

Research Article

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Engineering students' mathematical understanding based on the quality of mathematical connections activated to solve tasks about function's graph and its derivative

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Abstract

Background/purpose. One of the current problems facing mathematics students, graduates in mathematics, and engineering is the disconnection between the meanings, symbolic representations, and graphics of derivatives when solving problems, which hinders their understanding. This article analyzes engineering students' understanding activated by connections made to solve tasks on derivatives in a graphical context. To do so, networking between the Extended Theory of Connections and the Onto-semiotic Approach will be used.

Materials/methods. The methodology was qualitative and exploratory. A questionnaire was designed with three tasks on the meaning of the derivative and the graphs of the function f and the derivative f' . This questionnaire was administered in the context of participant observation to nineteen engineering students who volunteered. The collected and video-recorded data were analyzed using the theoretical tool from an onto-semiotic view.

Results. The results show that students who have a level 2 understanding of the derivative (graphically) because they sketch the graph of f' given the graph of f and sketch the graph of f from the graph of f' , establishing mathematical connections of implication, different representations, meaning, procedural, part-whole and the main connection of reversibility that allowed them to make the two graphs. Students who have level 1 understanding establish consistent connections, but do not argue or have details to correct in the graphs.]

Conclusion. [Other students have level 0 understanding because they did not make the graphs of f and f' because they did not activate mathematical connections but personal connections, assuming that the graph of the derivative is a reflection of the graph of the original function, they do not locate the relative extremes, inflection points, monotony due to poor conceptual understanding.

1. Introduction

The multiplicity of problematic phenomena related to the teaching and learning of mathematics has led to the emergence and consolidation of various theoretical frameworks in the field of Mathematics Education. For example, solving and posing mathematical problems are important lines of research because every person (student or teacher) must solve problems competently to demonstrate a type of understanding of mathematical objects (Cai & Rott, 2024; Galiç et al., 2025; Noviyanti et al., 2025; Rodríguez-Nieto et al., 2023a; Santos-Trigo, 2024). Also, problem-solving is fundamental in different curricular organizations because it is important for students to solve challenging problems in daily life (MEN, 2006; NCTM, 2000; Sudirman et al., 2025).

Other theoretical approaches, on the one hand, are concerned with analyzing people's mathematical activity in terms of practices, processes, objects, and semiotic functions (SFs), as is the case of the Extended Theory of Connections (ETC) and the Onto-semiotic Approach (OSA), in order to detail the most relevant aspects of sequenced mathematical practices that invite us to recognize how a mathematical problem is solved adequately or not (Font et al., 2016; Godino et al., 2019; Rodríguez-Nieto et al., 2023a). On the other hand, there is the APOS theory, which points out that a student understands a mathematical concept when they have the ability to transit through mental constructions such as action, process, object, and scheme through six mechanisms: internalization, encapsulation, decapsulation, reversal, coordination, and thematization (Arnon et al., 2014).

Specifically, Kenedi et al. (2019) argue that one of the fundamental competencies that students must achieve in their training process is the ability to establish mathematical connections, since by identifying the relationships between concepts, they can more easily understand mathematics and enhance the development of their skills in this area. For example, numerous works have been carried out from the ETC at different educational levels (Bahar et al., 2023; Businskas, 2008; Campo-Meneses & García-García, 2023; Dişbudak-Kuru & Işıkşal-Bostan, 2023; García-García & Dolores-Flores, 2021; Rafiepour & Faramarzpour, 2023; Rodríguez-Nieto et al., 2022a; 2023b), showing that connections are important and indispensable for solving mathematical problems where they connect and allow students to understand mathematical concepts in a functional way applied in everyday life, strengthening conceptual and procedural knowledge (Jäder et al., 2019).

In this context, it is recognized that mathematical connections constitute a connecting link between learning challenges, problem-solving, and a deep understanding of mathematical concepts. The need to promote meaningful connections between representations, processes, and meanings has motivated the development of various theoretical frameworks that allow for a more detailed analysis of how mathematical knowledge is constructed at different educational levels. The gap in the literature lies in the absence of studies that articulate the analysis of the quality of mathematical connections with the graphical understanding of the derivative concept in engineering students, especially through theoretical frameworks such as ETC and OSA. This research seeks to contribute precisely to this gap by offering an in-depth analysis of how these connections impact the conceptual and representational understanding of differential calculus. Furthermore, these graphical aspects of the derivative are problematic for students due to their complexity, and it is important to analyze them to improve understanding with consistent mathematical knowledge. Therefore, the following presents a literature review that systematizes the most relevant contributions on mathematical connections and derivatives.

2. Literature Review

It is worth mentioning that mathematical connections have been explored in different research projects focused on fostering the understanding of mathematical concepts and strengthening students' communication, argumentation, and ensuring that their procedures are rich in different representations (Berry & Nyman, 2003; Pino-Fan et al., 2017; Rodríguez-Nieto & Font, 2025; Son,

2022). Specifically, Businskas (2008) recognized the mathematical connections made by future teachers in solving problems about the quadratic equation; other studies have focused on connections and geometry (Eli et al., 2013; Olivero-Acuña et al., 2025), and in the case of Caviedes et al. (2024), who report that future teachers use numerical procedures and formulas more, ignoring geometric and intuitive procedures that would facilitate the task of quantifying the area because they do not establish connections between the different manifestations of the area. Regarding functions in mathematics, Hatisaru (2023) investigated the teaching of functions from an articulated approach of connections and the specialized knowledge of the teacher, addressing different representations in the Cartesian plane, sagittal diagrams, ordered pairs, and the difference between relation and function, among others.

Along these lines, Campo-Meneses et al. (2023) have investigated the mathematical connections on exponential and logarithmic functions activated by high school students and in-service teachers, who carry out processes of reversibility, representation, and significance in problem solving. It is worth noting that the broad research agenda reveals that mathematical connections are also important in the teaching and learning of Calculus and it has been found that students and some mathematics teachers have difficulties solving problems about derivatives and integrals because they do not connect different representations (graphical, verbal, numerical, symbolic, algebraic...) and maintain incomplete partial meanings that do not allow them to undertake a good signification process (Chandra et al., 2025; García-García & Dolores-Flores, 2021; Kula-Ünver, 2020; Pino-Fan et al., 2017; Rodríguez-Nieto et al., 2022a; 2022b; Rodríguez-Nieto et al., 2023a; Rodríguez-Nieto & Font, 2025; Sahin et al., 2015; Yavuz-Mumcu, 2018).

Likewise, in other research that does not necessarily deal with connections but with the understanding and graph of the derivative, they report that undergraduate mathematics students have difficulties connecting the graphical representation of the function and that of its derivative (Berry & Nyman, 2003; Borji et al., 2018; Sánchez-Matamoros et al., 2015; Ubuz, 2007). In fact, Ubuz (2007) considered that it was difficult for students to use graphical information to make sense of symbolic representations because they used excessively mechanized procedures. Furthermore, engineering students who also take Calculus courses do not understand the derivative because they have difficulties relating both the growth and decrease of f and the sign of f' and f'' , and they work more with graphical information and the specific analytical meaning of the derivative (Fuentealba et al., 2018; 2019).

Nemirovsky and Rubin (1992) recognized that most students draw the graph of f' in a similar way to the graph of f because they are unfamiliar with the properties of functions and their relationship with derivatives. Ferrini-Mundy and Graham (1994) argue that students want to find the equation of a given function only graphically but lack conceptual knowledge associated with the first and second derivative criteria. Similarly, it was identified by García-García and Dolores-Flores (2021) who showed that pre-university students need the algebraic expression of the function to make a graphical representation of itself and its derivative, otherwise they do not graph. In this sense, it is considered a key aspect to investigate mathematical connections based on the properties of graphs, especially how to move from the graphical representation of a function g to the graph of the derivative g' (García-García & Dolores-Flores, 2021). Therefore, Ikram et al. (2020) and Rodríguez-Nieto et al. (2023a) suggest that further research be carried out on this topic due to the diversity of students' knowledge with rigorous tasks.

After reading and interpreting the results of different works reported in the literature on connections and the derivative, it is noted that difficulties are present in understanding the derivative in the teaching and learning processes in mathematics courses for teachers, engineers, and related careers (Lumbantoruan & Manalu, 2024; Moru, 2020; Muñoz-Pinto et al., 2025; Rodríguez-Nieto et al., 2023a; Sahin et al., 2015). Explicitly, "both high school and undergraduate students are not

making sense of the rate of change and are not aware of the relationship between the derivative and the rate of change concepts” (Sahin et al., 2015, p. 184). The same is reported by Pino-Fan et al. (2017), who applied a classic application problem (about the speed at which a ball travels and reaches a given height in a given time) to future mathematics teachers and were unable to solve it because they had difficulty using the derivative as an instantaneous rate of change. Understanding these Calculus topics represents a challenge, since many students lack a solid background in functions, limits, continuity, and the interpretation of graphical representations, which highlights the urgent need to rethink the didactic strategies used in teaching these abstract concepts, and to emphasize graphical representations of functions and derivatives (Munyaruhengeri et al., 2023).

Due to these types of problems and especially the ones related to the students' difficulty in graphing f and f' , we propose to answer the research question: How does the quality of the mathematical connections that engineering students make impact their mathematical understanding when solving tasks about the graph of a function and its derivative?

2.1. Extended Theory of Connections (ETC)

From the perspective of networking between the ETC and the OSA, a mathematical connection is understood as the tip of an iceberg made up of a conglomerate of practices, processes, primary objects identified in the mathematical activity of a subject when solving a task, and SFs that relate them (Rodríguez-Nieto et al., 2022a). The theory of connections can be used according to three groups, and depending on the type of problems that are going to be proposed: the first group refers to intra-mathematical connections that are relationships between concepts, theorems, representations, and other mathematical ideas. The second group refers to extra-mathematical connections, which are activated in the resolution of application or non-mathematical problems by transferring the information from the problem to a mathematical model (Dolores-Flores & García-García, 2017). The third group is ethnomathematical connections understood as the relationship between mathematics practiced by people immersed in cultural groups (e.g., who work in carpentry, masonry, or any other trade) and institutional or public mathematics that are in curricular materials (Rodríguez-Nieto, 2021), see Figure 1.

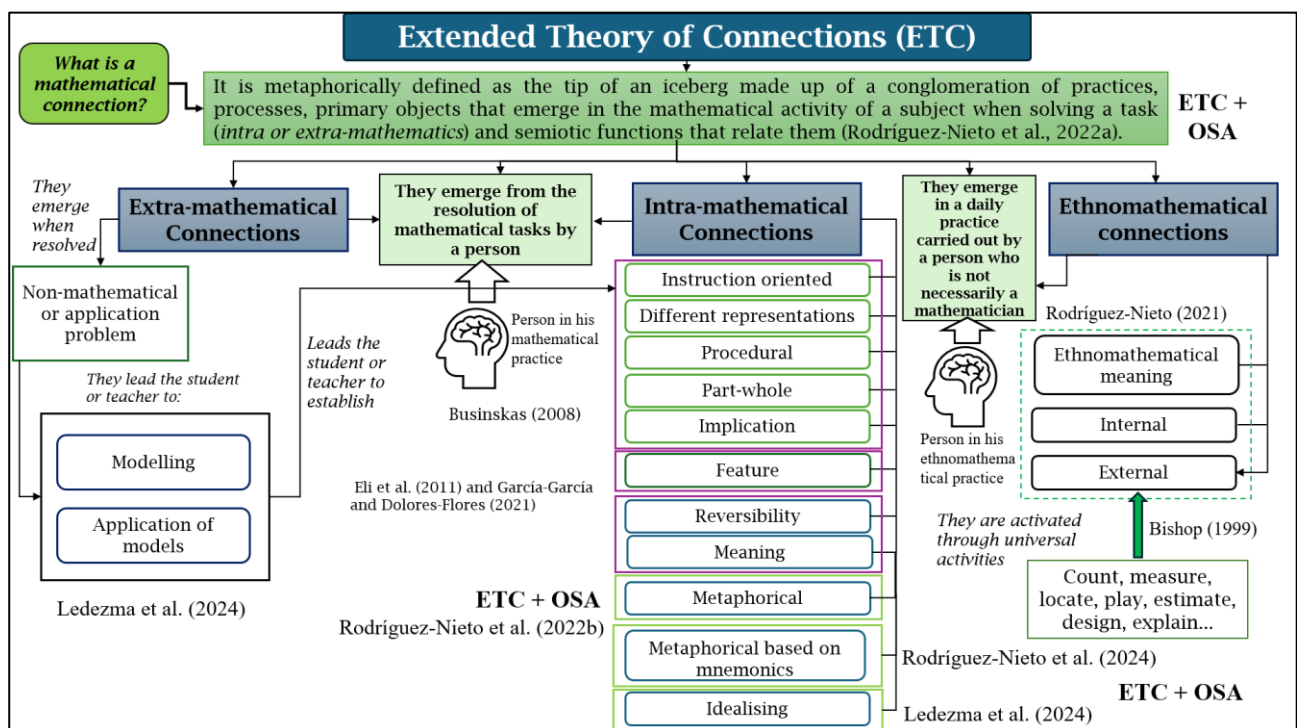


Figure 1. Synthesis of the ETC (Rodríguez-Nieto et al., 2024).

