal that female students who are prospective mathematics teacher exhibit a comprehensive and structured probabilistic thinking process in solving probability problems

involving the binomial distribution. Guided by Polya's problem-solving framework consisting of the stages of understanding the problem, devising a plan, and carrying out the plan the subject demonstrated critical, reflective, and metacognitive cognitive abilities that are essential not only for mastering mathematical content but also for developing pedagogical competence in teaching probability. In the understanding the problem stage, the subject was able to logically and systematically interpret the problem context, identifying key information such as the number of trials, characteristics of the target areas, and symbols related to binomial distribution. She also demonstrated the ability to distinguish between known and unknown components, and to connect the given information with fundamental concepts in probability theory, including the sample space, random variables, and events. This indicates that the subject had internalized abstract probabilistic concepts and was capable of applying them meaningfully in context-rich situations an ability that is crucial for future educators who must guide students through similar reasoning processes. At the devising a plan stage, the subject demonstrated strategic thinking by selecting the appropriate mathematical model namely, the binomial distribution based on her understanding of the problem as a series of independent Bernoulli trials. She was able to accurately identify and define relevant parameters (such as the number of trials and the probability of success), choose the correct formulas, and organize the steps of the solution in a logical and sequential manner. Her approach reflected not only strong analytical reasoning but also a high level of metacognitive awareness in planning and anticipating the demands of the problem. These competencies are foundational for effective mathematics instruction, where clear planning and the ability to explain problem-solving strategies are essential. During the carrying out the plan stage, the subject implemented her strategy with precision and demonstrated a deep conceptual understanding of the probabilistic procedures involved. She substituted values accurately into the binomial formula, applied combinatorial reasoning correctly, and explained each computational step with clarity and justification. Her reasoning extended beyond mechanical application, as she articulated the rationale behind her choices, simplified expressions appropriately, and interpreted the meaning of the final result. This blend of procedural fluency and conceptual depth illustrates a mature level of probabilistic thinking, which is especially important when teaching complex or abstract mathematical ideas.

Overall, the subjects' performance in the three stages of Polya's problem-solving process reflects the consistent, systematic, and integrated application of probabilistic thinking. The subjects demonstrated the ability to analyze contextual information, identify important elements of the problem, and select and apply appropriate mathematical models. The thinking process demonstrated includes higher-order cognitive activities, such as formulating solution strategies, estimating the likelihood of outcomes, and evaluating the validity and relevance of solutions based on the problem context. In the solution implementation stage, the subject also demonstrates good technical accuracy and conceptual depth, reflecting the integration of declarative, procedural, and strategic knowledge. The ability to process information, make inferences, and connect probability concepts to real-world situations indicates that the subject does not merely rely on formal calculations but also involves reflective and metacognitive dimensions in their thinking process. This is an important indicator of mature mathematical competence as well as pedagogical readiness to teach probability concepts effectively. The implications of these findings suggest that mathematics teacher education programmes need to intentionally instil and develop higher-order thinking skills, deep conceptual understanding, and reflective problem-solving practices. Particularly in the domain of probability, probabilistic thinking which encompasses intuitive understanding, formal reasoning, and the ability to interpret and model uncertainty in real life must be developed through curriculum design that is oriented towards conceptual understanding, problem-based learning strategies, and contextual and authentic learning experiences. Through this approach, prospective mathematics teacher will be better equipped to address student misconceptions, promote meaningful learning, and build a strong theoretical and practical foundation in probabilistic literacy that can be applied in everyday life.

## 7. Suggestion

Based on the results of research on probabilistic thinking process in solving probability problems in prospective mathematics teacher who have field-independent cognitive style based on the three stages of Polya, the following suggestions can be given: 1) For prospective mathematics teacher: It is expected to improve the ability of probabilistic thinking by paying more attention to the stages of understanding the problem, planning a solution strategy, and implementing the plan systematically. Exercises that focus on these three stages need to be increased so that prospective teacher can be effective facilitators in learning opportunities at school; 2) For lecturers or teacher of mathematics education courses: There is a need to design learning that integrates Polya's approach with an emphasis on developing probabilistic thinking. Learning strategies should also consider the cognitive characteristics of students, such as field-independent style, so that learning is more adaptive and in-depth; 3) For future researchers: It is recommended to explore the probabilistic thinking process based on all four stages of Polya and compare it with field-dependent cognitive style to obtain a more comprehensive picture of the differences in thinking strategies in solving probability problems, and 4) For curriculum development: it is necessary to consider explicit integration of strengthening probabilistic thinking skills in the prospective teacher education curriculum, including the provision of context-based problems that encourage the application of Polya in solving probability problems.

## Declarations

**Author Contributions.** Supratman: Conceptualization, Formal analysis, Investigation, Methodology, Project administration, Resources, Visualization, Writing - original draft, Writing - review & editing  
I Ketut Budayasa: Supervision, Validation Endah Budi Rahaju: Supervision, Validation

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### About the Contributor(s)

**Supratman**, is a Doctoral Student at the Doctoral Program of Mathematics Education, Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Negeri Surabaya, and a lecturer in the Mathematics Education study program, Faculty of Teacher Training and Education, Universitas Sembilanbelas November Kolaka, Indonesia. He focuses on mathematics education, Calculus, Trigonometry, Mathematics statistic, and Mathematics teaching methods. In addition, one of his studies has been published in an international journal indexed in Scopus. He can be contacted at email: [supratman21034@mhs.unesa.ac.id](mailto:supratman21034@mhs.unesa.ac.id) / [supratmanmathusnkolaka@gmail.com](mailto:supratmanmathusnkolaka@gmail.com)  
ORCID: <https://orcid.org/0009-0008-4356-0792>

**I Ketut Budayasa**, is a Professor at the Doctoral Program of Mathematics Education, Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Negeri Surabaya. I ketut Budayasa received his PhD program from Curtin University of Technology, Australia. He focuses on discrete mathematics, graph theory, Analysis of Mathematical Education Research, Methodology of Quantitative Research in Mathematics Education, Statistics and Probability, besides that his research has been published in International Journal Indexing in Scopus. He also actively publishes the research in books including discrete mathematics and graph theory and its applications. In addition, his research has been published in various reputable international journals indexed in Scopus, demonstrating his significant scientific contributions in his field. He can be reached at email: [ketutbudayasa@unesa.ac.id](mailto:ketutbudayasa@unesa.ac.id)  
ORCID: <https://orcid.org/0000-0002-5066-859X>

**Endah Budi Rahaju**, is Associate Professor at the Doctoral Program of Mathematics Education, Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Negeri Surabaya. Endah Budi Rahaju received his PhD program from the Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Negeri Surabaya, Indonesia. He focuses on Mathematical Thinking, Ethnomathematics, Mathematical Literacy, Knowledge and Praxis of in-service and pre-service Teacher, and Seminar of Mathematics Education, besides that his research has been published in International Journal Indexing in Scopus. He can be reached at email: [endahrahaju@unesa.ac.id](mailto:endahrahaju@unesa.ac.id)  
ORCID: <https://orcid.org/0000-0003-0807-519X>

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