Figure 1 recognizes that connections emerge both from the resolution of mathematical tasks and from daily and cultural practices that are not necessarily academic. Intra-mathematical connections are linked to internal structures of mathematical knowledge (representations, meanings, implications), while extra-mathematical connections arise when applying mathematics to real-life problems, guiding the modeling and use of models. Ethnomathematics, for its part, makes visible the situated knowledge of non-mathematicians who, through universal activities such as counting, measuring, or designing, activate ethnomathematical meanings, internal and external connections of knowledge from a cultural perspective. This framework not only broadens the understanding of mathematical thinking but also integrates cultural, practical, and semiotic dimensions, allowing for a more inclusive, meaningful, and socially contextualized mathematics. The typology of mathematical connections is presented below, making the theory of connections more pragmatic in its application.

1) Instruction-oriented: refers to the understanding of concept C based on two or more previous concepts A and B, required to be understood by a subject. They are manifested in two ways: a) association of a new topic with previous knowledge, and b) mathematical concepts and procedures connected to each other are considered prerequisites or skills that students must master before the development of a new concept (Businkas, 2008). This type of connection arises from a concern for more effective mathematics teaching processes, emphasizing continuity with prior experiences to foster the development of new ideas.

2) Modeling: it is understood as the relationship that a subject establishes between the world of mathematics and the real world (or the daily life of students) and between mathematics and other sciences. Specifically, it can be seen as the relationship established between the mathematical concept and a task in a real context (that occurs or may occur in real life) or an application task in some discipline other than mathematics, in which the subject, starting from the task, builds a mathematical model to solve it. When the subject builds the mathematical model, he uses various knowledge (mathematical or not) by executing multiple actions (algebraic, symbolic, graphic, etc.) to reach an answer consistent with the requirement posed (Campo-Meneses & García-García, 2023).

3) Different representations: these are identified when the subject represents mathematical objects using equivalent and alternate representations. Equivalent representations are transformations within the same representation register, while alternate representations involve a change of register (for example, from graphical to algebraic) (Businkas, 2008). The use of multiple representations is fundamental for a deep understanding of mathematical concepts, as it enables students to establish connections between different ways of interpreting the same object.

4) Procedural: these are identified when a subject uses rules, algorithms, or formulas to solve a mathematical task. This type of connection takes the form: A is a procedure to work with a concept B (Businkas, 2008). In addition, this category includes step-by-step procedures for constructing graphs, making drawings, or applying properties and theorems, as they involve a structured sequence of actions that allow one to operate with specific mathematical concepts.

5) Part-whole: these connections occur when logical relationships are established in the following two ways: 1) The generalization relationship is of the form A and is a generalization of B, and B is a particular case of A. 2) The relationship of inclusion is given when a mathematical concept is contained in another (Businkas, 2008).

6) Implication: they are identified when a concept P leads to another concept Q through a logical relationship ($P \rightarrow Q$) (Businkas, 2008).

7) Feature: it is identified when the person expresses some characteristics of the concepts or describes their properties in terms of other concepts that make them different or similar to the others (Eli et al., 2011).

8) Reversibility: they occur when a subject starts from a concept A to obtain a concept B and reverse the process, starting from B until returning to A (García-García & Dolores-Flores, 2021).

9) Meaning: it is identified when a subject “attributes a meaning to a mathematical concept, that is, what it means for him [...]. It includes those cases in which a student gives a definition that he or she has constructed for these concepts” (García-García & Dolores-Flores, 2021).

10) Metaphorical: understood as the projection of the properties, characteristics, etc., of a known domain based on bodily experiences to structure another less-known or abstract domain (Rodríguez-Nieto et al., 2022b).

11) Metaphorical connection based on mnemonics: this connection is “understood as the relationship established by the subject between a mnemonic rule (often a familiar resource) and a mathematical object, rule, or mathematical procedure to memorize and use strategically more easily” (Rodríguez-Nieto et al., 2024, p. 18). These types of connections are inclusive and recursive where three elements must be considered: a) keywords that are similar to the word (or term) being referred to; b) acronyms it is identified when the first letter of each word is used in a list to construct another word; c) acrostics which consist of constructing a sentence, where the first letter of each constitutes the term studied (Rodríguez-Nieto et al., 2024).

12) Idealising: this connection relates an ostensive to a non-ostensive. Its function is to dematerialize the ostensive and turn it into an ideal mathematical object (e.g., the bottom of a tank, rounded, is considered a circle/circumference) (Ledezma et al., 2024).

Recently, this theory of mathematical connections can be further expanded, and the work of Cantillo-Rudas et al. (2024) stands out, where they report new developments on the neuro-mathematical connections associated with the cognitive part of this theory and the evaluation of the brain areas activated in a person's mathematical activity.

2.2. Onto-semiotic Approach (OSA)

The OSA arose from the need to articulate and improve the theoretical and methodological tools of other specific theoretical frameworks, as well as the concern for the teaching and learning of mathematics. In this approach, mathematical activity (institutional or personal) is described and modeled in terms of practices and the configuration of objects and primary processes activated in these practices (Drijvers et al., 2013). Mathematical practice is “any situation or expression (...) carried out by someone to solve mathematical problems, communicate the solution obtained to others, validate it or generalize it to other contexts and problems” (Godino & Batanero, 1994, p. 334). These practices include objects used in a broad sense to refer to any entity that is involved in mathematical practice and can be identified as a unit (Font et al., 2013). These objects are six: 1) problematic situations, 2) representations, 3) definitions, 4) propositions, 5) procedures, and 6) arguments. These interconnected objects form the configuration of primary objects (Godino et al., 2019).

In mathematical practice, primary objects emerge and they can do so in different ways as the result of the several ways of seeing, speaking, writing, operating, etc., on primary objects, which allows us to talk about personal or institutional primary objects, ostensive or non-ostensive, unitary or systemic, intensive or extensive, and content or expression (Godino et al., 2007). Now, these objects or systems of interrelated heterogeneous objects make up the configuration, which can be institutional (epistemic) or personal (cognitive) (Godino et al., 2019).

The set of primary objects emerges in mathematical activity through the activation of primary mathematical processes (communication, problem posing, definition, enunciation, procedures (algorithms), and argumentation) derived from the application of the process-product perspective to

said primary objects, these precise ones occur together with those derived from applying the process-product duality to the five dualities mentioned above (institutional/personal, ostensive/non-ostensive, expression/content, extensive/intensive and unitary/systemic): personalization-institutionalization; materialization-idealization; representation-meaning; synthesis-analysis; generalization-particularization (Font et al., 2013; Godino et al., 2007), see Figure 2.

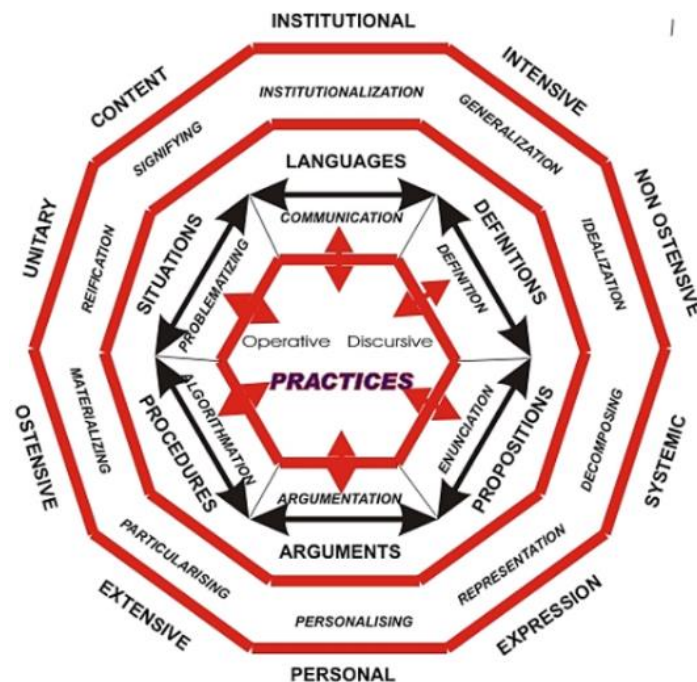


Figure 2. Schematization of mathematical knowledge from an onto-semiotic view (Font & Contreras, 2008).

Another component in OSA is the notion of semiotic function (SF) that allows practices to be associated with the processes and objects activated and the construction of an operational notion of knowledge, mathematical understanding, meaning, and competence (Godino et al., 2007). An SF is a triadic relationship between an antecedent (expression/object) and a consequent (content/object) made by a person (person or institution) according to a specific criterion or correspondence code (Font, 2007). SFs are inferred when mathematical activity is viewed from the expression/content duality. Particularly in Godino (2022), the importance of addressing the meanings of mathematical objects by subjects in given circumstances is expressed, referring to the semiotic-cognitive problem, which is related to the knowledge understood in the OSA as the "set of" relations that the subject (person or institution) establishes between objects and practices, relationships that are modeled through the notion of SF.

Networking between theories allows us to explore how their contributions can be linked while respecting their conceptual and methodological principles, facilitating the analysis of complexity in the teaching and learning of mathematics (Kidron & Bikner-Ahsbahr, 2015; Prediger et al., 2008). In this study, we considered the work of Rodríguez-Nieto et al. (2022a), who analyzed the relationship between ETC and OSA theories in the study of mathematical connections. They examined the nature of these connections and how they can be inferred from both theoretical perspectives. To do so, they conducted a content analysis of key publications from both theories, identifying principles, methods, and paradigmatic questions. They followed the typical steps for developing networking theories (Drijvers et al., 2013; Kidron & Bikner-Ahsbahr, 2015; Radford, 2008), selecting and describing episodes and classifying mathematical connections according to the ETC and the OSA.

The results showed that both theories agree on the use of content analysis, although they differ in their approaches: ETC employs a predefined typology of connections, while OSA uses various tools.

OSA analysis views mathematical connections as the tip of an iceberg composed of practices, primary objects, and SFs, providing a more detailed view. This study highlights that both theories complement each other for an in-depth analysis of mathematical connections, which will be applied in the study of engineering students' work.

In Rodríguez-Nieto et al. (2022a), it is stated that the notion of SF (OSA) is more general than the notion of mathematical connection (ETC) since the connections are considered particular cases of SFs of a personal or institutional nature. In the ETC, the mathematical connection may or may not be accurate, revealing from the perspective of OSA that when a subject makes a correct connection, it coincides with the institutional one. When it is incorrect, it is personal. These two theoretical frameworks are used because the ETC allows the use of a typology of mathematical connections, and with the OSA, it is possible to detail the connections in terms of practices, processes, objects, and SFs that relate them and to deepen a type of understanding of the subject with respect to a mathematical concept.

2.3. Mathematical understanding based on the quality of mathematical connections

The research questions formulated in the ETC can be located in the problem areas of the OSA or in the intersection of some of them, especially the cognitive semiotic. Mathematical understanding is the cognitive process that emerges in mathematical activity through which a subject X in his mathematical practice relates primary objects O (problem situations, concepts/definitions, propositions/properties, representations, arguments, procedures) among themselves, with other subjects or with everyday life, relationships established through SFs (grouped in mathematical connections of the ETC) (Rodríguez-Nieto et al., 2021a). Furthermore, understanding a mathematical concept allows the subject to use it competently in problem-solving.

In relation to mathematical understanding, the ETC has used important tools to nuance understanding and perform a connection-based analysis. In this context, the quality of mathematical connections is considered in terms of three levels and the perspective proposed by Mhlolo (2012), Mhlolo et al. (2012), and Rodríguez-Nieto et al. (2021b). The levels of quality of mathematical connections are: level 0 is identified when there is no mathematical connection in the person's (student or teacher's) response, that is, it is a personal connection rooted in procedural errors, representations, meanings, or semiotic conflicts; level 1 is recognized when a student or teacher makes mathematical connections without errors but does not argue their answers, and the student or teacher is located at level 2 if they make mathematical connections and consistently argue their answers.

Specifically, level 2 of the quality of mathematical connections is reached when the subject makes reasoned connections, for example, giving a meaning for the derivative, representing it graphically or symbolically, and, if possible, giving examples. The subject reaches level 1 of quality when they use a formula to derive a function, but do not say why they use it, and proceed mechanically. Finally, the subject is at level 0 of quality of connections when, for example, they say that the derivative is the tangent line to the curve without emphasizing the slope (rate of change) of said line, or by making errors in the procedure and meanings (Rodríguez-Nieto et al., 2021b).

Rodríguez-Nieto et al. (2021b) stated that, "quality level 2 mathematical connections ensure understanding, and quality level 0 mathematical connections are the reasons why students have difficulty understanding the derivative" (p. 1). In this way, the results will show that participants understand the derivative when they solve the task on this concept from a graphical perspective, carry out the appropriate procedures and graphs on the worksheet, and write their arguments for each step or mathematical practice performed.

3. Methodology

This research followed a qualitative methodology based on an analysis of mathematical connections from the perspective of Cohen et al. (2018), Rodríguez-Nieto et al. (2023a) and Rodríguez-Nieto and Font (2025), which was developed in four stages: 1) structuring a questionnaire with three tasks on the graph of the derivative; 2) selection of study participants who were engineering students from a university on the Colombian Caribbean coast; 3) application of the questionnaire to the students and finally, 4) the results were analyzed considering the ETC-OSA theoretical framework.

3.1. Questionnaire

A questionnaire that had been applied in the research by Rodríguez-Nieto et al. (2023a) was adapted, and a question was added that requests the meaning of the derivative, and in this study, emphasis is placed on understanding empowered by the quality of mathematical connections. Therefore, this questionnaire consists of three tasks: the first aims to explore the meaning of the derivative. The second is that students must sketch the graph of f' based on the information in f . The third requires students to sketch the graph of f , considering the information in f' (see Table 1).

Table 1. Questionnaire on the meaning and graph of the derivative

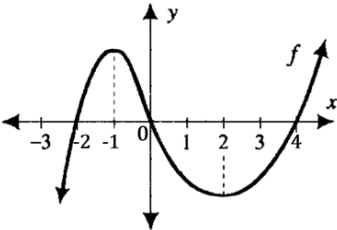
Tasks	Description
1	What does the derivative at a point mean to you? Explain your answer in detail. Given the graph of f (see Figure 3): a) Find the relative extremes of f and its relationship with f'. b) Find the intervals of growth and decrease of f and support it from f'. c) Determine the inflection point and its relationship with f' and f''. d) Identify the concavity intervals and their relationship with f''. e) Sketch the graph of f'.
2	
3	Given the graph of f' (see Figure 4): a) Find the relative extremes of f and its relationship with f'. b) Find the intervals of growth and decrease of f and support it from f'. c) Determine the inflection point and its relationship with f' and f''. d) Identify the concavity intervals of f and their relationship with f' and f''. e) Sketch the graph of f.

Figure 3. Graph of the function f (from Leithold, 1998).

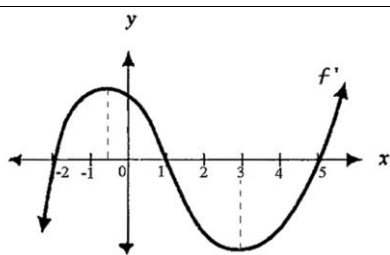


Figure 4. Graph of the function f' (from Leithold, 1998).

Source: authors' own elaboration.

This questionnaire of tasks was reviewed and validated by experts in the subject of mathematical connections, who have worked on the concept of derivative. One of the reasons for continuing research on this topic is because current research still affirms that students and some teachers have serious difficulties in understanding functions and their derivative, which affects the application of these concepts in solving real problems or in other subjects (Lumbantoruan & Manalu, 2024; Muñoz-Pinto et al., 2025; Rodríguez-Nieto & Font, 2025).

The instrument boasts content validity, supported by expert review and its use in previous studies. Reliability, although not quantified, is sustained by the consistency of the theoretical framework, the control over its application, and the transparency of the qualitative analysis.

3.2. Participants and context

Nineteen engineering students aged 16 to 17 participated in this research. They attended a private university on Colombia's Caribbean coast and were taking Differential Calculus as a core requirement for their degree programs, as suggested by the syllabus. In addition to being selected, these students expressed interest in participating in the project (which has educational, not economic, purposes), voluntarily (see Figure 5). An in-service mathematics teacher (IMT) also participated. He is the first author of this article and was responsible for designing and administering the questionnaire to the students.



Figure 5. Research participants. Source: authors' own elaboration.

The image shows that data collection was conducted in an authentic classroom setting, with students working individually on graphic tasks, which favors ecological validity, authenticity of the productions, and the possibility of rigorous qualitative analysis of mathematical connections. This visual documentation supports the methodological consistency of the study and reinforces the transparency of the research process.

3.3. Data Collection

For data collection, the questionnaire (see Table 1) was administered to the students for two hours, where they completed tasks on the meaning of the derivative, mainly as the slope of the tangent line to the curve at a point, and about the graphs of the function f and the derivative f' . The first author of this research managed the classroom and applied the tasks, suggesting that the

students explain their problem-solving processes in detail as if they were explaining them to someone else; that is, everything they wrote down on paper had to be argued or justified mathematically. It is worth noting that, during the completion of the tasks, the first author made participant observations (Figure 5) by making recordings, taking photos, and asking some key questions to delve deeper into the construction of the graphs.

3.4. Data Analysis

The data analysis will be carried out based on the method of analysis of mathematical connections based on the networking between the ETC and the OSA, considering the narrative, mathematical practices, cognitive configuration, SFs and mathematical connections (Rodríguez-Nieto et al., 2024), see Figure 6.

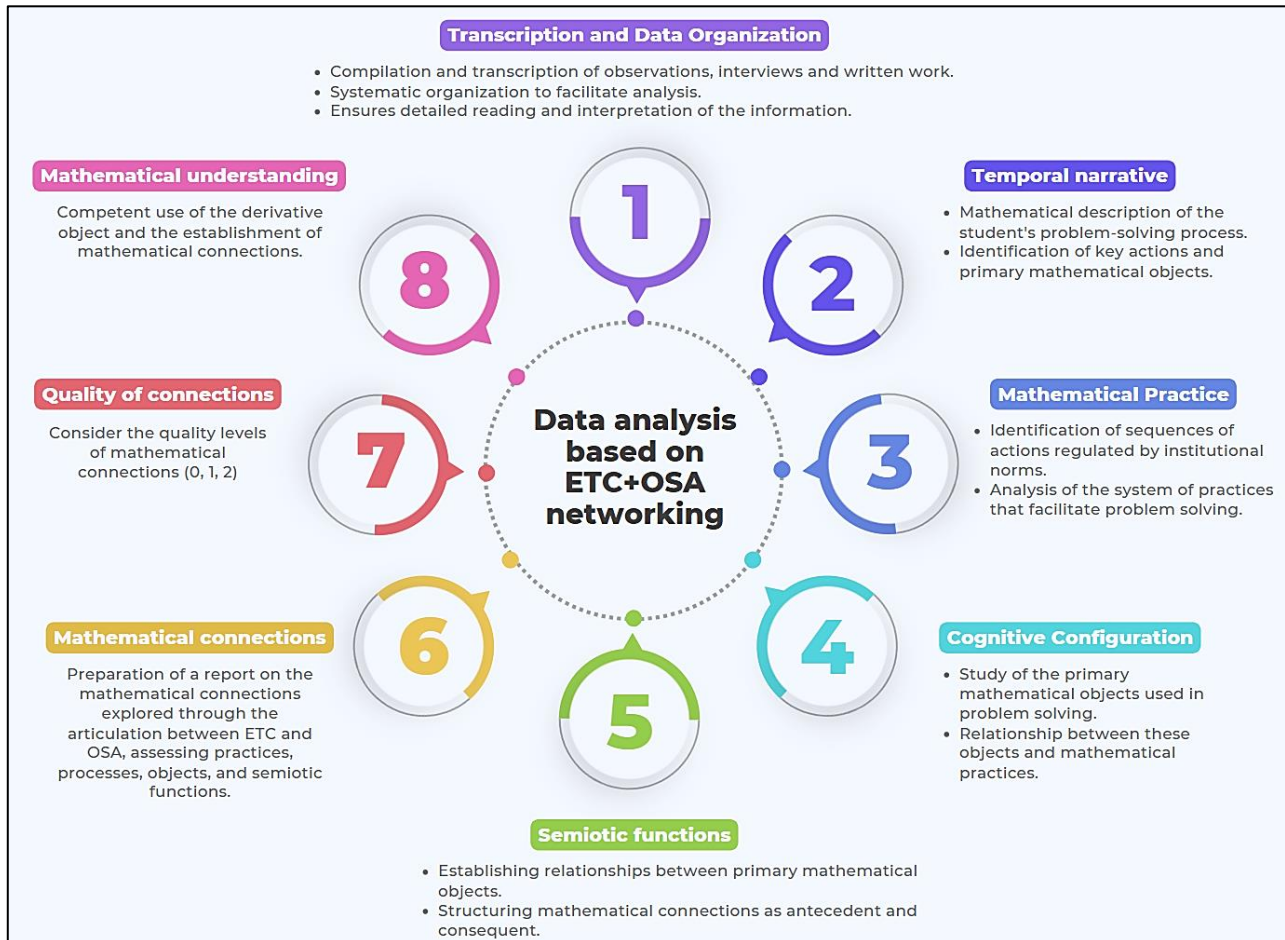


Figure 6. Phases for analyzing data based on ETC integrated with OSA. Source: authors' own elaboration.

Below, the results of mathematical connections established by the IMT and the engineering students are reported, following the analysis method presented in Figure 6. This ETC+OSA analytical approach enhances a deeper understanding of mathematical learning by integrating cognitive, institutional, and semiotic aspects. It also allows for the assessment of the quality of mathematical connections and generates useful knowledge for both educational research and for teaching and students learning about any mathematical concept.

4. Results

4.1. Temporal narrative

4.1.1. P1's Temporal Narrative for Task 2

In this case, only P1's narrative for Task 1 is presented due to space limitations: P1 was given the task of analyzing the function f and determining its growth and decline intervals. To solve the problem, he recognized that the derivative corresponds to the instantaneous slope of the curve's tangent line at any given point. In his reasoning, he expressed that "the slopes of the tangents here are negative, so the derivative must lie below the x-axis." Subsequently, he identified the critical points at $x=-1$ and $x=2$, which correspond to the relative extremes of the function, since at these points the derivative is zero, indicating local maxima or minima. Based on this, P1 determined the growth and decline intervals:

If $f(x)$ is increasing on $(-\infty, -1)$ and $(2, +\infty)$, then the derivative is positive and the slopes of the tangents are ascending.

If $f(x)$ is decreasing in $(-1, 2)$, then the derivative is negative and the slopes of the tangents are descending.

Furthermore, P1 identified an inflection point at $x=0$, since the second derivative vanishes, indicating a change in the concavity of the function. Through analysis of the second derivative, he determined the function exhibits upward concavity on the interval $(-\infty, 0)$ and downward concavity on $(0, +\infty)$, which allows him to determine the nature of the relative extremes:

If $f''(x) > 0$, the function is concave upwards and has a minimum.

If $f''(x) < 0$, the function is concave downwards and has a maximum.

Finally, P1 graphically represented the function and its derivative, using the information obtained to justify his analysis and answer the last part of the task, analyzing the graphical behavior of the first derivative f' .

4.1.2. P1's Temporal Narrative for Task 3

P1 was assigned Task 3 to analyze the function f and determine its growth and decrease intervals. To address the problem, he understood that the derivative represents the slope of the tangent line to the curve at a point. In his reasoning, he stated that "the slopes of the tangents here are negative, so the derivative must lie below the x-axis."

P1, observing the graph, identified the critical points at $x=-2$, $x=1$, and $x=5$, since at these values the derivative is equal to zero ($f'(x)=0$), indicating the presence of local maxima and minima. Based on this, he determined the growth and decrease intervals of the function:

$f(x)$ is increasing on $(-\infty, -2)$, $(1, 5)$, and $(5, +\infty)$, where the derivative is positive and the slopes of the tangents are ascending.

$f(x)$ is decreasing at $(-2, 1)$, where the derivative is negative and the slopes of the tangents are descending.

Furthermore, P1 identified the inflection points at $x=-0.5$ and $x=3$, since at these values the second derivative vanishes ($f''(x)=0$), indicating a change in the concavity of the function. Based on this, he concluded that:

$f(x)$ is concave downwards at $(-\infty, -0.5)$ and $(3, +\infty)$, suggesting the presence of local maxima in these intervals.

$f(x)$ is concave upwards at $(-0.5, 3)$, indicating the presence of local minima.

Finally, P1 graphically represented the function and its derivative, using the information obtained to justify his analysis and answer the last part of the task, explaining the behavior of the graph of f' and its relationship with f .

4.2. Mathematical practices

This section shows the mathematical practices of student P1 activated in the resolution of Tasks 2 and 3 (see Tables 2 and 3)

Table 2. Mathematical practices used to complete Task 2

Mathematical practice (Mp)	Description of practice
Mp1	Student P1 first read and then understood section a) of Task 2.
Mp2	They interpreted the slope of the tangent line to the curve at a point as the derivative.
Mp3	They recognized the critical points on the graph where the derivative becomes zero.
Mp4	They calculated growth intervals as: $(-\infty, -1)$ and $(2, +\infty)$.
Mp6	They determined the decrease interval determined the interval of decrease $(-1, 2)$.
Mp7	P1 read and understood part b) of the proposed Task 2.
Mp8	They located the Maximum point of the function. To do this, they pointed to the point (with abscissa at $x = -1$) on the graph of f and stated that, at that point, the graph of the derivative f' cuts the x - axis is equal to zero.
Mp9	They located the minimum of the function by identifying the point at $x = 2$ on the graph of f and stated that, at that point, the graph of the derivative f' cuts the x - axis is equal to zero.
Mp10	Student P1 first read and then understood section c) of the proposed Task 2.
Mp11	They stated that the inflection point is when the second derivative equals zero ($f'' = 0$), and this determines the change in concavity for the function. The minimum point of f' is identified with f'' .
Mp12	Student P1 read and understood part d) of the proposed Task 2.
Mp13	Based on the graph, the inflection point and the second derivative f'' , P1 explains that the function is concave downward at th interval $(-\infty, 0)$ and concave upward at the interval $(0, +\infty)$.
Mp14	P1 states that if $f''(x) > 0$, then the function is concave upwards and has a minimum.
Mp15	P1 states that if $f''(x) < 0$, then the function is concave downwards and has a maximum.
Mp16	P1 read and understood part e) of the proposed Task 2.
Mp17	The derivative function was graphically represented by students following these steps: first, a Cartesian coordinate system was established, after which the derivative plot was carefully constructed.

Mp18	The roots of the function were determined by analyzing its intersection points with the x-axis on the graph, that is, they located the points where the derivative equals zero ($x = -1$ and $x = 2$).
Mp19	They sketch the graph of f' in parts, considering the increasing intervals, the extremes of the function, and the inflection point.

Source: authors' own elaboration.

Subsequently, Table 3 shows the system of mathematical practices used by P1 to solve Task 3. With this information it is evident that this student carries out bidirectional processes moving from f to f' and vice versa (symbolically and graphically).

Table 3. Mathematical practices used to complete Task 3

Mathematical Practice (Mp)	Description of Practice
Mp1	P1 read and understood part a) of the proposed Task 3, which asked for the analysis of the function f to determine intervals of increase and decrease.
Mp2	P1 recognized that the derivative $f'(x)$ corresponds geometrically to the slope of the tangent line at any given point on the curve.
Mp4	The critical points were identified at $x = -2$, $x = 1$, and $x = 5$, corresponding to the values where the first derivative $f'(x)$ equals zero.
Mp5	Student P1 read and understood part b) of the proposed Task 3.
Mp6	They determined that the function is decreasing in the intervals $(-\infty, -2)$ and $(1, 5)$, since $f'(x) < 0$.
Mp7	They concluded that the function is increasing in the intervals $(-2, 1)$ and $(5, +\infty)$, based on $f'(x) > 0$ in those intervals
Mp8	Student P1 read and understood part c) about inflection points (Task 3)
Mp9	They identified the inflection points at $x = -0.5$ and $x = 3$, where $f''(x) = 0$.
Mp10	Student P1 read and understood part d) of the task involving concavity intervals (Task 3)
Mp11	They explained that the function is concave downward on $(-\infty, -0.5)$ and $(3, +\infty)$, based on the second derivative being negative $f''(x) < 0$.
Mp12	They stated that the function is concave upward on $(-0.5, 3)$, where $f''(x) > 0$.
Mp13	They related the sign of $f''(x)$ to the nature of the local extremum: concave down $\rightarrow$ maximum, concave up $\rightarrow$ minimum.
Mp14	They drew the graph of the function and of its derivative, using a Cartesian coordinate plane to represent the changes in behavior.
Mp15	They plotted the points where $f'(x) = 0$, marking $x = -2$, $x = 1$, and $x = 5$ on the graph of the derivative.
Mp16	They explained that in intervals where the function is increasing, $f'(x) > 0$, and the graph of the derivative lies above the x-axis.

Mp17	They explained that in intervals where the function is decreasing, $f'(x) < 0$, and the graph of the derivative lies below the x-axis.
Mp18	They interpreted the inflection points on the function f as local minima in the graph of f' .
Mp19	They used all graphical and analytical information to justify their conclusions and respond to the task, showing coherent reasoning throughout.

Source: authors' own elaboration.

4.3. Cognitive configuration

The cognitive configuration allows us to study how P1 interprets and performs Task 2, what knowledge is activated in terms of primary objects, how they relate them, and what meanings are given to the objects involved, which provides an important path for the construction of connections (Table 4).

Table 4. Cognitive configuration of P1 for Task 2

PO	Description
T Task 2 (T2)	
	<i>Verbal language:</i> slope, function, graph, derivative function, derivative at a point, tangent line...
	<i>Symbolic language:</i> f, f' , intervals: e.g., $(-\infty, -1)$; Points: $(0, 0)$; $x = -1$; $x = 2$; $x = 0$
	<i>Graphic language:</i> see Figure 7.

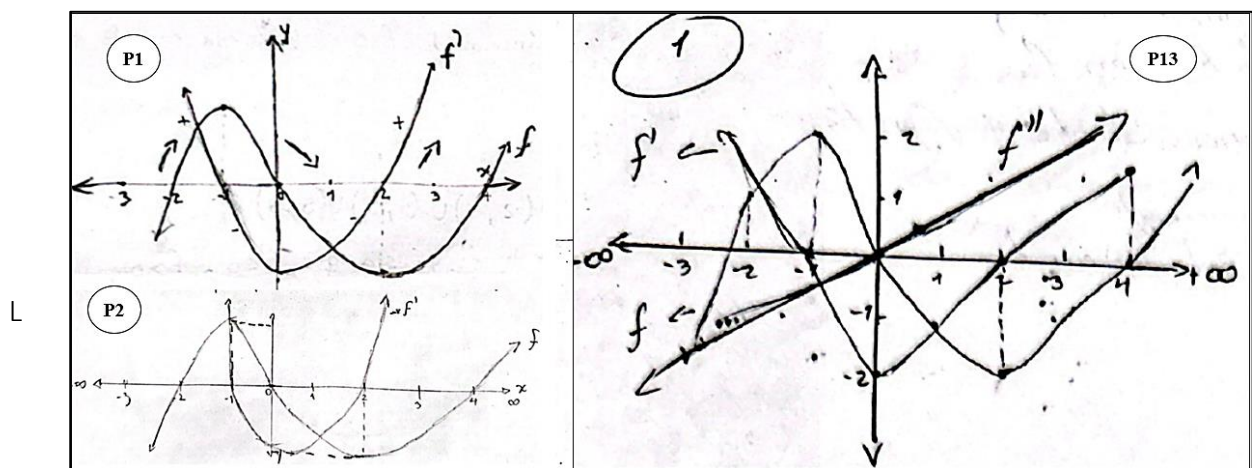


Figure 7. Graphic representation of the function and its derivative. Source: authors' own elaboration

Figure 7 shows a fundamental teaching and mathematical resource because it allows students to build a solid and articulated understanding of the concept of derivative, both in its geometric dimension (slope of the tangent) and in its analytical dimension (limit of the incremental quotient). Furthermore, it promotes graphical reasoning, qualitative analysis of functions, and the fluid transition between representation registers essential elements for meaningful learning of differential calculus.

Previous concepts/definitions: maximum or minimum, inflection point, critical point, line, graph, tangent, first and second derivative, derivative at a point, ...

D1: f' is like the slope of the tangent line to the curve at a point.

D D2: A critical point of a function f is a number c in the domain of f such that $f'(c)=0$ or $f'(c)$ does not exist.

D3: Let f continue at c . We call $(c, f(c))$ an inflection point of the graph of f , if f is concave up on one side of c and concave down on the other side of c .

Propositions:

a) (Pr1) the graph of f is increasing at the intervals: $(-\infty, -1)$ and $(2, +\infty)$.

b) (Pr2) The graph of f is decreasing at interval: $(-1, 2)$.

Pr c) (Pr3) the function has a maximum at $x = -1$ and (Pr4) has minimum at $x = 2$.

d) (Pr5) the inflection point is $x = 0$.

e) (Pr6) the function is concave downward at the interval of $(-\infty, 0)$ and (Pr7) is concave up at $(0, +\infty)$.

Main procedure 1 (Pcp1): Find the intervals of increase and decrease. To perform this procedure, the student used two auxiliary procedures (Pca):

(Pca1.1): They located the critical points (graphically finding the values of "x" where the slope is zero).

Pca1.2: Use of the sign of the slope to determine intervals of increase and decrease.

Pcp2: Determine the extremes in the graph of f . To perform this procedure, the student used two auxiliary procedures:

Pca2.1: P1 states that the function has a maximum when the derivative equals zero and uses the change of increasing to decreasing of the f .

Pc Pca2.2: P1 states that the function has a minimum when the derivative equals zero and uses the change of decreasing to increasing of the f .

Pcp3: Determine the inflection point of f .

Pca3.1: P1 determines the inflection point when the second derivative is equal to zero $f''(x)=0$ and uses the change of concavity to support the claim.

Pcp4: Determine the intervals of concavity of f .

Pca4.1: P1 determines the concavity intervals with the second derivative f'' in which its value determines whether it is concave upwards or downwards, taking the inflection point as a reference.

Pcp5: P1 sketch f' .

Pca5.1: Make a Cartesian coordinate system.

Pca5.2: Locate the abscissa points $x = -1$ and $x = 2$ which are the intersection points of the derivative with the x -axis.

Pca5.3: Sketch the graph of f' in parts, considering the increasing intervals, the extremes of the function, and the inflection point (see Figures 7 and 8).

3) Extremos relativos: $(-1, 2)$ Son los puntos en donde el valor de la derivada es cero, por lo tanto, se encuentran también máximos y mínimos de la función. 4) Intervalos de crecimiento y decrecimiento: $(-\infty, -1) \cup (-1, 2) \cup (2, \infty)$ $f' > 0$ $f' < 0$ Los valores positivos de la derivada determinan el crecimiento de la función, por lo contrario, los valores negativos al decrecimiento de la función. 5) Punto de inflexión: 0 $f'' = 0$ El punto de inflexión es cuando la segunda derivada se iguala a cero y esto en la función determina el cambio de concavidad. El punto mínimo de f' se identifica con f''. 6) Intervalos de concavidad: $(-\infty, 0) \cup (0, \infty)$ Los intervalos de concavidad se relacionan con f'' en que el valor de f'' determina si f es cóncava hacia arriba o hacia abajo, con esta información se puede identificar si la función presenta ya sea un máximo o mínimo. $f'' > 0$: f cóncava hacia arriba, presenta un mínimo. $f'' < 0$: f cóncava hacia abajo, presenta un máximo.	3) Relative extremes: $(-1, 2)$ These are the points where the value of the derivative is zero, therefore, the function's maxima and minima are also found. 4) Increase and decrease intervals: $(-\infty, -1) \cup (-1, 2) \cup (2, \infty)$ $f' > 0$; $f' < 0$ Positive values of the derivative determine the growth of the function; conversely, negative values determine the decrease of the function. 5) Inflection point: 0 $f'' = 0$ The inflection point is when the second derivative equals zero, and this determines the change in concavity for the function. The minimum point of f' is identified with f''. 6) Concavity intervals: $(-\infty, 0) \cup (0, \infty)$ Concavity intervals relate to f'' in that the value of f'' determines whether it is concave up or down. With this information, we can identify whether the function has either a maximum or a minimum. If $f'' > 0$; f is concave up, it has a minimum. If $f'' < 0$; f is concave down, it has a maximum.
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Figure 8. Evidence of procedures performed by P1. Source: authors' own elaboration

This written evidence (Figure 8) is key to analyzing and assessing meaningful understanding of derivatives and other concepts in differential calculus, as it shows how students articulate mathematical objects, make relevant connections, and meaningfully argue reversible processes that lead to connections of implication and reversibility.

A1: Thesis: The increasing intervals are $(-\infty, -1)$ and $(2, +\infty)$, and the decreasing interval is $(-1, 2)$.

Reason 1 (R1): If a function is continuous, the critical points separate intervals of increase and/or decrease.

R2: the critical points are $x = -1$ and $x = 2$ since for these values $f'(x) = 0$.

R3: there are three intervals and the sign of f' at each one is always the same. If the sign of the slope is negative, f is decreasing, and if is positive, f is increasing.

Conclusion 1: f is increasing at $(-\infty, -1)$ and $(2, +\infty)$, and decreases at $(-1, 2)$.

A **A2: Thesis:** The maximum point is at $x = -1$ and the minimum point is at $x = 2$.

R1: At $x = -1$ and at $x = 2$ the graph of f' cuts the x -axis, and the slope is 0.

R2: if f is increasing and at a point, it becomes decreasing, then there is a maximum, and, if f is decreasing and at a point, it becomes increasing, then is a minimum.

Conclusion 2: f has a maximum at $x = -1$ and a minimum at $x = 2$.

A3: Thesis: The inflection point is at $x = 0$.

R1: The inflection point is where the second derivative is equal to zero $f''(x) = 0$ and uses the change of concavity (changes from concave up to concave down).

Conclusion 3: At the abscissa $x = 0$ if there is an inflection point.

A4: Thesis: f is concave downward at $(-\infty, 0)$ and concave upward at $(0, +\infty)$.

R1: P1 considers that f is continuous and by finding the second derivative the inflection point is obtained at $x=0$, where it stops being concave downwards and becomes concave upwards.

R2: If $f'' > 0$; f is concave up, it has a minimum.

R3: If $f'' < 0$; f is concave down, it has a maximum.

Conclusion 4: Effectively, f is concave downward at $(-\infty, 0)$ and concave upward at $(0, +\infty)$.

Source: authors' own elaboration.

4.4. SFs as relations between primary objects

The idea of using the SF is to connect the signs (symbols, graphics, language, etc.) that allow subjects to interact with mathematical objects. This, in turn, directly favors the activation of connections because these depend on connecting objects (see Figure 9).

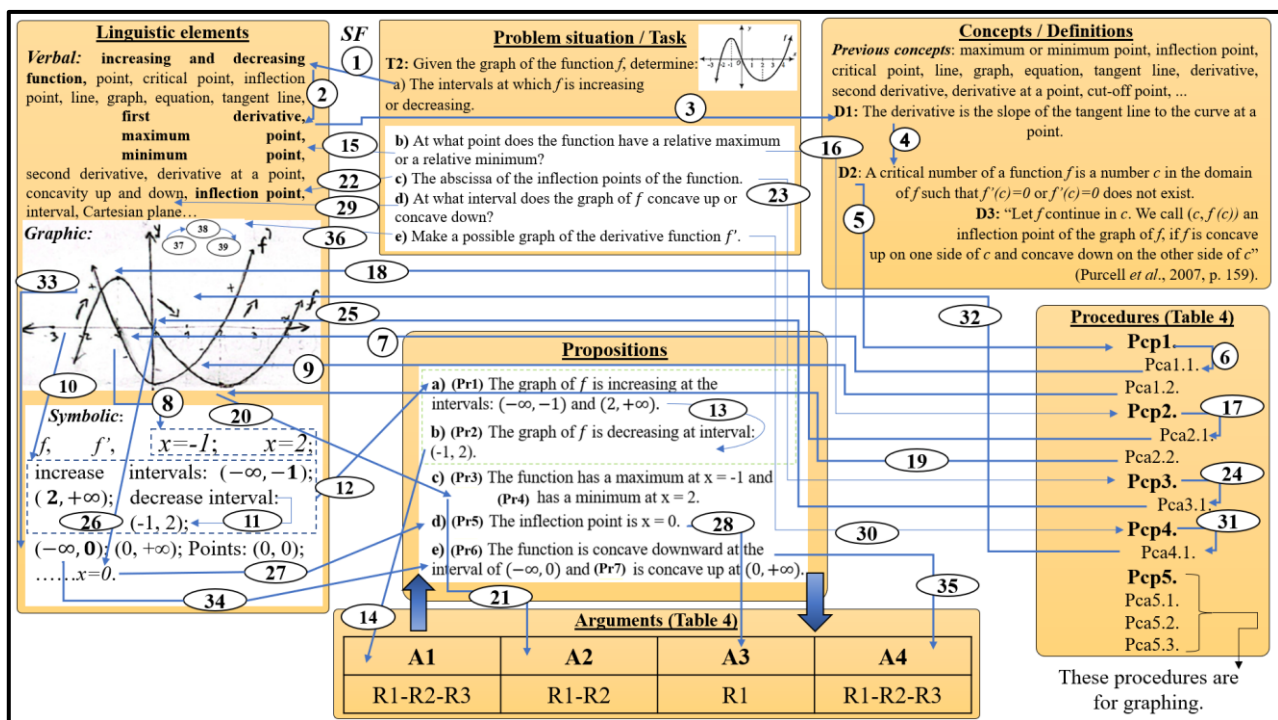


Figure 9. SFs established by P1 in Task 2. Source: authors' own elaboration

Figure 9 shows that solving a task such as "determining growth intervals of a function" is not a linear process. It involves coordinating prior knowledge, representations, procedures, definitions, and arguments. This way of organizing mathematical activity has yielded good results in terms of interpreting the SFs that relate to primary objects.

4.5. Mathematical connections activated by participants in Task 2

Table 5 presents the mathematical connections established by P1 when solving Task 2. Each row of the Table reflects a conglomerate of mathematical practices, processes, objects and SFs that, together, constitute a specific connection making the networking tools between the ETC and the OSA functional.

Table 5. In-depth Analysis of P1's Mathematical Activity in Task 2

Mp	Processes	Objects	SFs	Connections
Mp1	Meaning/understanding Problematization	P1 understood part a) the proposed Task 2 (T2)	SF1	Neuro-mathematical connection of term and symbol recognition
Mp2	Problem-solving Enunciation Signification	D1: The derivative is the slope of the tangent line to the curve at a point.	SF2 SF3	Meaning
Mp3	Problem-solving Enunciation	D2: A critical number of a function f is a number c in the domain of f such that $f'(c) = 0$.	SF4	Meaning (Figure 10)

1) La derivada es la ^{pendiente de la} recta Tangente en la función en un punto determinado de esta..
 The derivative is the slope of the tangent line to the function at a given point on it.

Figure 10. Evidence of the connection of meaning

Mp4	Problem-solving Representation (verbal, graphic, and symbolic).	Pca1.1: Find graphically the value of "x" in which the slope is equal to zero. They determined the intervals of increase $(-\infty, -1)$ and $(2, +\infty)$.	SF5 SF6 SF7 SF8 SF9 SF10	Procedural Different representations
Mp5	Problem-solving Representation (verbal, graphic, and symbolic).	They determined the intervals decrease at the interval $(-1, 2)$ of the function f , from the given graph and considering the sign of the slope of the tangent line.	SF11	Procedural Different representations
Mp6	Problem-solving Representation (verbal, graphic, and symbolic). Argumentation	Pca1.2: Use of the sign of the slope to determine intervals of increase and decrease of the function. A1 (Figure 7) If the function is increasing, the slopes of this tangent line are positive, that is to say, the derivative is positive.	SF12 SF13	Procedural Different representations Implication

Mp7	Meaning/understanding Problematicization	P1 understood part b) of the proposed Task 2.	SF14	Neuro-mathematical connection of term and symbol recognition
Mp8	Problem-solving Representation (verbal, graphic, and symbolic).	Maximum point of the function. P1 pointed to the point (with abscissa at $x = -1$) on the graph of f and stated that, at that point, the graph of the derivative f' cuts the x – <i>axis</i> is equal to zero.	SF15	Different representations
			SF16	
			SF17	Implication
			SF18	
Mp9	Problem-solving Representation (verbal, graphic, and symbolic).	Minimum point of the function. P1 pointed to the point (with abscissa at $x = 2$) on the graph of f and stated that, at that point, the graph of the derivative f' cuts the x – <i>axis</i> is equal to zero.	SF19	Different representations
			SF20	
			SF21	Implication
Mp10	Meaning/understanding Problematicization	P1 understood part c) of the proposed Task 2.	SF22	Neuro-mathematical connection of term and symbol recognition
Mp11	Problem-solving Representation (verbal, graphic, and symbolic).	The inflection point is found when f'' equals 0.	SF23	Different representations
			SF24	
			SF25	
			SF26	Implication
			SF27	
			SF28	
Mp12	Meaning/understanding Problematicization	P1 understood part d) of the proposed Task 2.	SF29	Neuro-mathematical connection of term and symbol recognition
Mp13	Problem-solving Representation (verbal, graphic, and symbolic).	The function is concave downward at interval $(-\infty, 0)$ and concave upward at the interval $(0, +\infty)$. The change in concavity occurs at the inflection point ($f''=0$).	SF30	Different representations
			SF31	
			SF32	
			SF33	Implication

Mp14	Problem-solving	A4 – R2	SF34	Implication
	Enunciation	If $f'' > 0$; f is concave up, it has a minimum.		
Mp15	Representation (verbal, graphic, and symbolic).	A4 – R3	SF35	Implication
		If $f'' < 0$; f is concave down, it has a maximum.		
Mp16	Meaning/understanding		SF36	Neuro-mathematical connection of term and symbol recognition
	Problematicization Representation (verbal, graphic, and symbolic).	P1 understood part e) of the proposed Task 2.		
Mp17	Representation (verbal, graphic, and symbolic).	Cartesian coordinate system.	SF37	Different representations
Mp18	Representation (verbal, graphic, and symbolic).	Locate the abscissa points $x = -1$ and $x = 2$ which are the intersection points of the derivative with the x -axis.	SF38	Different representations
Mp19	Generalization-particularization	Sketch the graph of f' in parts, considering the increasing intervals, the extremes of the function, and the inflection point.	SF39	Part-Whole
	Representation (verbal, graphic, and symbolic).			Different representations

Source: authors' own elaboration.

Table 5 is a powerful example of the potential of OSA to analyze mathematical activity as a complex network of practices and meanings. Reflecting on it helps us appreciate the semiotic richness of mathematical knowledge and the importance of considering the student as an active subject who constructs meaning from multiple registers and practices.

On the other hand, the SFs established by P1 to solve Task 3 are shown below and for reasons of space the cognitive configuration will not be presented in detail.

4.6. Level 2 of quality of connections and mathematical understanding

Figure 11 presents a semiotic analysis of a mathematical connection of meaning, focusing on a student's interpretation of the concept of derivative. The iceberg metaphor highlights the two connected mathematical objects: "the derivative at a point" and "the slope of the line tangent to the curve at that point." The correspondence code that allows this relationship to be established is explained.

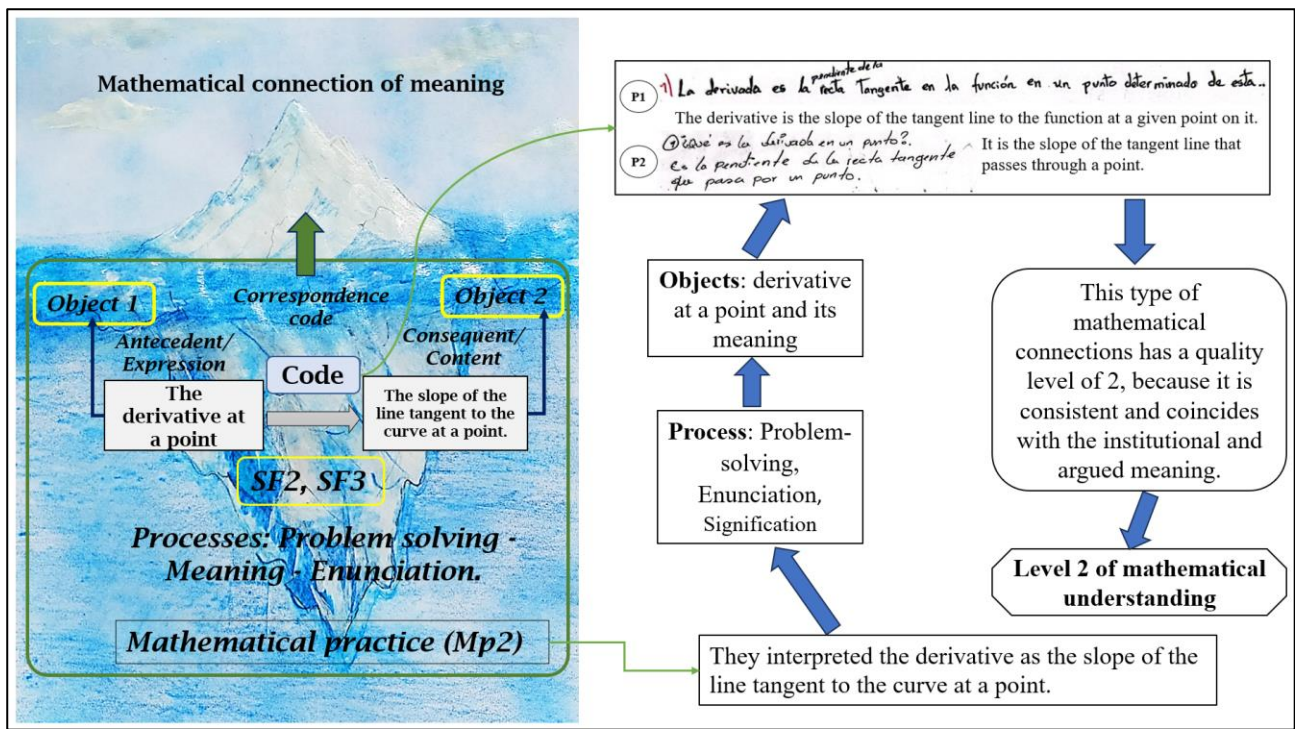


Figure 11. Example of analysis of understanding the meaning of the derivative. Source: Authors' own elaboration

The analysis shows that this connection occurs within the processes of problem-solving, meaning-making, and enunciation, specific to a specific mathematical practice (Mp2). The student's interpretation, although personal, is consistent with the institutionally accepted meaning of the derivative at a point, justifying its classification as a quality level 2 connection, corresponding to a level 2 understanding. This type of understanding assessment, based on the OSA, allows for the evaluation not only of the correctness of the statement, but also its semantic consistency, argumentative coherence, and alignment with school mathematical discourse. Student P19 connected two meanings of the derivative as the limit of the incremental quotient and as the slope of the tangent line (see Figure 12).

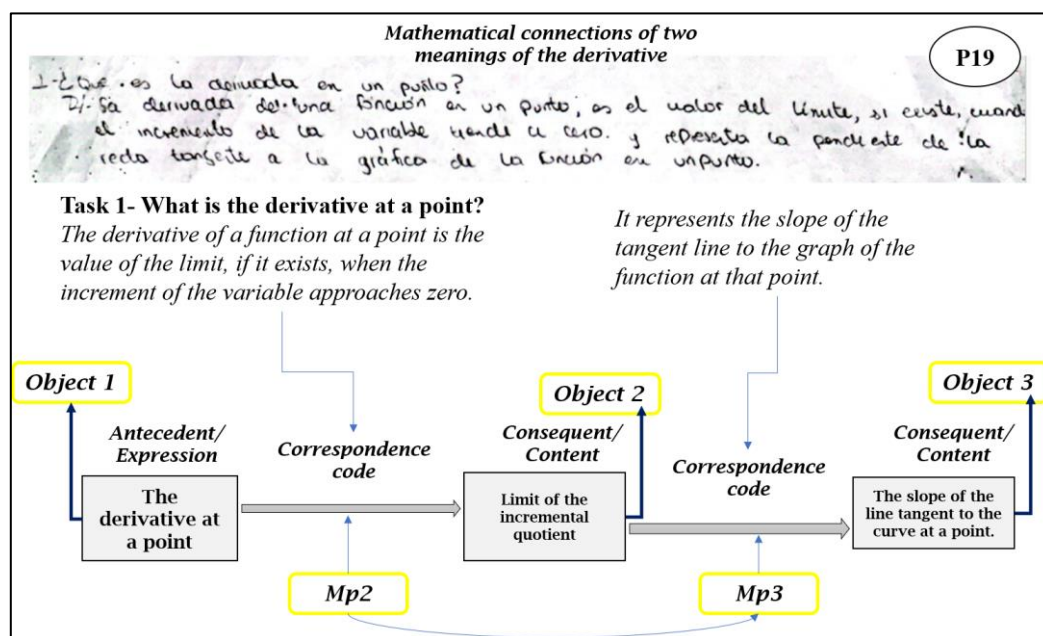


Figure 12. Level 2 connections between meanings of the derivative. Source: authors' own

In Figure 12, ensuring that the student understands both meanings of the derivative is a teaching that articulates the approximation process (limit) with the geometric interpretation (tangent slope), using multiple records, dynamic visualizations, and contextualized situations.

Table 6 shows the mathematical connections of meaning and their level of quality, revealing that none of the students had any misconceptions about the meaning of the derivative of the function at a point. This quality of connections indicates that the students understood the derivative and at least interpreted it as a limit, rate of change, or the slope of the tangent line to the curve at a point.

Table 6. Quality level of mathematical connections of meaning

	Level 2	Level 1	Level 0
Task 1: What does the derivative at a point mean to you?	P1, P4, P5, P6, P7, P8, P9, P11, P12, P13, P14, P15	P2, P3, P10, P18	

Source: authors' own elaboration.

Also at this level 2 of understanding are participants P2, P3, P5, P6, P7, P8, P10, P13, P16, P17 and P18, who proceeded differently in obtaining the answers to the sections of Task 2 until they sketched the graph of f' (see Figure 13).

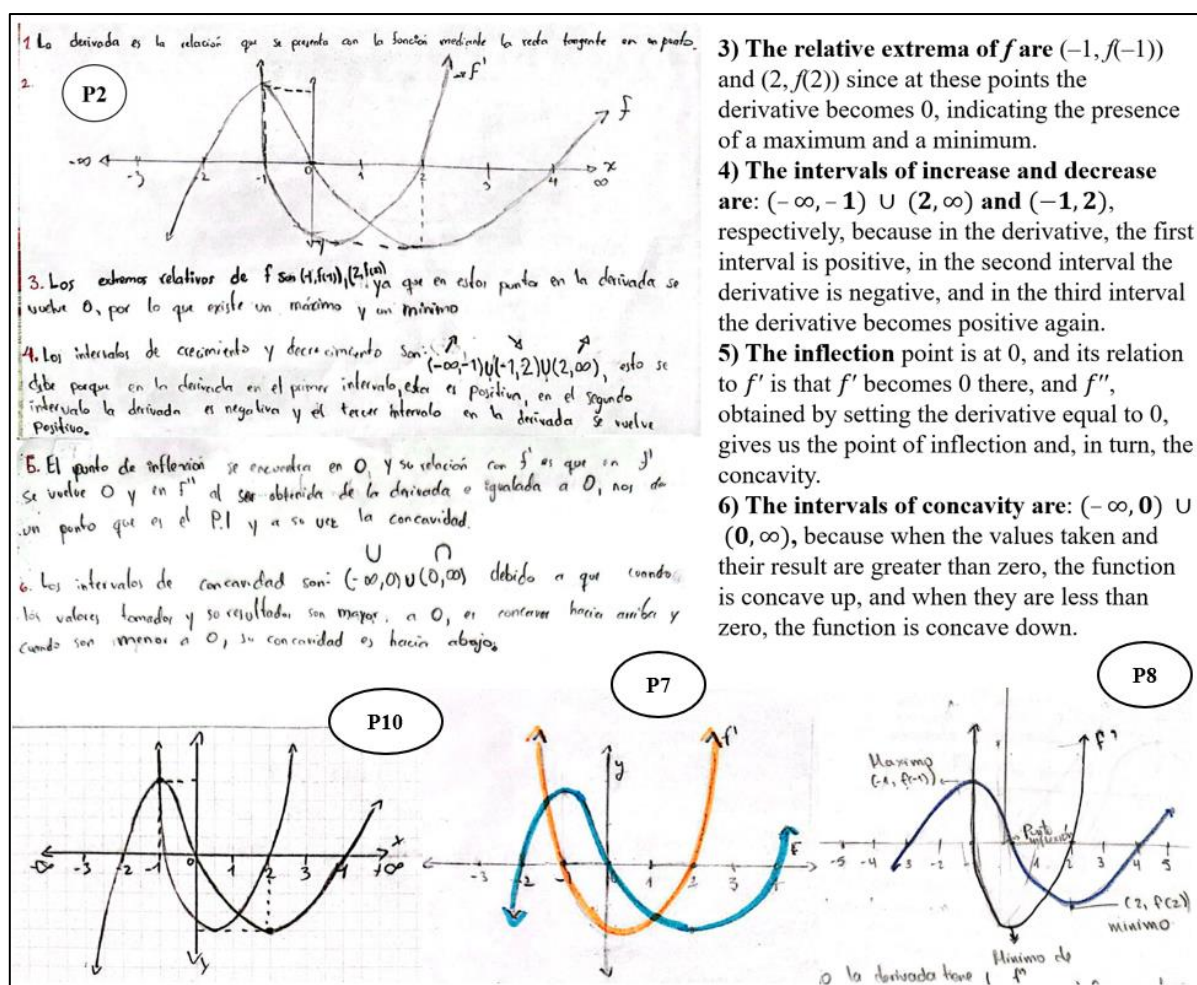


Figure 13. Written productions of some students who reached level 2 of understanding. Source: authors' own elaboration

Furthermore, P1 is also situated at level 2 of mathematical understanding because he activated connections with level 2 quality (consistent and justified) when solving Task 3. This understanding is widely supported by the mathematical practices presented in Table 3 and Figure 14.

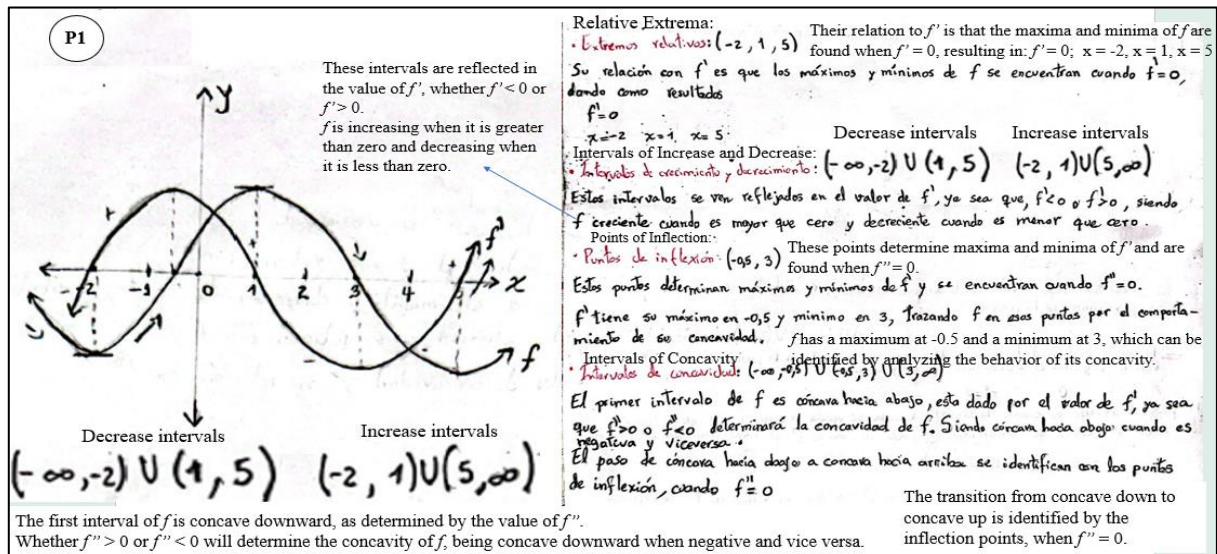


Figure 14. Evidence of the resolution and understanding of Task 3 by P1. Source: authors' own elaboration

Similarly, students P2, P5, P6, P8, P10 and P13 successfully solved Task 3 and reached level 2 of understanding since the quality level of their connections is 2. Figure 15 shows some examples of these connections.

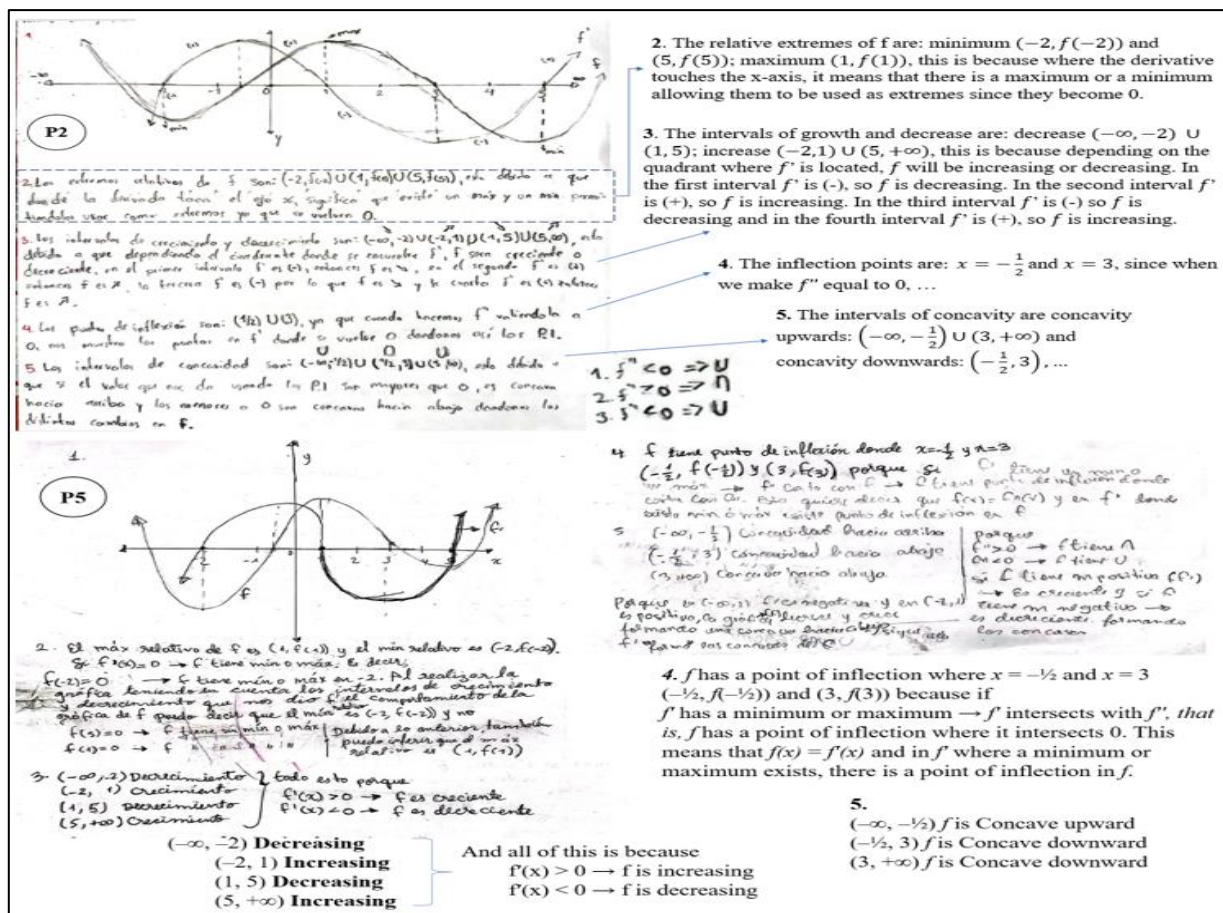


Figure 15. Evidence of connections and understanding of P2 and P5. Source: authors' own elaboration

This information is valuable because it reflects a progressive and well-argued mathematical reasoning process. It is observed how students (P2 and P5) use the information from the derivative to infer essential properties of the function f , which is central to understanding the behavior of functions in calculus.

4.7. Level 1 of quality of connections and mathematical understanding

This section describes student P19's quality level 1 mathematical connections. He constructed the graph appropriately, indicating intervals of growth, decrease, relative extremes, inflection points, and concavity intervals. However, in the justifications, he had some problems related to implication connections that he failed to make. These are indications that P19 needs to further strengthen the meanings of the derivative, specifically, as the slope of the tangent line. It is known that P19 made connections between meanings in Task 1 and was placed at quality level 2, but in the applications he needs to improve so that he understands the role of the slope in graphic situations (see Figure 16).

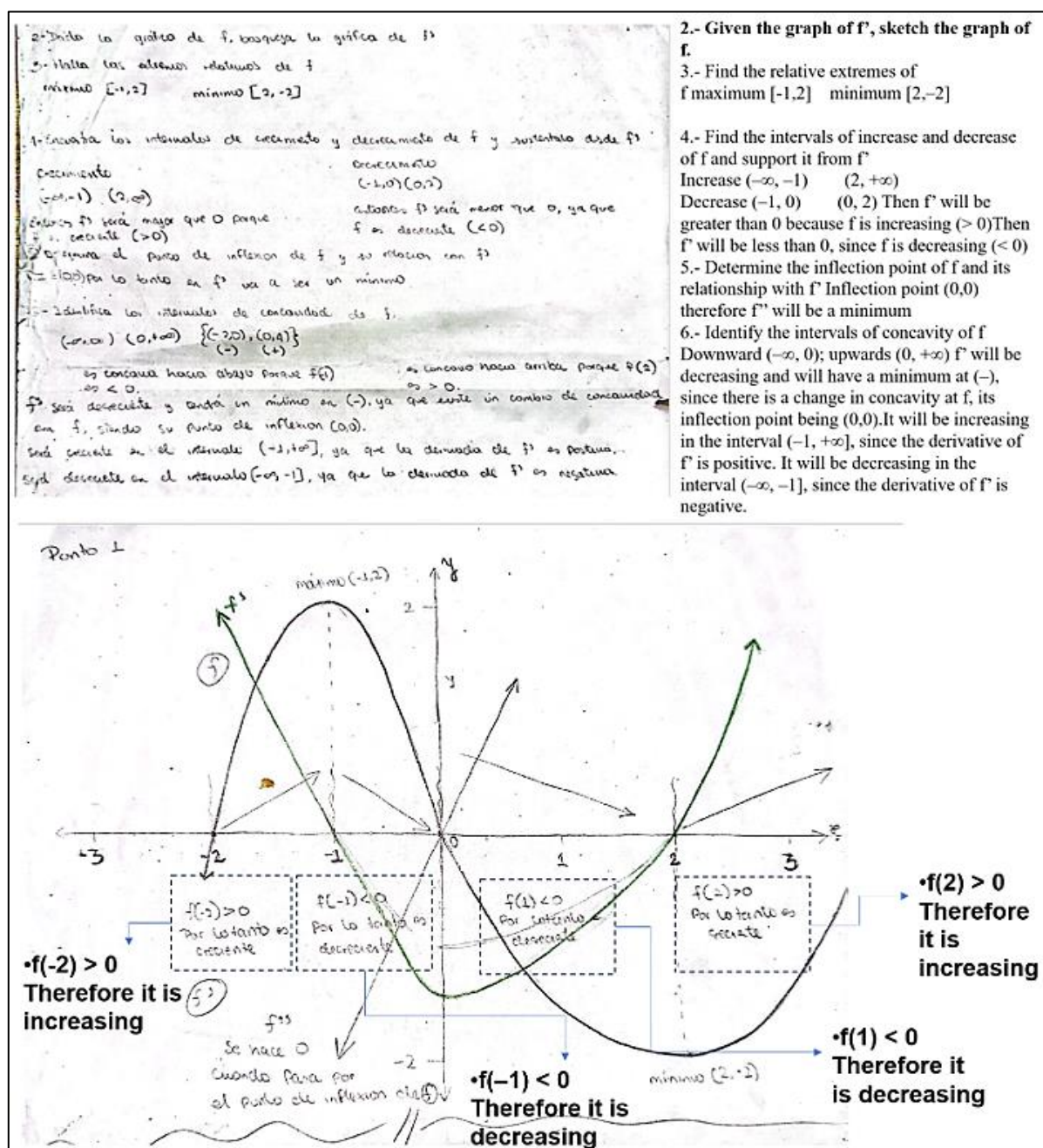


Figure 16. Evidence of P16 errors caused by the lack of implication connections. Source: author's own elaboration

In Figure 16, student P19 states that, " $f(-1) < 0$ therefore it is decreasing," here there seems to be a confusion between the function f and its derivative f' . For example, $f(-1) < 0$ means that the value of the function at $x = -1$ is negative, that is, the graph of f is below the x -axis at that point. Now, when P19 says "because f is decreasing," this does not justify that $f(-1)$ is less than zero. The derivative f' indicates the slope or rate of change of f , not directly the value of f .

To clarify this situation a little, P19 should take into account the following: The condition $f'(-1) < 0$ indicates that the function f is decreasing at $x = -1$, that is, the slope of the tangent line is negative there. However, saying that $f(-1) < 0$ requires checking whether the graph is below the x -axis at that point is not justified by the sign of f' . Therefore, the statement " $f(-1) < 0$ therefore it is decreasing" is not correct, because it confuses the value of the function with the behavior of its derivative. Therefore, considering the specific confusion between the value of the function f and the sign of its derivative f' , that is, a significant conceptual imprecision, but taking into account that the student makes most of the correct mathematical connections, argues his answers and appropriately relates concepts such as maxima, minima, growth intervals and concavity, his level of comprehension would be placed at a level 1 of quality of mathematical connections. It's worth noting that the student is close to level 2, but the confusion between function and derivative is enough to disqualify him from being considered at level 2 in mathematical understanding. Therefore, level 1 is the most accurate assessment for student P19. Students P4, P9, P11, P12, P14, P15, and P19 are at this same level of understanding.

On the other hand, students P7, P14, P15, P16, P17, P18 and P19 had difficulties in completing task 3, especially when sketching the graph of the function f from the graph of the derivative f' . It is worth noting that these students had correctly solved this task in terms of procedural connections, that is, establishing implication connections to find the intervals of growth and decrease, relative extremes, etc. Therefore, these connections are of quality level 1 (there was no adequate transition from the symbolic record to the graphic record) and are situated at a level of understanding 1. Furthermore, Figure 17 shows a comparison between the graphs of students P7 and P16 and a model of a correct graph for solving Task 3.

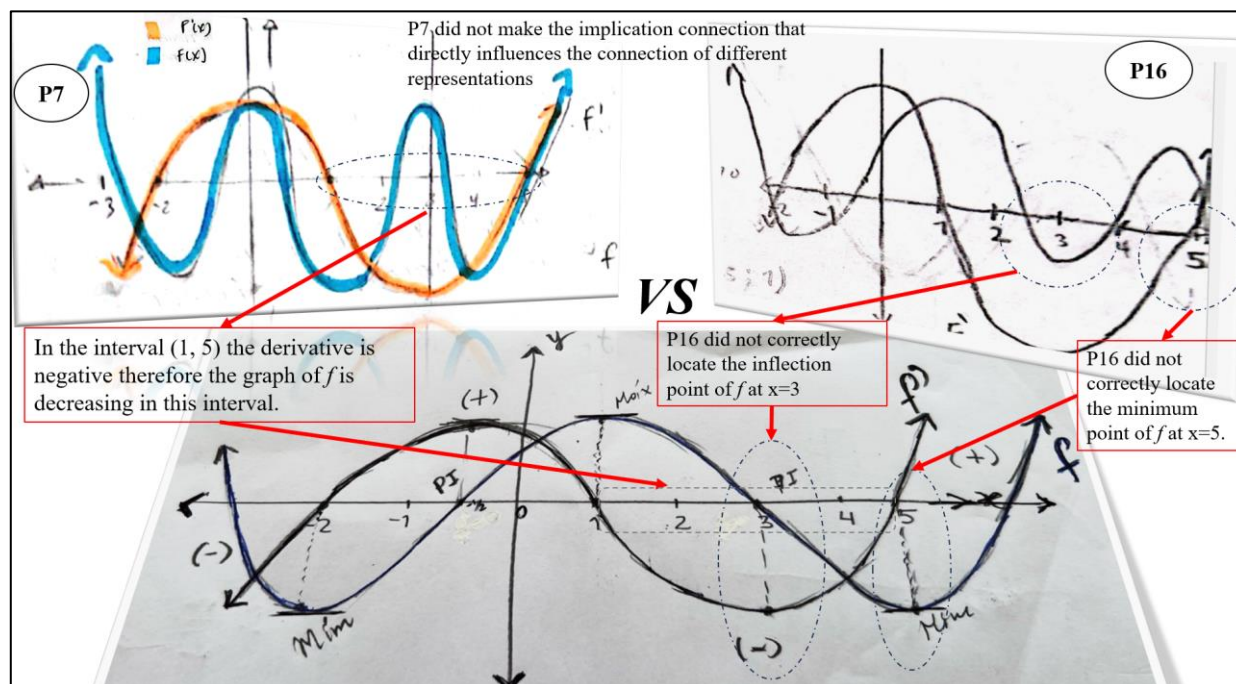


Figure 17. Comparison between the graphs of P7 and P16 with the graph suggested as appropriate.

Source: authors' own elaboration

This reflection on students' mathematical understanding does not end here because we still need to delve deeper into those mathematical connections at quality level 0 that are the causes of the errors made by students.

4.8. Level 0 of quality of connections and mathematical understanding

In relation to Tasks 1 and 2, the students did not reach this level of understanding; that is, at least they performed the procedures correctly. However, students P3, P4, P9, P11, and P12 responded inadequately to Task 3, which was caused by connections at level 0 quality, that is, incorrect procedures, the graphs did not correspond to what was requested in the task, and the justifications were inconsistent with institutional mathematics (Figure 18).

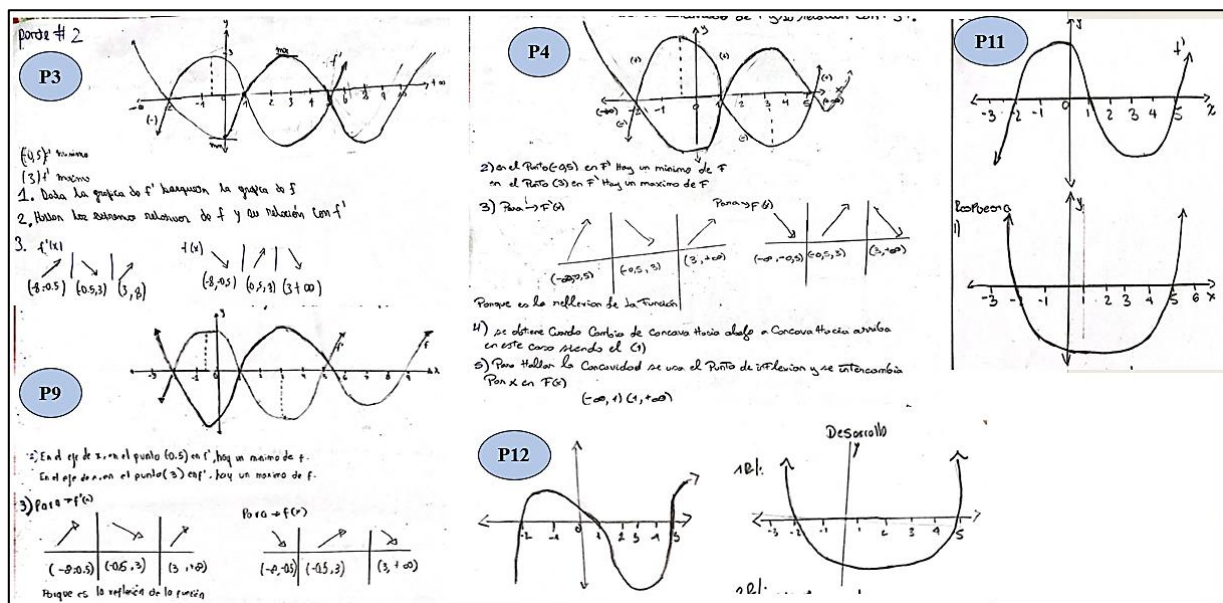


Figure 18. Evidence of Quality 0 connections

Source: authors' own elaboration.

The fact that the assignments require sketches of the graphs of the derivative f' or of the function f does not mean that any sketch can be made assuming that the curve will pass somewhere. This statement has triggered a series of important prior mathematical knowledge because students need to know logical relationships linked to the monotony of functions, relative extremes, concavity, mathematical calculations, and proper use of the Cartesian plane. Along these lines, some students, such as P3, P4, P9, P11, and P12, did not complete assignment 3 because they did not make consistent or institutional mathematical connections (implications, different representations, and part-whole).

Furthermore, these students place their connections at quality level 0 and therefore have a mathematical understanding that requires attention and reinforcement, which has been caused by personal connections linked to mathematical errors, for example, P3, P4 and P9 assume the graph of the derivative function f' as a reflection of the function f or vice versa, preventing them from recognizing the maximums, minimums, inflection points and, above all, they do not identify the growth and/or decrease intervals. Students P11 and P12 do not consider the implication connections conditioned by the graph of the derivative "made up of a set of slopes" that show a negative or positive sign (or zero) that implies that the graph of f grows, decreases, has a maximum, minimum or inflection point.

Other fundamental aspects found in this research is that students continue to have more preferences for the procedures when they are provided with the algebraic expression of the function or the derivative in order to resort to tables and numerical calculations to find points that lead them

to the construction of a graph (Borji et al., 2018; Fuentealba et al., 2019; García-García & Dolores-Flores, 2021; Ikram et al., 2020; Rodríguez-Nieto et al., 2023a). However, it is evident again that students must acquire the competence to sketch function graphs from concepts, properties, implication and reversibility connections.

From a mathematical point of view, the connections made by the students were important, but the most significant mathematical connection for solving Tasks 2 and 3 (correctly) involving the graphs of f and f' , without symbolic expressions, is the reversibility connection. Given the complexity of the derivative object, students have several difficulties in solving problems, but with reversible thinking activated by reversibility connections, they can achieve it, particularly, with the ETC+OSA theoretical framework, both the mathematical complexity of the task and the connections established to solve it are explained. Finally, Table 7 presents a summary of the levels of quality of mathematical connections, mathematical understanding according to the type of task.

Table 7. Levels of connection quality and mathematical understanding achieved by students by task

Level of quality of connections and understanding	Task 1	Task 2	Task 3
Level 2	P1, P4, P5, P6, P7, P8, P9, P11, P12, P13, P14, P15, P16, P17, P19 (15 students are equivalent to 78.9%)	P1, P2, P3, P5, P6, P7, P8, P10, P13, P16, P17, P18 (12 students are equivalent to 63.2%)	P1, P2, P5, P6, P8, P10, P13 (7 students are equivalent to 36.8%)
Level 1	P2, P3, P10, P18 (4 students are equivalent to 21.1%)	P4, P9, P11, P12, P14, P15, P19 (7 students are equivalent to 36.8%)	P7, P14, P15, P16, P17, P18, P19 (7 students are equivalent to 36.8%)
Level 0			P3, P4, P9, P11, P12 (5 students are equivalent to 26.3%)

Source: authors' own elaboration.

Table 7 provides an overview of student performance based on the quality of their mathematical connections and the level of understanding achieved in three tasks related to derivatives and their graphical representation. The data show that students who consistently solved Tasks 2 and 3, especially P1, P2, P5, P6, P8, P10, and P13, activated reversibility connections, which allowed them to adequately transition between the graphical representations of f and f' . This connection, considered crucial in the ETC+OSA theoretical framework, implies the ability to coherently relate the behaviors of a function and its derivative, using properties such as growth, decrease, maximums, minimums, and concavity, without relying exclusively on a symbolic expression. Thus, those who managed to establish and justify these connections from an argumentative and multirepresentational perspective reached Level 2 mathematical understanding.

It is worth mentioning the case of student P19, who, although not initially included in the level 2 group for Task 3, showed important signs of advanced understanding in other tasks and activated relevant connections such as those of meaning and representation. However, his transit to full understanding is limited by occasional conceptual errors such as confusion between the value of the function and the behavior of its derivative noted after Figure 17. Even so, P19 is in a borderline zone of level 2, which suggests that with adequate support he could consolidate his understanding. In sum,

Table 7 not only summarizes performance levels, but also clearly reflects the relationship between the activated connections, especially reversibility, and the development of a meaningful mathematical understanding.

In this context, it is essential to highlight the relevance of Task 1, which explicitly explores the meaning of the derivative at a point. This task not only allows students to identify prior conceptions but also acts as a key starting point for activating meaning connections one of the most powerful according to the ETC. Table 6 shows that most students (e.g., P1, P4, P5, P6, P7, P8) managed to activate level 2 connections, interpreting the derivative as the rate of change, the slope of the tangent line, or the limit of the incremental quotient. These interpretations, when coherent and reasoned, favor the understanding of subsequent tasks, as they provide a semantic scaffolding that allows linking representations, procedures, and properties of the derivative in more complex contexts. Thus, Task 1 fulfills a diagnostic and formative function, demonstrating that access to the institutional meaning of the concept is key to achieving high levels of mathematical understanding.

5. Discussion

This research provides a plausible answer to the question: How does the quality of the mathematical connections activated by engineering students impact their mathematical understanding when solving tasks involving the graph of a function and its derivative? This question was triggered by a review of the literature and classroom experiences where the authors of this article have identified students' difficulties when solving problems involving derivatives. In this context, the mathematical connections were first identified from an articulated view of the ETC and OSA, considering the practices, processes, objects, and SFs that relate them. Subsequently, each connection was analyzed in greater detail, classifying them according to their level of quality (0, 1, and 2), which directly influences each student's understanding of the concept.

This analysis further expands the view of analyzing mathematical understanding from the connections as other authors have done (Campo-Meneses & García-García, 2025; Fuentealba et al., 2019; Hiebert & Carpenter, 1992; Rodríguez-Nieto et al., 2021a), because, in this new research, the step-by-step procedures of each student are detailed, giving them special treatment with onto-semiotic tools, entering into each of the details that each form of resolution of the students deserves. So much so that, mathematical practices are the powerful basis of mathematical activity and help to identify whether the student is connecting or not. Furthermore, the onto-semiotic configuration depends on these practices which condition the procedures, propositions and the arguments that support them. In fact, to analyze the quality of students' connections, the primary object "arguments" is the one considered to know if the quality is 1 or 2. Previously, analyses of the quality of connections had been carried out (Mhlolo, 2012; Mhlolo et al., 2012; Rodríguez-Nieto et al., 2021b) but not from the ETC + OSA articulation.

Delving deeper into this discussion, the results of this research show that seven of the nineteen students activated the reversibility connection, that is, they moved from f to f' and vice versa using connections of implication, meaning, different representations, part-whole, and mainly reversibility, which for this type of tasks guarantees mathematical understanding. Several authors affirm that reversibility or bidirectional processes are complex to carry out (Ikram et al., 2020), because the student is not used to solving tasks where they are not provided with the symbolic expression.

The difficulties identified in this study when students approach tasks that involve interpreting the graphs of f and f' coincide with those pointed out in previous research (Fuentealba et al., 2018; Nemirovsky & Rubin, 1992). Although different authors have highlighted the importance of establishing reversibility connections and, more broadly, of using reversible reasoning to promote mathematical understanding (Ikram et al., 2020), the results of this research suggest that such a connection is not only key to understanding the concept of derivative, but can also be an essential

component in the application of alternative methods to calculate derivatives and integrals. Furthermore, the information provided in this article not only encourages the use of connections between different representations of the derivative but, through the proposed tasks, mathematical connections of meaning linked to the representations are promoted, seeking to counteract or mitigate the great difficulties that students and teachers have in solving problems about derivatives because they ignore the connections between multiple representations and meanings (Galindo-Illanes et al., 2025; Lumbantoruan & Manalu, 2024; Moru, 2020; Muñoz-Pinto et al., 2025; Rodríguez-Nieto et al., 2023a; Pino-Fan et al., 2017; Rodríguez-Nieto et al., 2024; Rodríguez-Nieto & Font, 2025; Sahin et al., 2015; Sánchez-Matamoros et al., 2015).

Based on the information provided and articulating the contributions of the ETC with the OSA, three general levels of mathematical understanding can be established. These levels reflect the quality of the mathematical connections made by the subjects and are suitable for exploring subjects' understanding of any mathematical object. These levels consider both the primary objects involved (concepts, procedures, representations, etc.) and the SFs that mediate the relationships and associated argumentation:

Level 0 or low: fragmented or superficial understanding (no meaningful mathematical connection)

The teacher or student fails to establish pertinent mathematical connections between the primary objects involved. Their responses may reveal procedural errors, incorrect representations, personal meanings without institutional support, or semiotic conflicts. The connections made lack consistency, which prevents them from constructing an adequate meaning for the concept in question. At this level, adequate mobilization of knowledge to solve problems is not achieved, and mathematical understanding is weak or absent.

Level 1 or intermediate: mechanical understanding (correct connections without argumentation or coherence)

At this level, the teacher or student makes mathematically correct connections from a technical or procedural perspective, but fails to justify or argue them consistently. Although they may use definitions or algorithms correctly, they do not reflect on the rationale for their use, nor do they connect multiple representations or meanings. Understanding is operational but limited, as it does not allow for extrapolation of knowledge to new situations or consolidation of in-depth learning.

Level 2 or high: meaningful and integrated understanding (argued and coherent connections)

The teacher or student makes reasoned, coherent, and consistent mathematical connections with the institutional meaning of the concept. They establish relationships between various mathematical objects (concepts, definitions, procedures, representations, examples, etc.) and articulate them through semiotic functions that facilitate the construction of meaning. This understanding allows for the flexible use of knowledge to competently solve problems, communicate ideas clearly, and justify the procedures employed. The connection is rich, contextualized, and sustained by reflective and institutional mathematical practice.

This study contributes a significant and novel perspective to the existing literature by offering an innovative articulation between ETC and OSA to analyze the quality of mathematical understanding in the specific context of graphical tasks involving functions and derivatives. Unlike previous studies that have separately explored the quality of connections (Mhlolo, 2012; Rodríguez-Nieto et al., 2021b) or students' difficulties with derivative representations, this research unites both approaches to generate a detailed theoretical framework that classifies students' mathematical understanding into three levels, according to the nature and coherence of their connections. The novelty lies not only in the integration of these two powerful theoretical frameworks but also in the methodological

depth with which students' step-by-step solutions are examined, placing particular emphasis on the role of SFs and argumentation. This two-layered analysis provides a richer understanding of how students construct meaning through their practices, offering a more nuanced interpretation of mathematical thinking that can inform both research and instructional design in higher education Calculus courses.

6. Conclusion

Based on the analysis, it is concluded that the quality of the mathematical connections activated by engineering students has a direct and significant impact on their understanding of tasks related to the graph of a function and its derivative. The study, based on networking between the ETC and the OSA, identified that those students who managed to establish level 2 mathematical connections that is, consistent, reasoned, and aligned with the institutional meaning of the concepts showed a deeper and more functional understanding of the concept of derivative. These students not only correctly represented the graphs of f and f' , but also justified their answers by articulating meanings, properties, representations, and reversibility processes.

In contrast, students who activated connections of quality 0 or 1 presented difficulties in solving tasks, either due to conceptual or procedural errors, or a lack of argumentation and coherence between the mathematical objects involved. In particular, the absence of implication and reversibility connections, essential for moving between the representations of f and f' , proved to be a determining factor in the poor understanding of the concept. These findings reinforce the need to foster didactic practices in calculus teaching that promote the construction of rich and meaningful mathematical connections, where students become active agents of their learning, capable of interpreting, justifying, and mobilizing mathematical knowledge in different contexts.

Furthermore, this research highlights the analytical and explanatory power of the articulation between the ETC and the OSA, as it allows for a comprehensive and nuanced assessment of students' mathematical activity. By examining not only the responses but also the nature and quality of the connections students make between representations, meanings, and procedures, the study offers a more holistic understanding of students' mathematical understanding. This dual theoretical framework not only deepens the analysis of student productions but also provides valuable criteria for the design of assessment tools and teaching interventions that can more effectively support the development of solid mathematical understanding in calculus. The findings obtained allow us to affirm that this study effectively answers the posed research question. The relationship between the quality of mathematical connections and students' level of understanding was clearly identified and empirically substantiated. Thus, it is demonstrated that the analysis of these connections is a valid and relevant way to understand how students construct meaning about the concept of derivative from a graphical perspective.

7. Suggestion

Future research could explore how mathematical connections develop in digital contexts or with the use of technological tools. It would also be relevant to investigate the impact of teacher training on promoting quality connections in the classroom. Finally, we suggest extending the analysis to other calculus concepts such as integrals and continuity to identify common patterns in understanding processes. Among the study's limitations is the small sample size since there were only 19 students, which restricts the generalization of the results. Furthermore, tasks were worked exclusively in a graphical context, excluding other forms of representation. Finally, the in-depth analysis focused on nineteen participants and specifically on only one (P1), so studies with larger samples are required to confirm or generalize the findings.

Although the participants are engineering students, the study does not explore how mathematical connections articulate with real-life problems or engineering disciplines. This limits the scope of contextual analysis and its applied relevance beyond the calculus classroom. However, it does provide a path to conceptual and practical preparation so that students are prepared to solve problems in the engineering context with a solid mathematical foundation.

Declarations

Author Contributions. CAR-N: conceptualization, methodology, supervision, writing – original draft, formal analysis; FMR-V: formal analysis, validation, writing – original draft; writing – review & editing, resources; VFM, SS and BMC-R: formal analysis, validation, writing – original draft; writing – review & editing, resources. All authors have made substantial contributions to this study and endorse its findings and conclusions in their entirety.

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Ethical Approval. The participants in this research project are aware of and agree to participate because it is academic and not for financial gain. Furthermore, the university supports this research because it improves students' understanding of calculus and quantitative reasoning concepts.

Data Availability Statement. Data supporting the findings and conclusions are available upon request from the corresponding author.

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